A Provincial View of China's External Imbalances

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Chapter 1

Introduction

1.1 Historical context

The rise of China has rightfully attracted attention. Its development has already started to change the geopolitical and economic "rules of the game". More than once though, predictions of the emergence of a new world hegemon have proved wrong (e.g. Japan in the 1980s). To which extent an economic and cultural dominance is still possible in today's world, as for instance Europe before WWI or the US following WWII, is an open question. Fears that China may soon reach that point are unfounded. Historically speaking, China is still even far from having regained its initial largely favorable position. It is well known that this country was far more advanced than Europe in the Middle Ages both from an economic and technological perspective. As a matter of fact, any economist faced with the choice of designing the future world power prior to the industrial revolution would have sovereignly pointed his finger east, not west.[1]

The fascinating aspect about China is that it forcefully shows what can happen when "things go wrong". Starting in the middle of the 19th century, China was ravaged by 100 years of nearly uninterrupted catastrophies, involving foreign aggressions (e.g. the Opium Wars and the Sino-Japanese Wars) as well as internal upheavals and civil wars (e.g. Taiping Rebellion, Xinhai Revolution, Communist Uprising). As the CCP (China's Communist Party) definitely came to power in 1949, the country could at best be qualified as a "subsistence economy". The Communist rule brought the system to collapse during the *Great Leap Forward* famines. The disruptions created by the *Cultural Revolution* were perhaps effective in destroying a substantial part of the Chinese cultural heritage and last remnants of social coherence but were far from bringing the expected economic take-off. Under Mao's rule, inequality may have reached an all time low (*sic*).[2]

Still, there were some minor successes achieved by the CCP. Gross National Product per capita is (optimistically) estimated to have grown at reasonable rate (4.2%) over the 1950-1975 period (Brandt and Rawski, 2008, chapter 1). Progresses were recorded in the heavy industry that initially benefited from Russian technology transfer. Putting the ideological aspect of training and school closures aside, access to education improved, particularly for girls and in rural area (Brandt and Rawski, chapter 7). Agricultural production timidly increased on average but could barely keep pace with population growth. Furthermore, it did not generate enough surpluses for industrialization and urbanization. Even in the 1970s, China was still massively importing grain and wheat that were vital for the population's survival (Brandt and Rawski, chapter 13). Prior to economic reforms, China's GDP per capita was lower than Nigeria's (Storesletten and Zilibotti, 2014).

[1] Albeit it is broadly accepted that the industrial revolution was key to the emergence of Europe, recent research has shown that fertility patterns may already have changed due to the Black Death, the ensuing switch toward land-intensive farming and the related increase in female labor demand (Voigtlaender and Voth, 2013a). European specificities such as increasing urbanization, frequent wars and intensive trade kept mortality high, reinforced the effect of the Plague and raised income, enabling the escape from the Malthusian trap (Voigtlaender and Voth, 2013b).

[2] Data are scarce but it seems that inequality within villages/cities was low. Interprovincial and urban/rural differences were already considerable (Brandt and Rawski, 2008, chapter 18).

The following decades are a case in point of "things going right". Even if the reform path was bumpy, cautious "trial and error" policies and a pragmatic approach to the economy were fundamental to the emergence of China. Main impulses came from the progressive incentivization of agricultural production, the decentralization of economic decisions and the opening up of the economy to foreign investment and technology (Brandt and Rawski, 2008). Western observers have been struck by the fact that such a fast development has been achieved in spite of institutions and a political system that have not been known to be supportive of growth before. Ironically, by stubbornly ignoring IMF's advices at the time, China fared well. At the initial stage, it refrained from the *Big Bang* strategy of the 1990s that crippled the GDP of most Eastern European economies (Brandt and Rawski, 2008, chapter 3). Then, it kept its capital account closed and did not borrow against its high growth potential, which largely spared it from the 1997 Asian Financial Crisis. However, one decision compatible with economics textbooks was to become a key aspect of the economic take-off: the accession to the WTO in 2001.[3] To many obervers, in 2007/08, its financial backwardness again minimized direct financial disruptions while the US and Europe were drawn into the Great Recession.[4]

The long run costs of warding off the crisis have already started to materialize in the form of high local government debt, increasing capital misallocation and losses on foreign assets. On top of that, it postponed the structural rebalancing of the economy. All the same, the drivers of Chinese growth are firmly entrenched in the principles of the market economy but mixed with "socialist" particularities. That is what makes China especially interesting for research. As of 2014, many see China as being at a crossroad. Without speculating on future development paths, I touch upon specificities of the Chinese economic system and discuss some of the factors that contributed to the emergence of large internal and external imbalances. Then, I put my dissertation into perspective and provide an overview of the results.

[3] In fact, WTO membership was a confirmation of an adopted long run policy rather than a paradigm change. China had already been relatively open to trade and have had low *de facto* tariffs since the mid-1990s due to exceptions for foreign-related firms, processing trade and strategic industries (Brandt and Rawski, 2008, chapter 16).

[4] China experienced no full-blown crisis but got ripples of the shockwave: growth slowed down and the current account was drastically reduced. The (volatile) stock market was hardly hit but the low participation of households and private firms in financial markets tamed the impact. Fiscal, and to a lesser extent monetary policy were massively used to avert a hard landing.

1.2 Internal and external imbalances in China

Since the mid-2000s, the issue of global imbalances has been a resurgent topic in academics and occasionally shaped the political agenda. At the world level, Western Offshoots (the US, Australia, Canada and New Zealand) and, to a lesser extent, Emerging Europe have been accumulating large current account deficits. These capital imports have largely been provided by China, the Middle East, Russia and Japan as they accumulated large current account surpluses (i.e. excesses of domestic saving over domestic investment). In particular, the fact that a developed country (the US) had a large saving-investment deficit provided – and even financed – by an emerging economy (China) fostered sustained research.

The factors driving internal and external imbalances in China are increasingly well-understood. Since Premier Jiabao's famous 2007 speech acknowledging the unsustainability of the Chinese growth model, the awareness that rebalancing is a central issue has established itself. The West has been critical about the slow pace of reforms. Often though, the interconnections of factors driving internal and external imbalances were not embedded in the analysis. Dealing with them separately may end up destabilizing the entire economic system. In fact, as I argue thereafter, the task of rebalancing the Chinese economy truly is daunting.

As suggested by Johansson (2012), the financial repression environment plays a key role in imbalances: the managed floating exchange rate system forces the PBC (People's Bank of China) to expand monetary mass. In the process of sterilization, harsh reserve requirements are imposed on state-owned banks to keep inflation in check (Lardy and Borst, 2013). In compensation, the PBC sets a higher bound on deposit rate and a lower bound on lending rate, thus guaranteeing a large spread favorable to SOBs (state-owned banks). As they earn low or negative real interest rates, households end up indirectly subsidizing lendings to SOEs (state-owned enterprises) via SOBs. Credit controls and entry barriers secure a dominant position of the "Big Four"on the banking market.[5] Capital account restrictions hinder agents to internationally escape the low interest environment. The intransparent and super volatile home stock markets are no credible alternative while bonds issuance has just been emerging as an alternative to bank financing (Hansakul et al., 2009). Thus, the financial repression and the exchange rate policy favor capital-intensive SOEs and the manufacturing sector as opposed to services (Johansson and Wang, 2011).

The mispricing of capital relative to labor is another factor contributing to external imbalances in China. State-controlled prices of specific factors of production are an implicit subsidy to the secondary sector (Huang, 2010). The pass-through of oil and electricity price variations to firms is limited (Lardy and Borst, 2013). Land is used as a strategic development tool by local authorities as property rights are weak and compensations low. Environmental costs are not priced in production and impair on households' life quality.

[5] *Bank of China*, *China Construction Bank*, *Industrial and Commercial Bank of China* and *Agricultural Bank of China*.

An essential aspect of internal imbalances is the primer of state and local investment over private consumption. In addition to the integration into the world economy, capital accumulation has been one of the pillars of China's growth (e.g. regional development plans or the massive stimulus during the Great Recession). Examples of ghost cities and train tracks to nowhere abund. Nevertheless, some argue that China still has a low capital to output ratio and huge infrastructure needs (HSBC, 2012). If the case for overinvestment is debatable, the capital misallocation issue is less controversial. SOEs are provided with more capital than private firms (Brandt and Zhu, 2010). Until recently, SOEs did not pay out dividends to the state, hindering any public good financing as compensation for their favorable position (Borst, 2012c). The bulk of ambitious investment targets are set by the central government but have to be implemented at the regional level with little help in financing.

The limited fiscal autonomy of local governments is another fundamental issue for internal rebalancing. They bear the rising burden of investment and social expenditures but have limited control on revenues. Local authorities can directly set up opaque and highly leveraged LGFVs (*Local Government Financing Vehicles*) to have access to cheap financing and issue bonds (Soh and Wang, 2011). Furthermore, high levels of SOE corporate debts (mostly bank loans) add to the local government-related implicit garantees and amount to substantial hidden extra-budgetary risks for the central government. Since its liberalization in the end of the 1990s, the housing market has become a major source of investment and provided a welcome store of value for trapped household savings. Preventing this (rational) bubble from getting out of control is made difficult by the fact that land rent and sales are a substantial source of financing for local governments (Borst, 2012a).

The tragic dichotomy of the rural and urban class contributes to imbalances as well. The industrialization of the East Coast has spurred the "largest voluntary migration" in human history (Chan, 2013). Remnants of the Communist era's land rights put a dent in agricultural productivity. Rural households and migrants lack most of the social security and insurances urban dwellers have access to (the *Hukou* registration system heavily restricts access to social security, insurances, basic political rights and higher education). Urbanization has thus remained a shallow driver of consumption (Borst, 2012d).

At last, we turn to the pivotal role of household saving and their influence on external imbalances. High level of household savings rate is likely to be sustained for a long time to come without further improvements in the social security system. The "breaking of the iron rice bowl" (SOE reforms of the 1990s) left households with lower insurance and pension benefits. The looming end of the demographic dividend is a ticking bomb and – at least partly – justifies high savings rates. Still, the potential contribution of reforms to rebalancing should not be overemphasized: alternative surveys found that more than half of all households have no or little savings (University and People's Bank, 2012). Massive saving accumulation seems to be a phenomenon limited to the urban upper-class.

Taken together, the preceding points help to understand why China has developed into the "factory of the world", an economy with high investment and even higher saving (i.e. current account surpluses). I will discuss how the literature rationalizes these patterns in the third chapter.[6] Paradoxically, as of 2014, China started to reform the economic model that has been serving its purpose so well for three decades. More than ever, daring to "take the visible hand off the bicycle seat" is a true challenge. On the one side, it may increase the likelihood of a hard landing of the Chinese economy in the short run (Yongding, 2012). On the other side, the *status quo* may imply a deepening of current internal and external imbalances, lower long run growth and a further increase in inequality, building the socle on which revolutions rise.[7]

Alexis de Tocqueville suggested that old regimes only fall to revolutions when they attempt reform but yet dash the raised expectations they have evoked (Economist, 2013). This seems particularly relevant these days as aspirations towards higher life standards are embedded in new generations. *"It is not during periods of great deprivation or rapid growth that political disturbances usually occur, rather it is when a period of growth and rising expectations are suddenly reversed"*.[8]

[6] I contribute by showing in a formal framework that the non-state and the international sector drive high saving while the state sector drives high investment (see chapter 3).

[7] In my opinion, the progressive relaxing of financial repression has to be concomitant with a higher flexibility of factor prices, a progressive income tax to finance social security reforms and most importantly, a major reform of the fiscal system (i.e. more control on financial resources at the local level). However, by doing that, the CCP may "pull the Dragon's tail" and initiate a reaction that could well lead to its own demise.

[8] James C. Davis quoted in (Borst, 2012b).

1.3 Thesis focus

Global imbalances occupied a prominent position in the academic debate in the 2000s. Over the last decade, the literature disproportionately discussed the issue through the lens of US interests. At the onset of the prophetic Great Rebalancing, the focus was increasingly laid on China. In policy discussions and economic studies, it tends to be considered as a uniform macroeconomic entity.

In 2010, Chinese provinces were, as one would expect, as large as typical developing countries (e.g. Indonesia, Thailand or Colombia). Fortunately, a substantial part of national statistics is available at the regional level as well. Putting aside quality issues for the moment, this provides us with a sample of 31 economies at different stage of development (from India to Slovakia).[9] Over more than three decades of economic reforms, in spite of sharing common characteristics typical of the national economic system (e.g. financial repression and a strong presence of the state), the provinces became fully-fledged emerging markets with rich variation in economic DNA. For example, the extent to which state-related enterprises and state-owned banks affect economic activity, the integration into the world market via international firms, the economic structure or even demographic factors typically vary a lot among regions. Therefore, they provide an ideal research laboratory to answer questions that are central to better understand the past, present and future contribution of China to global imbalances. It is the impetus that jump-started this thesis.

1.3.1 First paper

Chinese data are known to be noisy. The dissertation project therefore involved considerable start-up costs in terms of time used in data preparation work. In the second chapter, I document this effort and present some important stylized facts on regional macroeconomic data and capital flows in China. To my knowledge, if a large literature on differences in regional development and the potential explanatory factors exists, the issue of external regional imbalances has never been adressed systematically. While certainly not publishable *per se*, this chapter allowed me to outsource major data issues encountered in later works. In fact, it is this knowledge about available data and key empirical facts that paved the way for the more advanced empirical and theoretical studies of chapter three and four.

Concretely, I shed light on issues related to data quality and aggregation properties of Chinese regional macroeconomic figures. Large discrepancies between regional and national official components of GDP expenditure aggregates are observed. Investment errors are the key driver of differences between provincial and national output: regional "phantom investments" systematically exceeded national statistics since the mid-1990s. This clear trend has been a statistical

[9] See *The Economist*'s interactive maps for more (`http://www.economist.com/content/all_parities_china`).

companion of emerging China over the 1990s and 2000s. As a consequence, cumulated provincial net exports end up being lower than national values. For example, assuming regional data are correct, the historical 2007 8.8% peak in national net exports over GDP would be levelled down to 3.5%. Still, the match between aggregated local net exports and national values has been reasonable from the mid-1980s to the mid-2000s and dynamics of aggregates and national values are highly correlated, even in recent years.

In terms of GDP per capita, the clear north-south divide of the 1950s morphed into an east-west divide as coastal regions were the first to become integrated into the world economy. A decrease in terms of output per capita disparities among regions during the initial reform period is observed but it turns out to be largely driven by City-Provinces (i.e. Beijing, Tianjin and Shanghai). In fact, the cross-regional variability in relative output per capita has been on the rise since the 1990s at least, in spite of large migration flows, investment programs and the recent progressive integration of the hinterland into the world supply chain. Geographically, saving rates follow an east-west divide. Investment rates form a north-south divide. Historically, relative saving was highly related to the level of economic development. By contrast, relative investment was not.

I explore to which extent regional light intensity in China is related to real GDP figures at the provincial level. By using data from US Army satellites, I am able to show that once one controls for the huge differences in regional size, light intensity seems to be positively correlated with GDP in Chinese regions in a roughly concave fashion.

The principal part is devoted to external balance statistics. To start with, I focus on net exports surpluses and deficits (i.e. the difference between saving and investment). The coastal regions have indeed accumulated surpluses since the opening up of China. Interestingly, some other regions have even larger saving-investment gaps. City-Provinces and Manchuria started with large surpluses and converged towards a neutral position during reforms. Over the last decades, Central China has had roughly balanced net exports. West and South China have been running huge deficits.

An alternative indicator of external balance is international trade data. They have good aggregation properties since 1992. Imbalances in international trade are less dramatic than in net exports: with the exception of the East Coast, most provinces have been running small surpluses. Two lessons can be drawn from these empirical facts. First, large discrepancies in net exports seem to be driven by interregional flows. Second, in absolute terms, only the East Coast regions truly matter for international capital flows. This specialization pattern may explain why the variability of cross-regional international trade balance has been increasing since the 1990s.

1.3.2 Second paper

In the third chapter, I relate capital flows to regional productivity in a more formal framework using a small open economy model. I show that a *capital allocation puzzle* is present inside China: provinces that caught up relative to national productivity had surpluses of saving over investment (capital outflows) while the opposite happened for provinces that benefited less from economic reforms. This results is reminiscent of the findings of Gourinchas and Jeanne (2013) at the international level.

Starting from that empirical finding, I follow up by identifying the drivers of that pattern using a standard neoclassical model with two frictions in the mould of Gourinchas and Jeanne. The first one takes effect at the aggregate level of the economy and influences capital accumulation. It is an investment wedge that affects gross return on aggregate capital. It is identified in matching an empirical with a theoretical decomposition of investment rate. I find an investment puzzle: regions that caught up relative to the rest of China seem to have higher wedge (lower investment rate), while provinces that fell behind implicitly subsidized investment. This is a first blow to the baseline neoclassical framework and stands in sharp contrast with international patterns.

The second friction (the saving wedge) is comparable to a tax on capital income of households. It is identified in matching an empirical with a theoretical decomposition of cumulated relative capital flows (i.e. net exports). As on the international level, we find a saving puzzle: the relationship between productivity catch-up and saving wedges is negative and very significant. Provinces that caught up are the ones that implicitly subsidized saving, causing a saving glut that translates into capital outflows. This is a second blow to the neoclassical model. As opposed to investment wedges, saving frictions are the main driver of the *capital allocation puzzle*.

In a next step, I investigate whether the estimated long run wedges are related to usual suspects proposed in the literature. The regional cross-sectional variability of the wedges seems useful in shedding light on the general patterns of capital flows. Some characteristics related to the investment structure of the economy robustly account for a high part of the cross-regional variation in investment wedges: a high share of the state in investment in fixed assets or in construction gross output value and a marked presence of the formal, state-near financial sector – loans in financial institutions – seem to foster investment.

Turning to saving wedges, there seems to be an ubiquitous effect of the state's involvement in the economy (e.g. state-owned share of gross industrial output value) in repressing saving. By contrast, a greater importance of multinational firms, privately-owned enterprises and a larger industrial sector are all associated with higher saving compared to the neoclassical model. Financial development – deposits and loans in financial institutions – seems to put a dent in saving.

I conclude that the *capital allocation puzzle* is driven by both the visible hand (the state) and the private sector. By constructing non-state sector net exports, I show that more marketized regions with a strong presence of private and international firms (i.e. the East Coast and the City-

Provinces) have a large non-state saving surplus while other regions have balanced non-state net exports on average. In fact, massive state net exports deficits are largely responsible for large capital imports (i.e. a negative saving - investment balance) in the Chinese hinterland. An important part of this chapter is devoted to robustness checks using alternative data, parametrization and extensions of the baseline methodological framework.

1.3.3 Third paper

The last subproject of my thesis is undertaken jointly with Prof. Mathias Hoffmann. While the methodology used in chapter three enabled to identify the long run drivers of capital flows for Chinese regions, the cross-sectional information was obviously limited by the number of provinces. Exploiting time-variation in external balance and explanatory factors might be informative. Furthermore, a theory-based decomposition of the different driving forces of regional net exports would enable us to be more specific in testing factors proposed by the literature. We build on a recent paper by Hoffmann (2013), to study the dynamics of China's province-level net exports and to correlate regional patterns of external adjustment with different characteristics in a theory-based panel framework. This gives us detailed insights into what types of frictions drive the accumulation of internal imbalances.

Our analysis follows the tradition of the intertemporal approach to the current account (Sachs et al., 1981; Bergin and Sheffrin, 2000; Kano, 2008). We apply and extend the empirical framework used in these studies to intranational data by introducing a saving wedge. Concretely, we end up with four channels of net exports adjustment. The first term reflects the intertemporal consumption smoothing channel that is emphasized by basic versions of the neoclassical model (intertemporal variation in quantities or net output). The second term is the effect on intertemporal substitution of expected changes in the local price of non-tradables (i.e. intratemporal substitution). The third and fourth terms capture how variation in the domestic real rate of interest and in the impact of the excess return on the foreign bond – the saving wedge – respectively affect province-level capital flows.

We back up in-sample expectations with a reduced-form VAR and identify three (a priori unknown) deep parameters of the model (coefficient of relative risk aversion, discount factor and degree of financial repression) based on a three-dimensional grid-search procedure. Our simple model can account for 85 percent of the variation in a panel of 30 province-level net exports over the 1985-2010 period. By using the four channels of our structural panel, we focus on characteristics that the literature has emphasized as potentially important in explaining China's persistent surpluses since the 1990s: i) the relative importance of private and state-owned enterprises (SOEs) and the differential access of these types of firms to finance, ii) a province's degree of integration into the world economy in terms of openness to FDI or trade, iii) sectoral composition and iv) demographics.

We find that there are major differences in the patterns of adjustment across provinces. In particular, the relative importance of SOEs and private enterprises for the local economy has a major bearing on the patterns of external adjustment. Intertemporal variation in net output is particularly important as a driver of capital flows in provinces with a strong presence of private firms, as is the domestic interest rate. This pattern is consistent with theories that see financial repression as a major source of China's persistent surpluses: under financial repression, private firms do not have access to bank finance and therefore have to finance investment from retained earnings. As a result, surpluses are better at predicting decreases in net output (via increases in investment) in provinces with a large share of private firms. The absence of access to international finance also means that the domestic (financially repressed) interest rate is the relevant driver of saving decisions of households and private firms.

Furthermore, we show that a higher integration into the world economy – international openness and FDI – is strongly related to a rising importance of the international interest rate channel and to a decrease in intertemporal variation in quantities (net output). Foreign participation thus possibly alleviates financing issues of the private sector. We also find that variation in non-tradable prices is an important driver of net export variation in less developed regions, suggesting that housing is particularly important as a savings vehicle when there is a lack of investible assets.

By reconstructing Chinese net exports from the inside, we find that most of the 2000s run-up and the successive adjustment is driven by intertemporal variation in net output. During this period, financial repression makes a persistent and positive contribution to China's surplus. Variation in the world interest rate plays only a relatively minor (but increasing) role overall while internal price pressure on non-tradable goods has a major dampening effect on China's burgeoning external surplus.

Chapter 2

Regional External Imbalances in China: Data and Stylized Facts

2.1 Introduction

Global imbalances occupied a prominent position in the academic debate in the 2000s. That issue came back on the agenda in the wake of the eurozone crisis. Over the last decade, the literature disproportionately discussed the issue through the lens of US interests. At the onset of the prophetic Great Rebalancing, observers and experts increasingly laid the focus on China. In policy discussions and economic studies, it tends to be considered as a uniform macroeconomic entity.

Enlightened observers are keen to remember how disparate the regions are. A large literature on differences in development between coastal and inner provinces exist and the potential factors explaining them have been largely discussed. Still, surprisingly little attention has been devoted to within-country discrepancies in external imbalances. In our opinion, a better knowledge of the available data and the exposition of key empirical facts could pave the way for further empirical and theoretical works.

This paper sets the stage for a better understanding of Chinese – and presumably global – imbalances. We discuss issues related to regional data availability and quality of output as well as three indicators of external balance: net exports, international trade and interregional capital flows. Our second contribution is to shed light on their cross-regional patterns. Concretely, we focus on geographical distribution, persistence over time, variability and on their correlation with regional level of economic development.

We find that regional investment has been increasingly overestimated compared to national values. As a consequence, aggregate provincial net exports are lower than national data suggest. Furthermore, while the northern part of the country seemed more developed in the 1950s, the eastern part now leads in terms of real GDP per capita. By using nighttime light intensity, we find that the broad patterns of development correspond to official data.

Moreover, we observe a complete scope of different and persistent regional saving-investment balances with South and West China being typical capital importers while the East Coast has been exporting capital all along. Focusing exclusively on international trade reveals that only a few provinces in the eastern part of the country are the drivers of the Chinese contribution to global imbalances. Our indicator for interregional flows suggests that regions in Manchuria, the *Bohai Economic Rim* and the *Yangtze Delta* have been providing capital to the rest of China over the 1992-2004 period.

The structure of the paper is as follows. To begin with, we discuss the related literature and general data issues in the rest of Section 2.1. Next, in Section 2.2, we turn to the components of regional economic aggregates (i.e. output, saving, investment and net exports). Then, we use satellite nighttime light data to test the quality of provincial output aggregates in Section 2.3. In Section 2.4, we investigate patterns of international trade data. Finally, we propose an indicator for interregional capital flows in Section 2.5 and conclude in Section 2.6.

2.1.1 Related Literature

This paper is linked to four distinct strands of the Chinese-specific literature: the quality of data, the imbalances among regions in level of development, the factors driving these regional discrepancies and the patterns of provincial trade. To our knowledge, no study has exclusively focused on cross-provincial patterns in external imbalances.[1]

A small circle of scholars investigated data quality. This issue is as old as the Communist Party itself: the *Great Leap Forward* famines of 1958-1962 were a direct consequence of over-reports of grain output by local officials for fear of the Anti-Rightist movement (Cai, 2000). The resilience of the Chinese economy during the 1997 Asian Financial Crisis raised scepticism about growth figures (Smith, 2001). More recently, in 2011, excess coal reserves and electricity production did not seem to corroborate the optimistic GDP growth figures in some regions. There have been reports of officials using electricity consumption, rail cargo volume or bank loans as indicators of economic activity rather than official GDP figures (Bradsher, 2012).

Output statistics mostly came under close scrutiny: Adams and Chen (1996) reassessed real GDP growth over the 1978-1994 period using energy consumption. It halved official growth figures. In the same spirit, Rawski (2001) argued that official 1997-2001 GDP figures have been overrated. Rawski and Mead (1998) found that data massively overestimated Chinese farm workers (the notorious "phantom farmers") from 1979 to 1993, causing a large bias in sectoral output estimation. Following official instructions of the National Bureau of Statistics (NBS), Holz (2004b) reconstructed private consumption between 1997 and 2001 but it rarely matched official data.

Rawski (2000) points to the long Chinese tradition of literacy and to the systematic record-keeping of the socialist state. Central planning necessitates an extensive array of data. Chow (2006) opines that statistics are by and large reliable. Still, there is a general agreement that the 1990s saw a near collapse of the statistical system. Xiaolu and Lian (2001) observe that during the era of central planning and the initial phase of reforms, dominant state-owned enterprises (SOEs) managed their production according to official plans, which made data collection easy and transparent. Incentives to overreport GDP originated in the 1990s, when this indicator became a central criterion to assess performance of local officials. Incentives for misreporting are not obvious: some regions may benefit from substantial help if they do not live up to expectations while others may be penalized (Cai, 2000). The suggestion that *"officials make statistics and statistics make officials"* has been made by former officials of the NBS themselves (Rawski, 2000).

Another difficulty arises from the frequent redefinition of variables. Reporting categories change over time and sometimes cause statistical breaks in time series. Until 1993, China used

[1] In their influential paper, Song et al. (2011) found a positive relationship between relative net surpluses (S-I over GDP) and domestic private employment share at the provincial level between 2001 and 2007. In a risk sharing paper, Li (2010) noticed great discrepancies in the average provincial net exports over GDP.

the *Material Product System* typically adopted by planned economies (Campbell, 1985).[2] The introduction of the *System of National Accounts* enabled to better keep track of the growing economy. The problems of the 1990s to capture the rapid growth in private productive units led to two major revisions of the statistical system (1993 and 1998). The revised laws reduced the role of the reporting system in favor of censuses as basis for revision of yearly data (Holz, 2004a). There is little evidence that regional data are worse than at the national level: the *2004 Economic Census* validated provincial GDP data and invalidated national ones (Holz, 2008).

On the one hand, the domination of the Communist Party seems to guarantee a stability over time of institutions. On the other hand, the statistical system was not adapted to adequately record the take-off of the private sector in the 1990s. All things considered, there are many reasons to doubt the exactness of the NBS data. All the same, one has to acknowledge that general data coverage is impressive. With respect to statistical standards, the World Bank report (WB, 2002) ranked China as a typical top-tier developing country (as e.g. South Africa or Russia).

There is a large literature on the level and dynamics of output and income disparities in China. Fleisher and Chen (1997) found a conditional convergence of per capita production across provinces from 1978 to 1993.[3] Historically, a large part of regional income disparity seems to be attributable to the rural-urban income gap (Chang, 2002). As noticed by Kanbur and Zhang (1999), the inland-coastal contribution to total inequality hugely increased in the 1990s. Since the reforms started, the coastal regions grew faster while Central China lost momentum and converged toward Western China (Yao and Zhang, 2001). An acceleration of the process was observed since the mid-1990s.

The Maoist development strategy was to eradicate regional industrial disparities and investments were mostly promoted in interior provinces for strategic reasons (Yang, 2002). Ideally, each region had to be able to survive in autarchy. Limited investment in transportation and heavy restrictions on labor mobility were ubiquitous (Brandt et al., 2012). At the same time, the low prices imposed on agricultural production and resources harmed interior provinces and the rural population. Jian et al. (1996) found that real income among provinces was stable from 1952 to 1965, diverged during the Cultural Revolution (1965-1978) but began to equalize in the initial reform period thanks to a rise in rural productivity. Disparities started to grow again with the integration of coastal provinces into the world economy in the 1990s. As argued by Brun et al. (2002), it seems that spillover effects from coastal regions have not reduced inequalities in the 1990s.

The factors explaining these differences are numerous. Using data from Chinese cities, Alder et al. (2013) found that the establishment of special economic zones (SEZs) led to a substantial increase in the level of GDP. Other studies point to the key role of preferential policies. Coastal

[2] The economy then only consisted of "socialist productive enterprises" and households. Services – most of them were free – and smaller independent enterprises were part of the "non-productive sector".

[3] They controlled for physical investment share, employment growth, human capital investment, FDI and coastal location.

regions were the first to benefit from special tax treatments and FDI-friendly legislations (Demurger et al., 2002). They have higher TFP level (Brandt et al., 2012), possibly due to their openness to FDI and trade (Jiang, 2011). Demurger (2001) used provincial data over 1985-1998 and identified differences in geographical location and infrastructure endowment (transport and communication). According to Brandt and Rawski (2008, chapter 17), the pre-reform importance of the state in a region may matter as it could influence successive capital allocation and economic policy.[4]

Human capital disparities certainly play an important role as well: Hannum and Wang (2006) found large and rising geographic inequality in access to education in China. They link it to rising economic inequality and large disparities in educational spending due to fiscal decentralization. Liu and Li (2006) found that growth imbalances over 1984-1998 were strongly related to the financial source or ownership type of capital. Domestic bank loans and foreign-owned enterprises are important in coastal provinces while state appropriation or state-owned enterprises rule the roost in inner provinces. Jia (2012) found that history matters as prefectures where treaty ports had been established between 1840 and 1910 developed faster during the reform period. As successful explanatory factors, she suggests human capital (education) and social capital (attitudes, norms and culture towards the market economy).

At last, we turn to the literature on discrepancies in trade patterns. Naughton (2003) found evidences of a large inter-provincial trade in 1992 that seemed to be dominated by intra-industry trade in manufactured goods. Barriers to movement in factors of production and trade in services as well as intermediate goods were still large. The evolution of intranational trade between 1987, 1992 and 1997 is tackled by Poncet (2003) who found an increasing importance of provincial boundaries. Indeed, some observers early argued that reforms could have encouraged the creation of local clusters protecting their market with the blessing of regional authorities. Poncet (2005) mentioned the fear of high unemployment and the protection of loss-making state-owned firm as well as fiscal revenues maximization as potential reasons. Girardin and Owen (2011) discuss the highly asymmetric regional distribution of the share of multinational enterprises in international trade. They argue that China's high trade surplus is associated with FDI inflows and international production structures. At last, we recommend Gaulier et al. (2011) for a careful discussion of the different components of Chinese foreign trade and their dynamics.

2.1.2 General remarks

When not mentioned otherwise, data used in this chapter are from the *National Statistical Yearbooks* of the People's Republic of China and from the *Provincial Statistical Yearbooks* of the

[4] For example, southern provinces benefited from large state-driven investments in the 1960s and 1970s as a result of the *Third Front Campaign* that aimed to relocate industrial and military sites in the secure hinterland. These regions thus already had high state share compared to the East Coast when reforms started.

22 provinces, 5 autonomous regions and 4 municipalities of Mainland China.[5] The *China Data Center* (CDC) of the University of Michigan provides electronic access to the yearbooks and made main statistics conveniently available.[6] For most provinces, our online access only covers regional statistical yearbooks in the 1990s and 2000s. Thus, it happens that the data are sometimes incomplete. We will primarily rely on data directly retrieved from recent online yearbooks and complete possible gaps with CDC sheets. This allows us to take account of revisions as much as possible. When necessary, we briefly discuss particular data issues at the beginning of each section.

Table 2.1 summarizes our subdivision of China into six larger entities and provides some basic statistics (respective share in output and population). We try to keep the balance between geographical and economic coherence.[7] Figure 2.1 provides an overview of the relative real output share for recent decades. The striking patterns are the rising GDP share of the East Coast and the decline of Metropolises (City-Provinces). Moreover, migration flows largely contributed to an increasing share of total population of the East Coast and the Metropolises, principally at the cost of the Center, Manchuria and the South (see Table 2.1). Interestingly, the West seems to be an exception as its relative GDP was on the rise in the 2000s while its population share staid relatively constant.

2.2 Regional expenditure components

2.2.1 Aggregation properties

Before discussing empirical facts, we first want to focus on the aggregation properties of the data. Ideally, the sum of provincial aggregates should be roughly equivalent to national values, keeping in mind that measurement errors and sample gaps probably hinder a perfect match. The question of interest is to which extent discrepancies appear to be systematic. It is an important step towards understanding key patterns of provincial aggregates. In the end, it should enable us to have a better grasp of potential consequences for our later findings.

To get an intuitive error indicator, we substract national values from cumulated provincial ones and weigh the result with national GDP for each year. A positive result thus means that

[5]The autonomous regions are Tibet, Xinjiang, Guangxi, Inner Mongolia and Ningxia. The cities of Beijing, Tianjin, Shanghai as well as the region of Chongqing are municipalities. Thereafter, the term province will be used as general qualifier.

[6]`http://chinadataonline.org/`. The CDC reports data values as soon as they are published in the corresponding yearbook. Although data have sometimes been subject to official revisions in later years, the CDC did not systematically adapt past values.

[7]Our subdivision of China goes one step further than usual in the literature, which typically focuses on three blocks: East, Center and West. For our part, we think that considering separately the South and the West may be informative. Another deviation is to separate Metropolises (City-Provinces) from other (larger) coastal regions. At last, we consider the Chinese Rustbelt (Manchuria) separately from other coastal regions. The attribution of Shanxi and Inner Mongolia to the West is more controversial. We make that choice on the ground that they share strong economic similarities with other western regions (large mineral resources, high investment and a strong presence of the state sector).

provincial data overestimate national values. Inevitably, as the data availability across regions improves over time, we expect initial gaps to fade out towards the end of the sample. The errors of GDP are decomposed into private consumption, government consumption, investment and net exports errors.

In Figure 2.2, one sees that aggregated provincial output has been underestimated compared to national values with an average of 7% in the initial part of the sample (1975-2000). It is at least partly explained by the fact that some large provinces only have GDP data from 1978 onward.[8] By taking missing regions into account – substracting their 1978 weight from the average error – and extending the sample (1952-1999), the pattern would look similar: a sizeable part of national output hovering around 5% is not captured in provincial statistics. Surprisingly, this pattern reversed during the 2000s: provincial aggregates toppled national GDP value in 2003 and ended up overestimating it by 11% of GDP in 2010. No new region entered the sample over this period. Obviously, a yet unidentified factor has been at work that either led provinces to report too high output, national statistics to report too low output or a combination of both.

We delve into the components of output in the hope of identifying the factor responsible for the marked trend in output errors. Private consumption errors are relatively stable.[9] After having reached roughly the same value as its national counterpart in the early 1980s, they followed a U-pattern reaching -7% in 1999 and converged towards near neutral position by 2007. Government consumption errors follow a positive trend from under- to more precise estimation.[10] In a word, both private and government consumption do not seem to have played a major role in the recent trend in output errors. As for investment, many incomplete provinces enter the sample simultaneously in 1978. Still, the improvement in aggregate errors is only gradual.[11] Over the sample period, investment errors follow a clear positive trend: they reversed from more than -11% in the 1970s to nearly zero a decade later. In 2010, the overestimation was massive (13% of GDP). Therefore, it seems that errors in investment induced the trend in output errors.

Obviously, provincial data have poor aggregation properties. The trend in investment error is particularly troubling as it contaminates output and biases net exports far more than errors in saving, which are large but roughly stationary. Being a small part of output compared to other components, net exports are severely affected. Between 1985 and 2005, the sum of regional external positions was relatively close to the national counterpart with an average error of 0.31%. Net exports became increasingly underestimated at the provincial level in the following years to reach 5.9% of GDP in 2010. National data suggest a net exports surplus of around 4%. As a result, regional data would imply that China in fact run a small deficit in 2010! In the same

[8] There is a high heterogeneity in the availability of GDP: at reporting start in 1978, Guangdong and Sichuan accounted for no less than 5.4 and 5.1% of national output. The impact of smaller provinces is negligible: Hainan and Ningxia made up about 0.5 and 0.4% of GDP in 1978. Tibet only starts in 1992 with an impact of 0.1%.

[9] They decrease as Guangdong, Hainan, Sichuan and Ningxia enter the sample in 1978, followed by Jiangxi in 1980 and Tibet in 1992.

[10] There are no differences in sample entry compared to private consumption.

[11] Jiangxi, Guangdong, Hainan, Sichuan and Ningxia start in 1978. Tibet in 1992.

spirit, assuming regional data are correct, the historical 2007 8.8% peak in national net exports over GDP would be levelled to 3.5%. At first sight, it seems to be supportive of Zhang (2008) who convincingly made the point that misinvoicing of exports and imports has been biasing the Chinese trade balance upward in the 2000s (i.e. only if local statistics are less affected). Still, the dynamics of aggregates and national values are roughly similar: the correlation of their absolute first difference between 1985 and 2010 amounts to 0.80.

As a consequence of the problems encountered in this section, one has to remain cautious when interpreting the results of this study, particularly with statements concerning absolute values. An empirical check of the exactness of regional macroeconomic figures would necessitate considerable resources. Potential indicators could be light intensity, electricity consumption or cargo transports. At this point, we have no choice but to rely on a fingers-crossed argument. In Section 2.3, we use light intensity to evaluate GDP data quality. Thereafter, the focus lies on cross-section characteristics of Chinese provinces. Thus, the errors should not invalidate our conclusions as long as they occur randomly or with the same intensity. In the following sections, we remain cautious and typically concentrate on the relative ranking of provinces rather than interpreting absolute values.

2.2.2 Nominal and real GDP per capita

Nominal GDP data are available from 1952 onward for 26 out of 31 regions and for all but Tibet by 1978. Nevertheless, the surprisingly good availability should be relativized: the transition from a planned economy – in which the private sector was practically inexistent – to a market economy certainly makes any direct times series comparison hazardous. Thereafter, we divide GDP per capita by the national value reported for the corresponding period for two reasons. First, it enables us to ignore methodological changes affecting level values across the board (e.g statistical system reforms). Second, it makes comparisons easier.

Chinese population data are a topic of their own. Two main problems are plaguing them: the underreported birth numbers as a consequence of the one child policy (Scharping, 2001) and the *"largest (voluntary) migration in human history"* (Chan, 2013). We tried to address the second issue. Basically, three sources of population estimates exist. The Hukou *Household Registration System* population data is reported by the *Public Security Authorities*.[12] It can be considered as a *de jure* statistic because it does not capture migration flows adequately. Typically, richer coastal provinces have an underestimated population and hinterland provinces a too high population (Chan and Wang, 2008). An alternative is the use of regular sample surveys of around 1% of the population and population censuses (1982, 1990, 2000 and 2010). They should better approximate resident population but unfortunately, the time of the survey as well as the definition of permanent residents and migrants is not always consistent over time. They are usually referred

[12]The Hukou aims at limiting rural migration by restricting access to welfare goods and services for non-urban residents such as health care, insurances or education (Chan, 2010)

to as *de facto* data.

The yearbooks population data often are a combination of the three sources that we have already mentioned. We carefully compared CDC data, recent yearbooks, sample surveys, censuses and existing studies to at least avoid sudden jumps due to changes in definition and assemble our own population time series. We tried to consider de facto data as much as possible, particularly for provinces traditionally heavily influenced by migration.[13]

To start with, we focus on relative output inside China in the initial years of the communist era. In Figure 2.3, we provide a map of the mean of nominal relative output per capita from 1952 to 1959. A clear north-south divide in output per capita relative to national value stands out. The Metropolises (Beijing, Tianjin and Shanghai) were the richest provinces by far, with values more than two and a half times national ones. Manchuria (Liaoning, Jilin and Heilongjiang) was economically more developed as well. Apart from large oil resources, it potentially benefited from the Japanese occupation from 1931 to 1945 in terms of a massive increase in railway network (Chou, 1971) and an early heavy industrialization (Eckstein et al., 1974). It was to become the main industrial base of China for the next decades. Other resource-rich regions (mainly coal) such as Inner Mongolia and Shanxi had higher level of GDP per capita. In the western part of the country, where mineral resources are also relatively abundant, provinces were more developed as well (e.g Xinjiang with its large oil reserves). The rest of the country was more dependent on agriculture. This broad pattern in relative GDP persisted until the onset of economic reforms in the 1970s.

In a next step, we focus on the recent geographical distribution of real GDP per capita.[14] In Figure 2.4, we observe a major shift of regional patterns compared to the 1950s. The north-south divide morphed into an east-west one.[15]

First, the large gap between the Metropolises and national values has narrowed considerably. Second, with Liaoning (1.41), Jilin (1.13) and Heilongjiang (0.89), Manchuria lost some ground but still figures in the upper part of the ranking. Third, a well-known and largely discussed surge in output quickly followed the creation of SEZs and the progressive opening to FDI and foreign firms on the East Coast. Not surprisingly, these regions are highly active in international trade. As a result, the wealthiest provinces are now located on the coastline. Guangdong (1.18) in the *Pearl River Delta* is known to have benefited from investment windfalls thanks to its proximity

[13]Central China as well as Chongqing and Sichuan have been the main outflow regions. Shanghai, Guangdong and to a lesser extent other eastern provinces have been net recipients (Chan, 2013).

[14]We compute real GDP from 1984 onwards using official regional CPI data as a proxy for the development of prices over time and the price of a living expenditure basket of Brandt and Holz (2006) to correct for the initial difference in price level. Revised data have been used as much as possible. The choice of CPI as deflator is justified as follows: first, it is broadly available. Second, no (official) explicit GDP deflator on the regional level exist (see Brandt et al., 2012). Third, alternatives such as retail price index, price of investment in fixed asset, services price index or producer price index of manufactured goods seem too specific to be used as output deflator.

[15]Clearly, this shift in patterns is to be taken cautiously as it remains unclear to which extent it only is a byproduct of the transition from the *Material Product System* to the *System of National Accounts*.

to Hong-Kong and Macau. In the same spirit, Fujian (1.31) has close links with Taiwan. Another production hub is the *Yangtze River Delta* comprising Shanghai (1.76), Zhejiang (1.63) and Jiangsu (1.63). In the north, the area bordering the *Bohai Sea* is a major economic hotbed with Beijing (1.72), Tianjin (2.03), Hebei (1.03) and Shandong (1.39). Fourth, most regions of the West lost their relatively comfortable position: with the exception of Inner Mongolia (1.55), data reveal that Xinjiang (0.77), Qinghai (0.67), Gansu (0.50), Ningxia (0.84), Shanxi (0.79) and Shaanxi (0.83) are all below national value. Statistics for Tibet (0.46) have only been available since the mid-1990s. Fifth, Southern China still is the least developed cluster: agrarian and rugged regions like Guizhou (0.40), Yunnan (0.45) and Guangxi (0.67) as well as Hainan island (0.62) lag behind. Notable exceptions are the large Sichuan (0.72) and the recently created municipality of Chongqing (1.08), two regions on the come that are increasingly being integrated into the world economy. At last, the populous, largely agricultural provinces of Central China are below national values as well (Henan (0.88), Hubei (0.91), Hunan (0.73), Anhui (0.72) and Jiangxi (0.72)).

To shed light on the evolution of disparity in output across provinces over time, we compute the cross-sectional standard deviation and the coefficient of relative variation of relative nominal GDP per capita from 1952 to 2010 weighted by population share (Figure 2.5).[16] The same indicator is computed for real GDP starting in 1984. The disparities dropped dramatically in the aftermath of the *Great Leap Forward* (1958-1961) and the slump it provoked. Obviously, provinces were levelled and convergence took place (*sic*). Pre-reforms levels were nearly reached by the end of the *Cultural Revolution* (1966-1976). Then, we observe a sustained decrease in discrepancies as the first reforms begin to take effect. It seems to corroborate findings of the literature discussed in Section 2.1.1. As an explanation, the initial flexibilization of agricultural production (the *Household Responsibility System)* and the decentralization of economic planning are typically invoked (Brandt and Rawski (2008), chapter 1 and 19). Hinterland regions may have particularly benefited from these reforms.

Cross-provincial disparities in relative terms then started to increase again in the 1990s, possibly driven by the (inequal) emergence of foreign trade and FDI investment as key drivers of economic development. Interestingly, the cross-regional volatility is on a downward trend again since the mid-2000s and reached its lowest level ever. It contrasts with the recent increase in regional income inequality recorded in the literature. However, the pattern is not robust to the exclusion of Beijing, Tianjin and Shanghai. In that case, the same indicators would suggest a sustained increase in discrepancies across provinces since 1990.

[16]The practice of dividing them by national values enables us to ignore the tremendous growth experienced by all of these economies and focus on the disparities relative to the nation as a whole. Measuring income inequality is not the goal of this study and it should be obvious that a decrease in our indicator does not mean a change in inequality *per se*. We only investigate cross-regional variation at the macroeconomic level.

2.2.3 Saving and investment

In order to set the stage for the external imbalances part in Section 2.2.4, we look into the patterns of the components of net exports (i.e. saving and investment). Both are expressed relative to output to facilitate comparison across provinces and periods. During the pre-reform era (1952-1978), high saving to GDP ratios were typically observed in the Metropolises and Manchuria as well as in some western provinces (e.g. Inner Mongolia and Shanxi). The reform era (1979-2010) saw the emergence of a coastal saving glut while southern and western provinces experienced far lower rates (Figure 2.6). Although these regions already had low rates before reforms, the pattern has become even more clear-cut. Northern regions lost the lead and joined central provinces in the middle group. Obviously, an east-west divide is observable. Relatively wealthier provinces seem to have higher saving to output ratio.

In a next step, we focus on investment. Geographical patterns differ markedly. West and South China experienced high rates in the pre-reform era relative to the East Coast. In Figure 2.7, it seems that a south-north divide materialized in the following decades with the bulk of southern provinces switching to the lowest investment category. Some hinterland provinces may have benefited from the *Western Development Plan*.[17] Officially launched at the turn of the new millenium, it actually encompasses the South as well. The CCP intends to follow a similar strategy for Manchuria (*Northeast Area Revitalization Plan*[18]) and the Center (*Rise of Central China Plan*[19]).

In Figure 2.8, we observe that the rank correlation between saving rates and real GDP per capita oscillates around a mean of 0.7 over the sample and is very persistent. Thus, historically, provinces with high saving rate have had a higher output in per capita terms, even prior to economic reforms. By comparison, there is no persistent link of investment rate with the distribution of GDP (the mean correlation barely differs from 0). In Figure 2.8, we observe that the relationship was clearly negative in the 1960s and 1970s. This may be a manifestation of the visible hand allocating investment to less developed regions (e.g. the *Third Front Campaign*). The initial phase of reforms pushed the correlation to 0.6 in the mid-1990s but the positive relationship waned and reached negative pre-reform levels in the 2000s.[20] Starting the rank correlation later – in 1980 instead of 1953 – enables to add new regions to the sample but does not influence patterns.

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[17]Massive investments in infrastructures, mainly transportation and power generating facilities (`http://www.chinagate.cn/features/Western_Development/node_7084033.htm`).

[18]For more, see `http://english.gov.cn/special/ne_index.htm`.

[19]For more, see `http://www.bjreview.com.cn/17thCPC/txt/2007-10/16/content_80952.htm`.

[20]Interestingly, the drop in correlation of investment seems to correspond to the "Go West" strategy initiated by the Chinese government in the late 1990s. In that sense, the visible hand may be back. The progressive redistribution of FDI toward inner provinces is another potential explanation for this pattern: FDI to GDP ratio typically peaked in the mid-1990s for most eastern regions while it started to rise only recently in less developed provinces.

2.2.4 Regional external balance

In the regional expenditure approach statistics, each entity is considered as a country with its own GRP (gross regional product). Net exports figures should therefore be informative about external balance at the regional boundary (i.e. interregional and international flows in goods and services).[21] Thereafter, net exports and output of the 31 regions are aggregated into six larger areas as defined in Table 2.1.

In Figure 2.9, a clear pattern emerges from the comparison of different periods in the sample: prior to reforms, more industrialized Manchuria and the Metropolises had large surpluses. Theses regions experienced a large drop in subsequent decades and ended up being net importers of capital in 2010. Remarkably, central provinces have had roughly balanced position all along. Eastern regions already had large surpluses averaging more than 10% of their GDP in the 1960s and 1970s. The initial reform era brought them to a near neutral position by the mid-1980s. Since the opening up to foreign trade, this region steadily increased its surplus to reach more than 10% in 2006-2008. Southern China initially markedly reduced its deficit but this trend reversed in the 1990s. Historically, it has been a net importer of capital (16% of output in 2010). Similarly, the West has had large negative net exports all along as well with a negative mean value of 12% from 1975 to 2010. For more, see Figure 2.10 where we provide a detailed geographical representation of the mean value of relative net exports at a disaggregated level.[22] All in all, our results suggest a large volume of interregional capital flows among provinces, a large cross-sectional variation in net position as well as a high persistency in the patterns of relative external balances.

To assess the relative contribution of regions to dynamics in national net exports, we focus on the six clusters defined before but without normalizing by output (Figure 2.11). Interestingly, the rapid increase in the Chinese current account in the 2000s seems to have been primarily driven by the East Coast. By contrast, all other regions have had deteriorating external balance. By cumulating them (*AllnoEC* line on the graph), one can observe a striking divergence in the patterns of capital flows: increasing net surpluses in the eastern part of the country have been concomitant with increasing net deficits in the rest. Obviously, a substantial part of negative flows originated in the South and the West. To which extent this "Great Divergence" is driven by statistical errors is an open question.[23]

Historically, as for relative saving, net export surpluses were related to the relative provincial

[21] Regional flow of funds data indeed confirm the presence of international and interregional capital flows in net exports but only half of provinces – typically more developed ones – differentiate them. Definitions of net exports made available by the statistical bureaus are identical across provinces and are similar to those used in *National Accounting*.

[22] A clear pattern would emerge from the comparison of different periods in the sample at the provincial level: in the 1950s, northern regions, Manchuria and the Metropolises mostly were capital exporters. Progressively, surpluses converged east and five decades later, from Heilongjiang to Guangdong, large surpluses are observed on the entire East Coast (with the exception of Jilin and Beijing that run a small deficit). Northern regions and Manchuria experienced a drop in net exports. Over the last decades, central regions had near neutral values.

[23] The aggregation issue is obvious: by adding the green (*East Coast*) and the grey line (*AllnoEC*), we do not obtain the blue line (*China*).

output per capita (i.e. the relatively wealthier regions had large surpluses). In Figure 2.12, it seems that the rank correlation dwindled during initial reform years. Still, it has been converging back to high levels since the mid-1990s. As for the preceding sections, we focus on the pattern of cross-provincial variability over time.[24] The (population-weighted) standard deviation of net exports was volatile in the pre-reform area (Figure 2.13). Then, it was decreasing in the 1980s and stable in the 1990s. It progressively bounced back to higher level as the Chinese economy became more and more integrated with the world supply chain.[25]

2.3 Light-predicted regional GDP

2.3.1 Data and methodology

Recently, using light intensity has been proposed as a strategy to test and improve the quality of official GDP data (Chen and Nordhaus, 2010). Obviously, many channels associated with economic structure could cast doubt on the validity of this proxy. On top of that, for technical reasons, measured light is only an imperfect indicator for true light. The key advantage is that as errors in GDP statistics and light measurement are likely to be uncorrelated, one could get a better indicator by simply combining them (Henderson et al., 2011). Keeping the numerous problems plaguing Chinese data in the back of our mind, two issues may be of interest. First, we want to explore to which extent regional light intensity in China is related to real GDP figures at the provincial level. Second, in our opinion, explore patterns of real growth and their potential changes among regions is important. It may raise a red flag and point to some possible regional misreports of output.

We use satellite data over the 1992-2010 period capturing light intensity in 64 categories from 0 (no light) to 63 (maximal intensity).[26] Light data are from US Air Force.[27] We follow the methodology used in a working version of Alder et al. (2013) where light data are used to assess the effect of economic reforms in China at the city level.[28] Whenever two satellites were at disposal for the same year, the more recent one was chosen. In terms of real GDP and light, all regions (31) are available. Our sample thus stretches from 1992 to 2010. As one can observe in Figure 2.14, the levels of official real GDP statistics and light intensity are highly positively correlated.[29]

GDP is predicted using the following model :

[24] In this section and the following ones, we do not compute the coefficient of variation because it fluctuates hugely when the cross-sectional mean approaches zero.

[25] This "U-turn" pattern is more marked when considering non-weighted variability. Results are little affected by the Metropolises. The later apparition of some large provinces is negligible in terms of additional variation.

[26] Some pixels take higher values but have been censored.

[27] Freely avaible on `http://ngdc.noaa.gov/eog/`.

[28] We gratefully thank the Chair of Macroeconomics and Political Economy of the University of Zurich for providing us with the light data structured by region.

[29] Note that one would need to calibrate the different satellites using a specific region to be able to compare light data. We refrain from it but use time fixed-effects in the empirical analysis to address that issue.

$$\ln(GDP_{i,t}) = \beta_0 + \beta_1 \ln(Area_{i,t}) + \beta_2 \ln(Area_{i,t})^2 + \sum_{k=3}^{K=10} \beta_k LightShare_{k,i,t} + \tau_t + \varepsilon_{i,t}$$

where *LightShare* corresponds to the share of the regional area with light intensity between 1 to 10, 11 to 20, 21 to 30, 31 to 40, 41 to 50, 51 to 60, 61 to 62 and 63 and more. Zero light intensity is the reference share. In this way, we allow for the relationship between light intensity and GDP to be non-linear. The variable *Area* is the relative size of a region. Because of the huge discrepancy in area among Chinese provinces, we allow it to enter with a quadratic term as well.

Results are available in Table 2.2. Both controls for size are highly significant. They suggest a concave functional form. Regions smaller than Yunnan have an increasing positive relationship between size and economic size while for bigger regions (e.g. Inner Mongolia or Xinjiang), the effect decreases but still stays positive. Figure 2.15 provides a representation of the coefficients of light intensity. Their impact on real GDP seems to be non-linear: it rises sharply from the first to the second category, loses significancy for the third and fourth categories and peaks in the fifth one. In the three subsequent categories, the effect then slowly decreases. All in all, the coefficients on light intensity are positively, roughly concavely related to economic activity and five of the eight categories are found to be highly significant. We suspect that the lack of evidence for the impact of middle categories could be linked to the limited number of observations (589) in the panel.[30]

2.3.2 Results

The predicted GDP obtained from the preceding model is positively and highly significantly correlated with official data (correlation of 0.83). Henderson et al. (2011) used a comparable methodology to improve data of countries having a score of 3 or less out of 10 possible points in a World Bank report on statistical capacity (WB, 2002). China has a somewhat higher score of 5. All the same, one could consider a statistical worst-case scenario and pretend that it belongs to the country group with weak statistical resources. By assuming GDP growth to be perfectly measured in countries with good indicators, Henderson et al. (2011) found an optimal weight of 0.56 on official national output and 0.44 on light-predicted output. Therefore, one could use a similar weight to construct an alternative GDP for Chinese regions.

To make comparison easier, we discuss differences between official GDP figures and light-predicted output. Table 2.3 summarizes the results. The rank correlation of average growth rate over the sample period is 0.50. A few outliers emerge: Shanghai had an official growth rate

[30]Using city-level data, Alder et al. (2013) found all light coefficients to be highly significant. Interestingly, the impact of the middle-categories (21-30 and 31-40) was also found to be slightly lower than the 11-20 one. Another difference is that coefficients were increasing systematically over light intensity in higher categories. Their specification did not feature a quadratic size term.

slightly below average over the period while light-GDP suggests that it belongs to the fastest-growing regions. The discrepancy is comparable for the island of Hainan. By contrast, Inner Mongolia was at the top in official data but does not seem to have outperformed other regions according to the alternative GDP indicator. The same is true for the small region of Ningxia. In Figures 2.16, one can compare average growth rates of official and light-predicted real GDP over 1992-2010. According to the new indicator, the *Bohai Economic Rim* and the *Yangtze Delta* area have been the fastest-growing clusters. As discussed before, some hinterland regions seem to have grown far slower than official data suggest (Inner Mongolia, Ningxia and Shaanxi). Broadly speaking, there seems to be no massive regional patterns of over- or underestimation. In this respect at least, regional GDP data seem to be of reasonable quality.

2.4 Regional international trade statistics

2.4.1 Aggregation properties

International trade flows data at the provincial level reported in dollars enable us to focus exclusively on external imbalances of regions versus the rest of the world. Before 1992, we rely on CDC data. From 1992 on, two sets of data exist. The first is international trade by location of foreign trade managing unit (by location of importers/exporters). It is based on the place where corporations are registered by local *Custom Houses*. In the second one, imports are ventilated by place of destination (i.e. consumption or final destination) and exports by place of origin. For provinces containing a high number of import/export companies such as Beijing or Shandong, the difference between both concepts is considerable. Since 1992, CDC relied on the first concept. We adopt the second one as it should better reflect the final direction of flows in goods and services.

No specific data on provincial expenditure aggregates in dollars such as output exist. With the help of national trade values available in both currencies, we derive an implicit exchange rate and use it to to convert trade data into Renminbi.[31] As in the preceding sections, aggregation properties of the data can be inspected by dividing the difference between cumulated provincial and national value by national GDP. The data availability is poor for initial decades.[32] The bulk of the considerable errors stems from regional imports: they are too low and this error increases even as the sample availability improves over time. Exports are only slightly underestimated. As a consequence, the provincial net trade saldo is massively overestimated during the time period preceding economic reforms. Estimation improves in the second half of the 1980s and regional aggregates have been corresponding to their national counterparts since 1992. In conclusion, data only seem reliable for the 1990s and 2000s.

[31] Some regional yearbooks made results in dollars available but only for recent decades. To keep the sample as large as possible and avoid exchange rate biases, we apply the national trade exchange rate to all results.

[32] 8 provinces out of 31 are available in 1952, 18 in 1964 and 26 in 1979. The full sample is available from 1992 onwards.

2.4.2 International capital flows

In Figure 2.17, we provide a representation of international surpluses and deficits during the reform period. The rising surplus of the East Coast, which peaked at 16% in 2007, is not surprising given its well-known high involvement in international trade. Manchuria recorded large surpluses in the 1980s before converging to a near neutral position. Similarly, foreign trade is considerable in the Metropolises: they exhibited a high surplus of 14% of GDP from 1952 to 1990 before experiencing a sharp deterioration, which bottomed out at -19% in 2004. Other clusters are less involved in trade: Central, South and West China had an average small surplus of around 1.5, 0.7 and 1.6% over the 1975-2010 period. There is a striking difference compared to external balances at provincial boundary from Figure 2.9: with the exception of the coastal part of China, most regions have a roughly neutral international trade balance. It suggests that interregional – rather than international flows – may be responsible for the large external deficits observed in some areas.

All things considered, as for net exports, there are large and enduring differences in the exposure to international trade and the relative trade position across provinces. However, in absolute terms, only East Coast regions seem to matter at the international level. This is well-illustrated in Figure 2.18, where we provide absolute supraregional aggregates of trade balance.

In Figure 2.19, we focus on the geographical distribution of surpluses and deficits for the reform period and find a clear pattern: with the exception of Beijing, all eastern regions have a highly positive mean trade balance. There is more variation in the remaining provinces. Virtually all regions registered a positive or neutral mean saldo over the period apart from Beijing, Hainan and Tibet. Interestingly, some western regions run international surpluses while, as we saw in Figure 2.10, their total balance in net exports was strongly negative (e.g. Xinjiang, Qinghai and Ningxia).

The correlation of the international trade saldo relative to GDP with real GDP per capita has been lower during the reform era than before: in Figure 2.20, it hovered around zero in the 1990s and 2000s. As a consequence, the link seems less strong than for net exports. In Figure 2.21, the (population-weighted) cross-provincial variability follows an upward trend. The increase is particularly strong in the 2000s. It seems consistent with the pattern of regional specialization discussed in the literature.

2.5 Interregional capital flows indicator

2.5.1 Methodology

As data on net exports at provincial boundary (Section 2.2.4) as well as international trade (Section 2.4) are available, an approximation of interregional flows in goods and services is possible by substracting the latter from the former. Nevertheless, one should bear in mind that the data availability problem is more acute due to the linear combination of both variables. From 1992 onwards, aggregation errors in international trade are negligible. Errors for net exports have been fluctuating around zero before worsening between 2005 and 2010. For these reasons, we restrict our sample to the 1992-2004 period. Given the fact that international trade data are reliable and regional net exports are on average underestimated by 0.08% of GDP over 1992-2004, we expect our indicator to be too low (i.e. some provinces having a neutral or small negative balance could actually have a positive one).

To begin with, we investigate aggregation properties to assess our estimation.[33] Another natural test for the goodness of our indicator is to confront it with official statistics on physical capital flow transactions. Unfortunately, regional flow of funds data are not broadly available. For the 1998-2002 period to which we have access, only 16 out of 31 provinces offer a sharp distinction between capital flows stemming from outside the province (domestic) and from the rest of the world. We compute the correlation over time for each region between our estimates of interregional capital flows and the values of the flow of funds data. Our methodology fails for 5 provinces but seems to reasonably capture the dynamics and the absolute level of capital flows for 11 regions.[34]

2.5.2 Interregional capital flows

Results are in Figure 2.22. Net exporters of capital are concentrated in the northeast of the country: Manchuria, the *Bohai Bay* region (Beijing, Tianjin, Hebei and Shandong) as well as Shanghai and Jiangsu. To a lesser extent, some central provinces (Henan, Hubei) have been accumulating internal surpluses as well. Most of the western and southern part of the country experienced capital inflows. In those regions, as suggested in preceding sections, small international surpluses are overturned by huge interregional capital imports. Interestingly, some provinces with large international trade surpluses on the southern part of the East Coast cluster end up being internal net capital importers (e.g. Zhejiang, Fujian and Guangdong).

2.6 Conclusion

This paper discusses key empirical facts of regional Chinese external imbalances. We shed light on issues related to data quality and aggregation properties of Chinese regional macroeconomic

[33] A few tests derived from simple theoretical considerations enable us to test our proxy. To begin with, the interregional indicator using national values should not deviate persistently from zero. Indeed, no systematic difference is observable as an average of 0.29% of GDP over the 1992-2004 period is obtained. Another test consists in simply adding the absolute balance of all provinces (if intranational trade was sensibly estimated, the sum of regional values should be approximately zero). It validates our methodology as errors in 1992-2004 were modest: 0.37% of GDP on average.

[34] The average 1998-2002 correlation of both indicators over 11 regions is 0.89. We compare our own indicator with flow of funds tables for Hebei (1997-2009) and Henan (2000-2009) that regularly publish data. The level and dynamics of interregional capital flows seem to be well-captured by our proxy.

figures. Large discrepancies between regional and national components of the expenditure aggregates are observed. Investment errors are the key driver of differences between provincial and national output: regional "phantom investments" have systematically exceeded national statistics since the mid-1990s. This clear trend has been a statistical companion of emerging China over the 1990s and 2000s.

As a consequence, provincial net exports end up being lower than national values. Still, cumulated local net exports roughly correspond to national values from the mid-1980s to the mid-2000s. However, it should be borne in mind that the fact that national data are better than provincial ones is not established. Consequently, our findings could be interpreted the other way round since national data have been taken as reference point. All in all, data quality certainly is an issue, as shown by the impact of the *2004 Economic Census* on the statistical system.

In the 1950s, the geographical distribution of output per capita followed a different pattern than today. A clear north-south divide was observable and morphed into an east-west divide as coastal regions were the first to become integrated into the world economy. We observe a large decrease in terms of output per capita disparities among regions during the initial reform period but it does not survive the exclusion of the Metropolises from the sample. In spite of large migration flows, investment programs and the recent progressive integration of the hinterland into the world supply chain, we find that cross-regional variability in relative output per capita has been on the rise since the 1990s at least. Saving rates follow an east-west divide. During the last three decades, investment rates in the northern and western part of the country clearly overtook values of other regions, forming a north-south divide. Historically, relative saving was highly related to the level of economic development. By contrast, relative investment was not.

The historical distribution of regional net exports surpluses and deficits (i.e. the difference between saving and investment) is informative. The coastal regions indeed accumulated surpluses since the opening-up of China but already had large ones before reforms. Interestingly, some other regions have even larger saving-investment gaps. In the communist era, the Metropolises and Manchuria accumulated huge surpluses and converged towards a more neutral position during reforms. Over the last decades, Central China has had roughly balanced net exports. West and South China have been running huge deficits independently of the adopted economic system. From the 1950s onwards, positive net exports have been persistently associated with high output per capita, albeit with varying intensity: the initial domestic reforms lowered the strength of the relationship but it then increased following the uneven integration of regions into the world economy in the 1990s and 2000s. The cross-provincial variability in relative net exports dwindled considerably from the 1970s to the mid-1990s but has been on the rise since then.

By using light intensity data from US Army satellites, we were able to show that once we control for considerable differences in regional size, light intensity seems to be positively correlated with GDP in Chinese regions in a roughly concave fashion. Light-induced GDP leads to minor changes in the growth pattern of provinces: the *Bohai Rim* and Hainan island grew comparatively faster while other regions like Inner Mongolia, Shaanxi or Ningxia may have developed less rapidly than official data suggest.

International trade data aggregate correctly since 1992. Imbalances in international trade are less dramatic than in net exports: with the exception of the large deterioration of the Metropolises and Manchuria of the early 1990s and the steadily increasing export surpluses of the East Coast, most provinces have been running small surpluses. Thus, the bulk of the large discrepancies in net exports seems to be driven by interprovincial flows. In absolute terms, only the East Coast area truly matters for international capital flows. This specialization pattern may explain why the cross-regional variability of international relative flows has been increasing since the 1990s.

We propose an indicator for internal trade that seems to do a reasonable job in approximating internal capital flows between 1992 and 2004. It suggests that provinces of Manchuria, the *Bohai Economic Rim*, the *Yangtze Delta* and some Central China provinces provided capital to other provinces, particularly in the western and southern part of the country. Over this period, the international export-oriented southeast coastal regions were internal net importers of capital flows at the domestic level in spite of their large international trade surpluses.

Table 2.1: Definition of regional clusters and relative size

	East Coast	Manchuria	Metro	West	South	Center
	Hebei	Liaoning	Beijing	Shanxi	Chongqing	Henan
	Shandong	Jilin	Tianjin	Inner Mong.	Sichuan	Hubei
	Jiangsu	Heilongjiang	Shanghai	Shaanxi	Yunnan	Hunan
	Zhejiang			Ningxia	Guangxi	Anhui
	Fujian			Gansu	Guizhou	Jiangxi
	Guangdong			Qinghai	Hainan	
				Xinjiang		
				Tibet		
	East Coast	Manchuria	Metro	West	South	Center
nGDP 1965	25.1	16.7	14.0	12.9	9.4	21.9
rGDP 1984	31.9	13.3	9.8	10.1	14.2	20.6
rGDP 2010	43.2	9.2	7.3	10.0	11.6	18.7
pop 1965	31.9	9.1	3.5	11.1	19.3	25.1
pop 1984	30.7	9.0	2.9	11.6	20.2	25.6
pop 2010	33.2	8.2	4.2	12.0	18.4	24.1

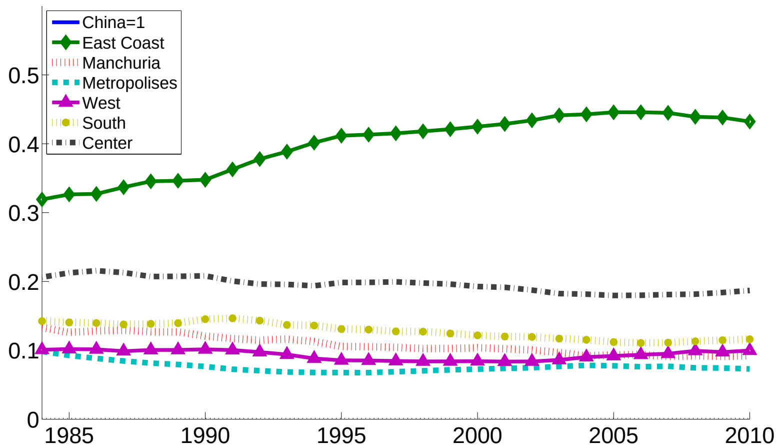
nGDP is nominal output (current prices in RMB), *rGDP* is real output (1984 national RMB), population data are from the *Household Registration System*, sample surveys, censuses and own calculations.

Table 2.2: Light regression

	DepVar: $\ln(realGDP)$		
	coeff	t-val	p-val
$\ln(Area)$	2.22	5.28	0.00
$\ln(Area)^2$	-0.32	-3.78	0.00
LS_{1-10}	2.08	4.13	0.00
LS_{11-20}	6.01	2.83	0.00
LS_{21-30}	3.65	0.44	0.66
LS_{31-40}	4.98	0.39	0.70
LS_{41-50}	9.44	2.82	0.00
LS_{51-60}	8.40	3.72	0.00
LS_{61-62}	7.03	2.55	0.01
LS_{63}	6.71	1.59	0.11

Pooled OLS estimates using 589 observations, adjusted R^2 of 0.673, time fixed-effects included, inference based on standard errors clustered by province (Liang and Zeger,1986).

Figure 2.1: Relative real output share, 1984-2010



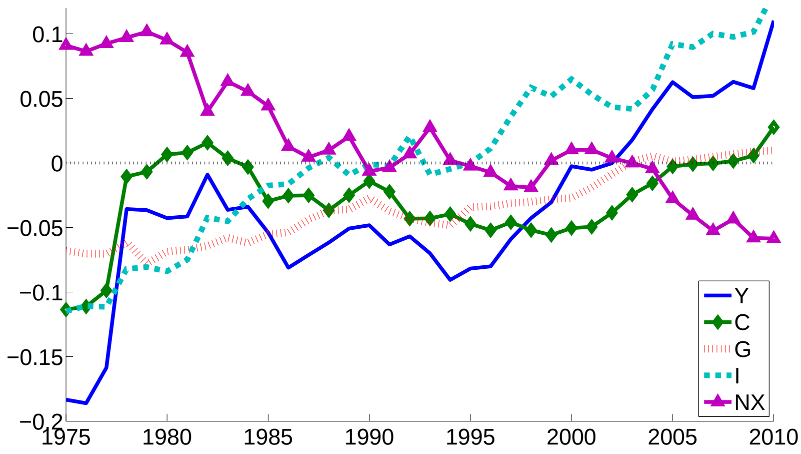
Real GDP of regional clusters relative to cumulated real GDP as defined in Table 2.1. Deflators computed with official provincial CPI data and Brandt and Holz (2006) for initial price level.

Table 2.3: Official and light-predicted mean real GDP growth, 1992-2010

	Official	Rank	LightPred.	Rank
Beijing	0.12	10	0.14	7
Tianjin	0.14	2	0.22	1
Hebei	0.12	14	0.14	8
Shanxi	0.11	17	0.11	14
Inner Mongolia	0.15	1	0.07	28
Liaoning	0.10	23	0.11	13
Jilin	0.12	13	0.10	15
Heilongjiang	0.09	30	0.09	20
Shanghai	0.11	20	0.20	3
Jiangsu	0.13	7	0.20	2
Zhejiang	0.13	5	0.17	5
Anhui	0.11	16	0.13	10
Fujian	0.13	4	0.11	12
Jiangxi	0.12	15	0.09	21
Shandong	0.12	12	0.18	4
Henan	0.12	9	0.15	6
Hubei	0.11	21	0.09	18
Hunan	0.11	18	0.09	23
Guangdong	0.13	3	0.13	11
Guangxi	0.11	19	0.09	19
Hainan	0.09	29	0.14	9
Chongqing	0.12	11	0.09	17
Sichuan	0.10	22	0.08	26
Guizhou	0.10	25	0.08	25
Yunnan	0.09	31	0.08	24
Tibet	0.10	28	0.06	31
Shaanxi	0.12	8	0.10	16
Gansu	0.10	27	0.07	27
Qinghai	0.10	24	0.06	30
Ningxia	0.13	6	0.09	22
Xinjiang	0.10	26	0.07	29
Mean	0.11		0.11	

Official real GDP growth rate and light-predicted real GDP growth rate.

Figure 2.2: Aggregation errors of output (expenditure approach)



Aggregation errors computed as provincial sum minus national value over national GDP. A positive result means that cumulated provincial data overestimate national values.

Figure 2.3: Nominal output per capita relative to national value, 1950s average

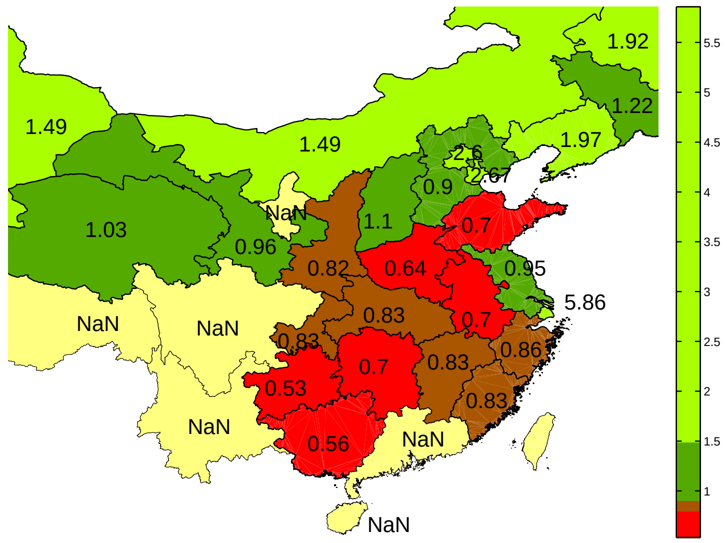


Figure 2.4: Real output per capita relative to national value, 2010

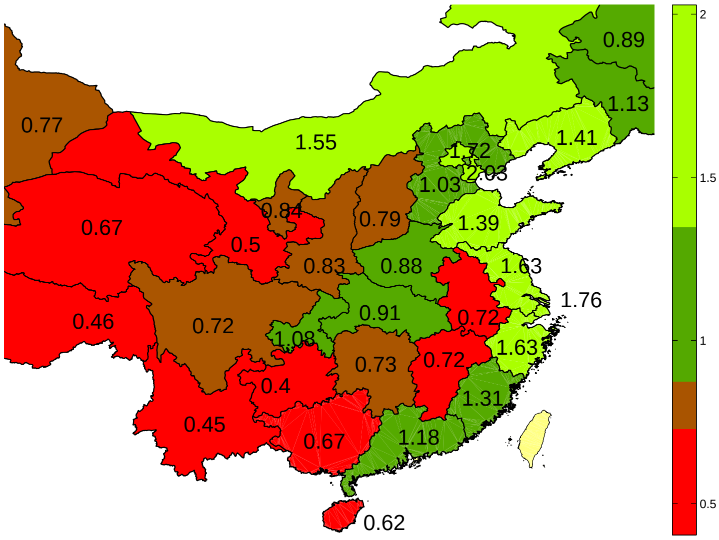
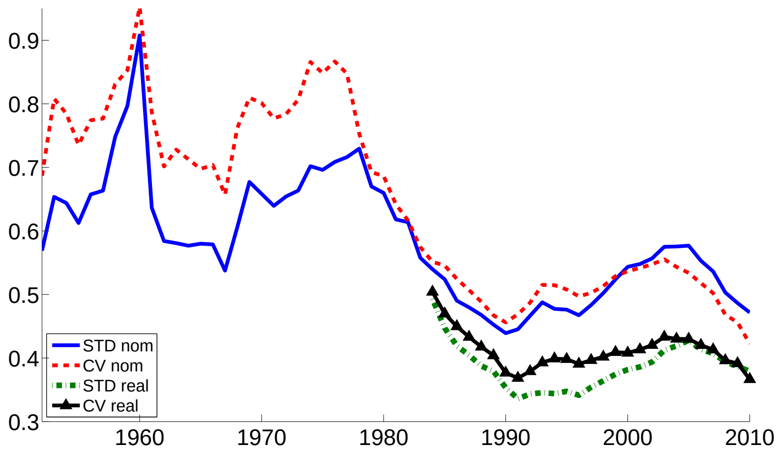


Figure 2.5: Cross-provincial variability of GDP per capita



Cross-sectional standard deviation (*STD*) and coefficient of relative variation (*CV*) of the relative nominal and real GDP per capita. Contribution of provinces weighted by time-varying population share.

Figure 2.6: Mean of saving over GDP, 1979-2010

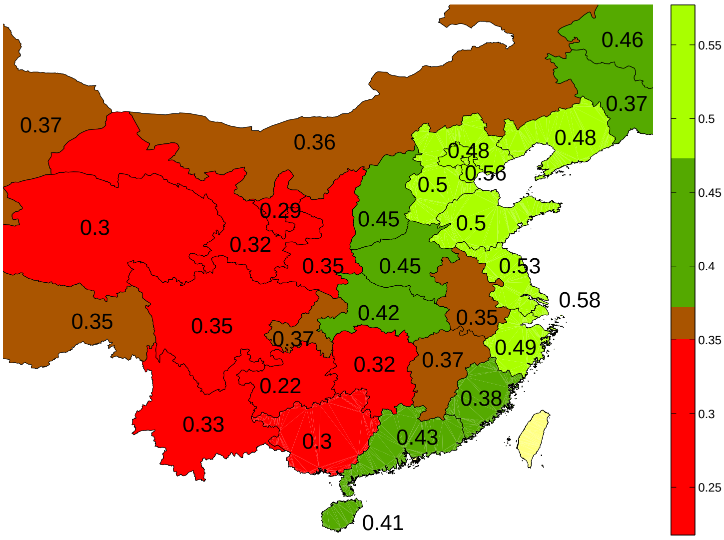


Figure 2.7: Mean of investment over GDP, 1979-2010

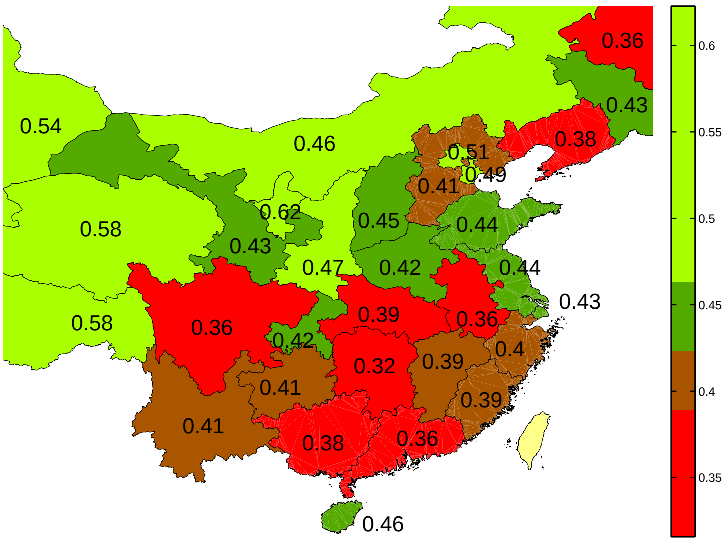


Figure 2.8: Rank correlation of S/Y and I/Y with real GDP per capita

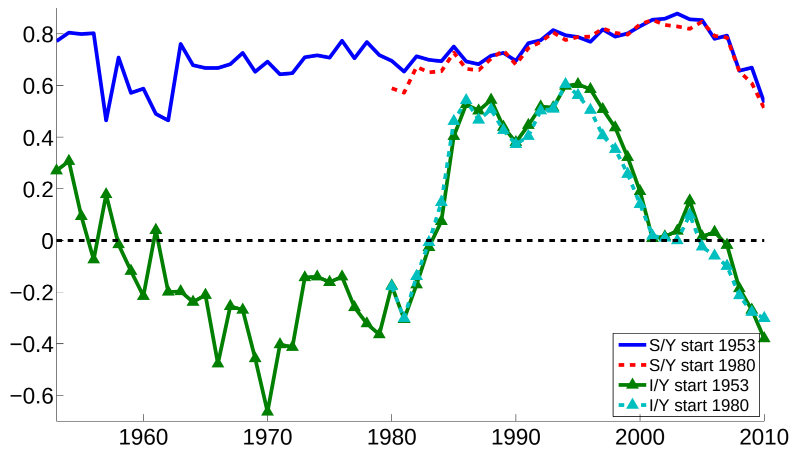
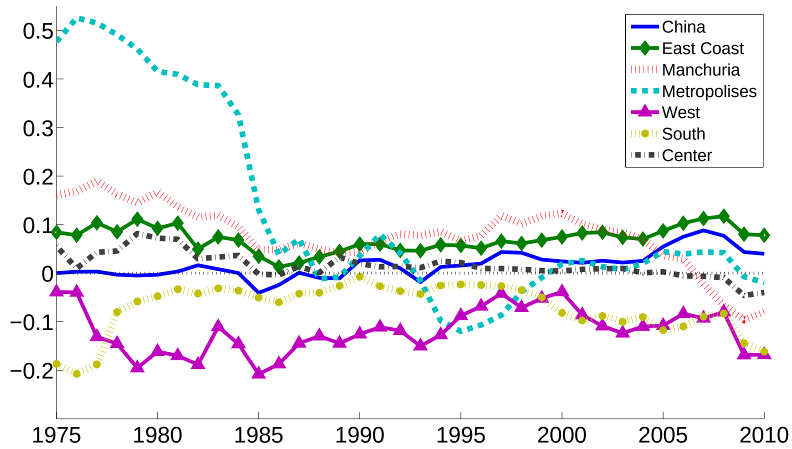


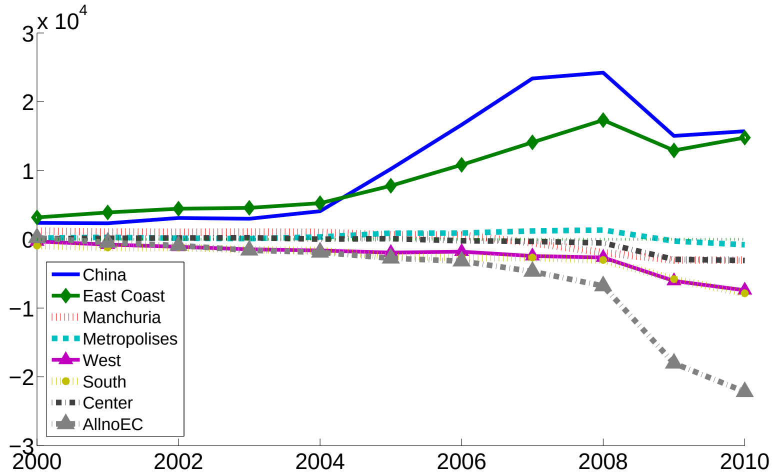
Figure 2.9: Net exports over GDP, 1975-2010



Aggregate net exports over aggregate GDP for the six clusters defined in Table 2.1.

Figure 2.10: Mean of net exports over GDP, 1979-2010 (in %)

Figure 2.11: Nominal net exports (100 million RMB), 2000-2010



Absolute cumulated net exports of the six clusters defined in Table 2.1, in current RMB. *AllnoEC* is the sum of all regions without the East Coast area.

Figure 2.13: Cross-sectional standard deviation of net exports over GDP (population-weighted)

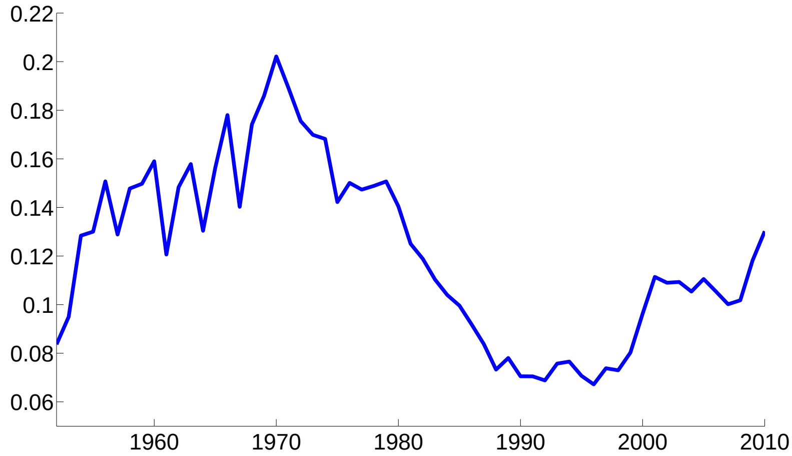


Figure 2.12: Rank correlation of net exports with real GDP per capita

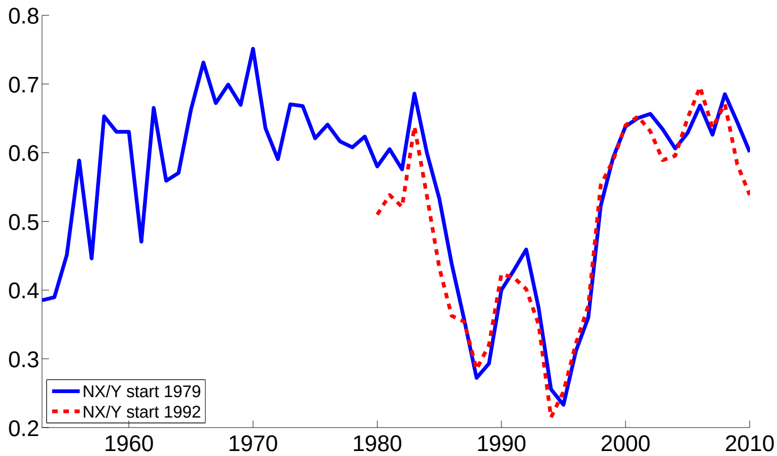
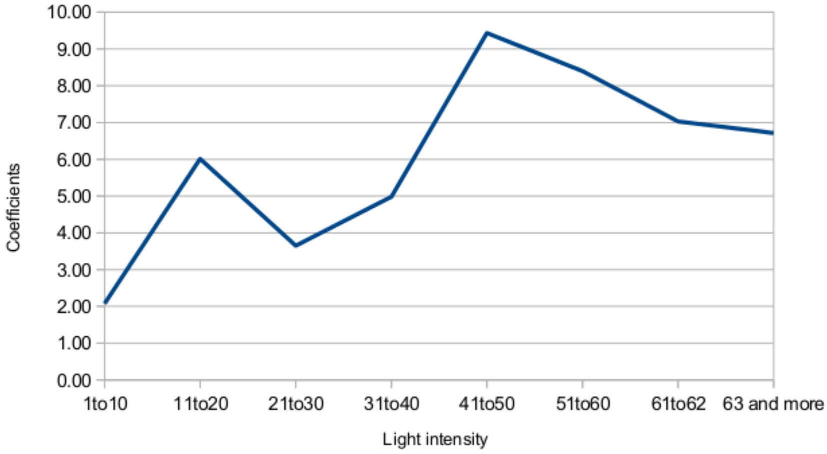
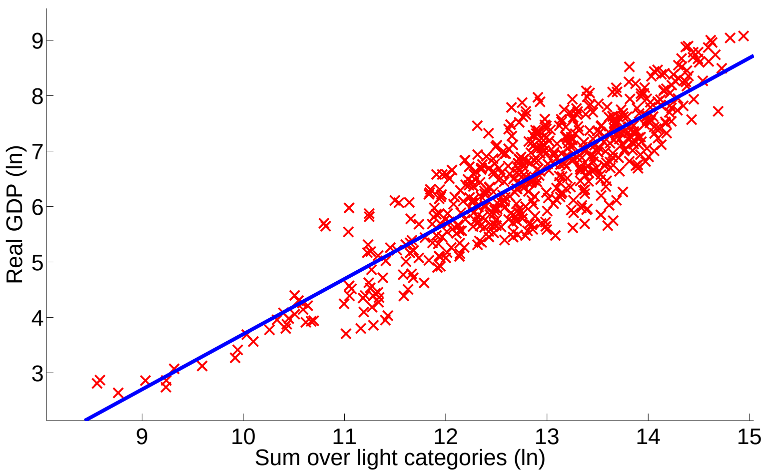


Figure 2.15: Coefficients of light intensity categories



Coefficients of the light intensity categories of the regression $\ln(GDP_{i,t}) = \beta_0 + \beta_1 \ln(Area_{i,t}) + \beta_2 \ln(Area_{i,t})^2 + \sum_{k=3}^{K=10} \beta_k LightShare_{k,i,t} + \tau_t + \varepsilon_{i,t}$.

Figure 2.14: Real GDP vs light intensity (non-calibrated)



Light intensity is the sum of light value (0 to 63) across pixels within provincial boundaries.

Figure 2.16: Average real GDP growth: data (first) vs light-predicted (second), 1992-2010

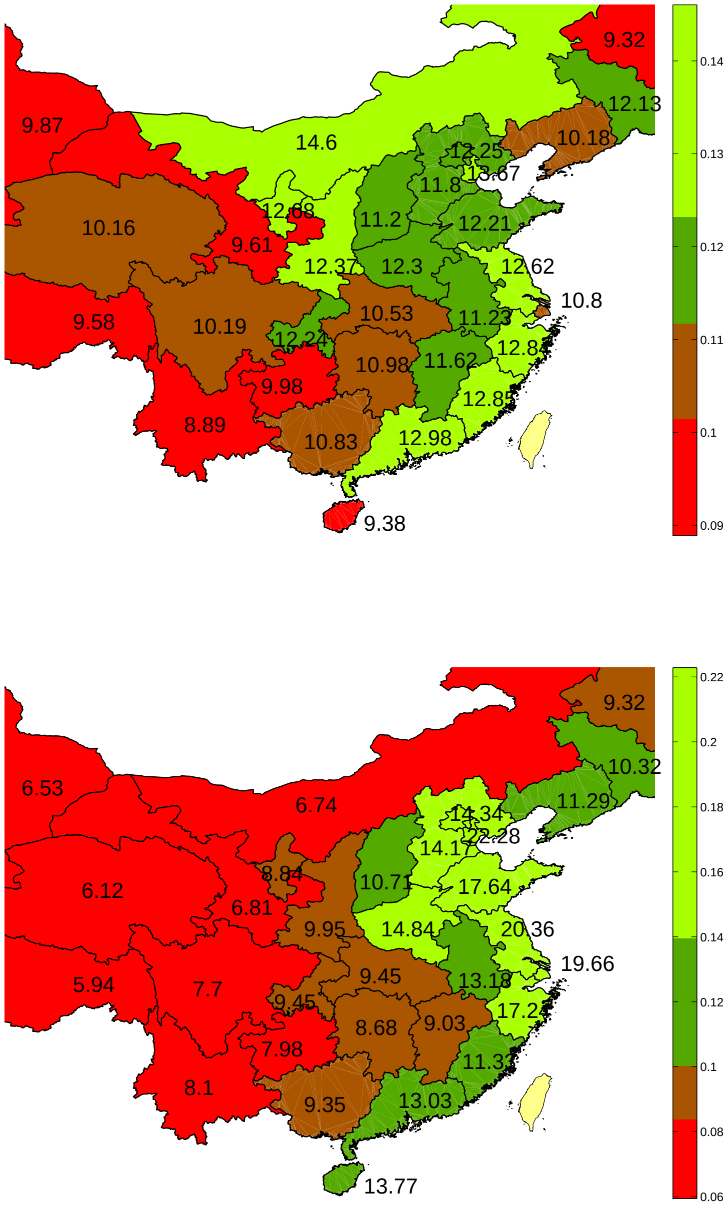
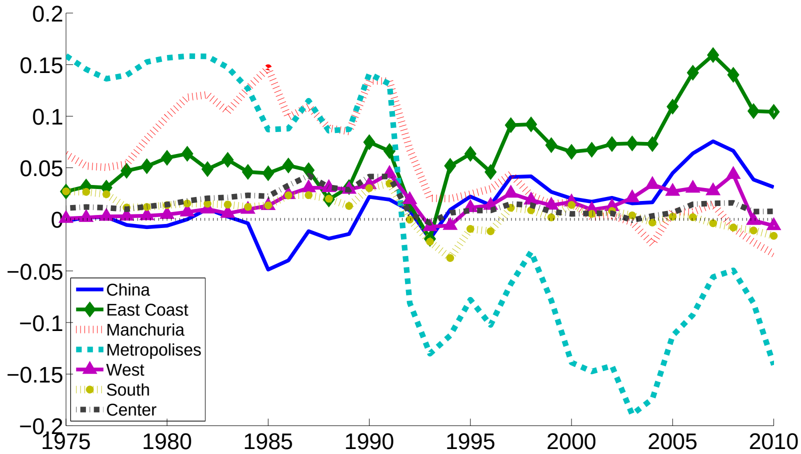
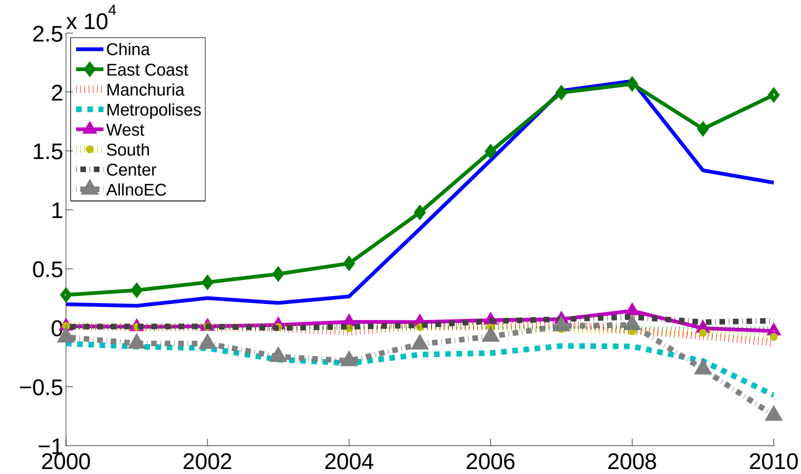


Figure 2.17: International trade balance over GDP, 1975-2010



Trade data in dollars converted using implicit national exchange rate. CDC data until 1991, trade by place of destination and origin from 1992 to 2010.

Figure 2.18: Nominal international trade balance (100 million RMB), 2000-2010



Trade data in dollars converted using implicit national exchange rate. Trade by place of destination and origin.

Figure 2.19: Mean of trade balance over GDP, 1979-2010 (in %)

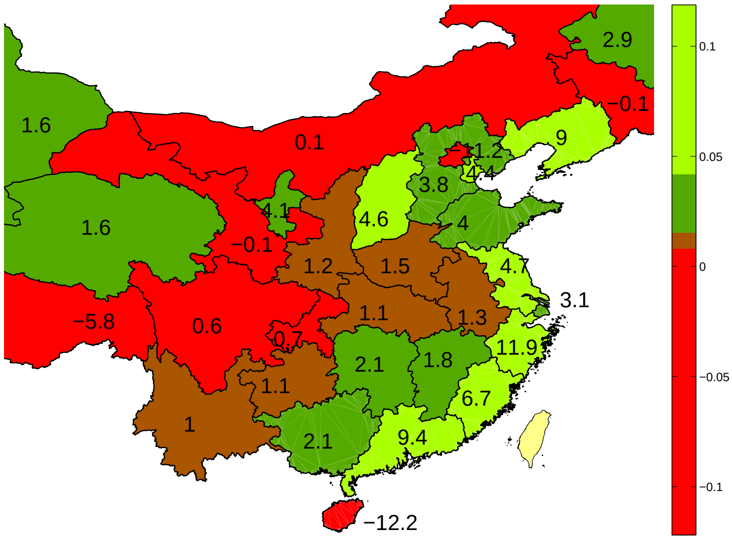


Figure 2.20: Rank correlation of trade balance with real GDP per capita

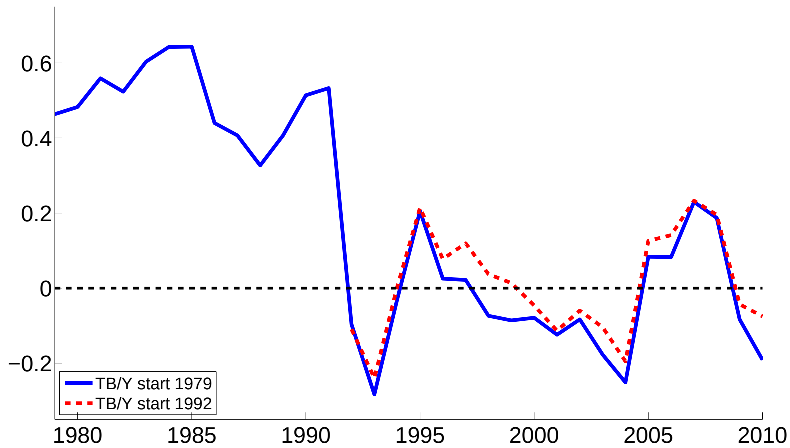


Figure 2.21: Cross-sectional standard deviation of trade balance over GDP (population-weighted)

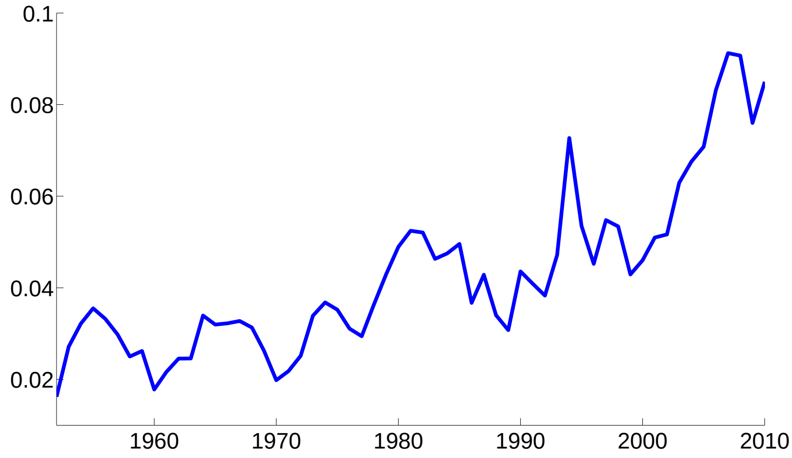
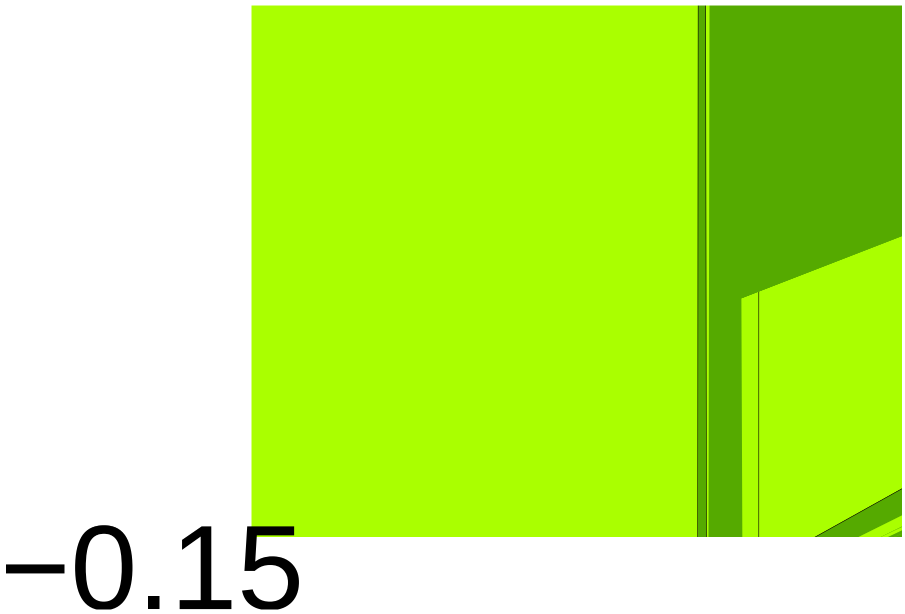


Figure 2.22: Mean of interregional capital flows over GDP, 1992-2004 (in %)



Chapter 3

Capital's Long March West: Saving and Investment Frictions in Chinese Regions[1]

[1]This chapter is currently under submission at the *American Economic Journal: Macroeconomics*.

3.1 Introduction

Since the mid-2000s, global imbalances have been a resurgent topic in academics and occasionally shaped the political agenda. The far-reaching implications of that issue fostered sustained research. Still, the debate is far from over and most policy recommendations had much bark but little bite.[2] While imbalances undoubtedly came down since the Great Recession, the fundamental factors driving them are still subject to debate. Understanding to which extent external imbalances are symptomatic of differences in underlying economic structure and policies is key to assess when – and under which conditions – a convergence towards more sustainable patterns would be possible. Over the last decade, the literature disproportionately discussed that issue through the lens of US interests. At the onset of the prophetic "Great Rebalancing", observers and experts increasingly laid the focus on China.

The driving forces of Chinese imbalances are increasingly well-understood. Since Jiabao's famous 2007 speech acknowledging the unsustainability of the Chinese growth model, the awareness that rebalancing is a decisive step towards sustainable economic development has established itself. Often though, the interconnections of factors driving internal and external imbalances were not embedded in the analysis. For a discussion on Chinese internal and external imbalances, we refer to the first chapter of the thesis (Section 1.2).

With tremendous growth experienced over three decades, continental Chinese regions begin to matter. Some of them already are GDP equivalent to big developing countries in terms of PPP international dollar. For instance, as of 2010, Sichuan overtook Malaysia, Yunnan was roughly comparable to Vietnam and Henan reached the level of Thailand. The output of Guangdong in the *Pearl River Delta* is expected to overtake Indonesia's one in the coming years.[3] More developped coastal regions long have the weight of small industrialized countries: Shandong and Jiangsu stand for Switzerland, Zhejiang for Austria and Fujian for Ireland. On top of that, they have become highly integrated into the world economy: in terms of total value, Jiangsu exports roughly as much as Taiwan while Zhejiang is comparable to Thailand. The province of Guangdong is on the same scale as South Korea in total export terms. These regions thus play an important role in the world economy. Moreover, their contribution to global imbalances is substantial, as is their potential impact on future adjustments.

While Chinese external imbalances have been largely discussed by practitioners, the academic literature on global imbalances rather focused on theoretical explanations. This paper is a first try in bridging the gap between both. We discuss potential frictions in saving and investment that emerged during the transition from a largely agricultural and planned to an industrial and more market-based economy. We relate capital flows (i.e. cumulated net exports) to regional

[2] The blunt proposals to force countries with "excessive" current account surplus to adjust using the WTO as a legal platform are a case in point.

[3] For more, see interactive maps available on *The Economist*'s website (`http://www.economist.com/content/all_parities_china`).

productivity in a formal framework. We show that a *capital allocation puzzle* is present inside China: provinces that caught up relative to national productivity had surpluses of saving over investment (capital outflows) while the opposite is observed for provinces that benefited less from economic reforms. This result is reminiscent of the findings of Gourinchas and Jeanne (2013) at the international level.

Starting from that empirical finding, we follow up by identifying the drivers of that pattern using a standard neoclassical model with two frictions in the mould of Gourinchas and Jeanne. The first friction (the investment wedge) takes effect at the aggregate level of the economy and influences capital accumulation. It affects gross return on aggregate capital and is identified by matching an empirical with a theoretical decomposition of investment rate. We find an investment puzzle: regions that caught up relative to the rest of China seem to have higher wedge (lower investment rate), while provinces that fell behind implicitly subsidized investment. This is a first blow to the baseline neoclassical framework and stands in sharp contrast with international patterns.

The second friction (the saving wedge) is comparable to a tax on capital income of households. It is identified in matching an empirical with a theoretical decomposition of cumulated relative capital flows (i.e. net exports). As on the international level, we find a saving puzzle: the relationship between productivity catch-up and saving wedges is negative and highly significant. Provinces that caught up are the ones that implicitly subsidized saving, causing a saving glut that translated into capital outflows. This is a second blow to the neoclassical model. As opposed to investment wedges, saving frictions are the main driver of the *capital allocation puzzle*.

In a next step, we investigate whether the estimated long run wedges are related to usual suspects proposed by the literature. The regional cross-sectional variability of the wedges seems useful in shedding light on the general patterns of capital flows. Some characteristics related to the investment structure of the economy robustly account for a high part of the cross-regional variation in investment wedges: a high share of the state in investment in fixed assets or in construction gross output value and a marked presence of the formal, state-near financial sector – loans in financial institutions – seem to foster investment.

Turning to saving wedges, there seems to be an ubiquitous effect of the state's involvement in the economy (e.g. state-owned share of gross industrial output value) in repressing saving. By contrast, a greater importance of multinational firms, privately-owned enterprises and a larger industrial sector are all associated with higher saving compared to the neoclassical model. Financial development – deposits and loans in financial institutions – seems to put a dent in saving.

We conclude that the *capital allocation puzzle* is driven by both the visible hand (the state) and the private sector. By constructing non-state net exports, we show that more marketized regions with rapid TFP growth and a strong presence of private and international firms (i.e. the East Coast) have large non-state saving surpluses while other regions have balanced non-state net exports on average. The neoclassical model would predict the opposite pattern. In addition to that, we find that massive state net exports deficits are largely responsible for large capital imports (i.e. a negative saving - investment balance) in the Chinese hinterland.

The paper is structured as follows. First of all, we briefly review the assumptions of the model and discuss data issues in Section 3.2. Second, in Section 3.3, we establish the existence of a capital allocation puzzle inside China and investigate to which extent discrepancies in external balance arise from frictions in aggregate investment, aggregate saving or possibly both. Then, in Section 3.4, we relate the wedges to a large number of "usual suspects" put forward in the literature and compute state and non-state net exports. In a next step in Section 3.5, bearing in mind that the data are known to be noisy, we discuss the effects on our general results of the use of alternative data. In Section A.1.3 in appendix, the sensitivity of the estimates to alternative parameter values is discussed. Eventually, Section 3.6 concludes. While we deliberately keep the baseline framework straightforward, we propose extensions of the model in Section A.1.4.

3.1.1 Related literature

Since the mid-2000s, the issue of global imbalances has been largely discussed. Given the large set of factors determining saving, investment as well as financial and physical capital flows, it is no surprise that a vast theoretical and empirical literature has emerged and proposed numerous – mostly non-exclusive – explanations of the phenomenon. Thereafter, we focus on the Asian side of the coin (i.e. the "saving glut" of Bernanke (2007)).

A prominent strand of the literature invokes differences in financial development. Caballero et al. (2008) highlight the central role played by heterogeneity in countries' ability to produce financial assets for global savers. Mendoza et al. (2009) introduce differences in the enforceability of financial contracts. The limited access to credit of high-productivity private firms could force them to rely heavily on self-financing and spur a rise in saving, as argued by Song et al. (2011). Bacchetta and Benhima (2010) rationalize the observation of high saving and high investment by arguing that if liquid assets are needed at some stage of the production process (e.g. to finance working capital), then credit-constrained firms increase foreign bond holdings following a productivity shock. In Jin et al. (2012), financially constrained economies have a greater share of savings arising from middle-aged than from young agents. As the former save more in response to lower interest rates, it helps rationalizing asymmetric responses in capital flows to growth shocks observed in the data.

While highly plausible for China, the empirical validity of the financial development argument does not seem to be established at the international level. Gruber and Kamin (2009) use a large panel of countries over the last decades and find little explanatory power of various financial development indicators on relative current accounts. Financial repression is seen as a potential cause of external imbalances in China (Johansson, 2012). There indeed seems to be more evidence about the influence of indicators broader than financial development: in an international panel setting, financial repression seems to be strongly related to current account surpluses as argued in Johansson and Wang (2012). Huang and Wang (2011) found that financial repression seemed to have positive effect on Chinese economic growth in the 1980s and 1990s. Interestingly, the effect turned negative in the 2000s.[4]

Since the seminal work of Hsieh and Klenow (2009), the issue of capital allocation in China has increasingly attracted attention. Brandt et al. (2012) measure the impact on aggregate TFP of distortions in factor allocation across provinces and sectors. They find that capital misallocation between the state and the non-state sector, rather than labor frictions, has been driving the recent increase in distortions. They considerably impact on aggregate TFP growth. A 2005 firm survey conducted by Dollar and Wei (2007) finds that private firms have difficulties to finance their working capital and rely more on retained earnings and informal channels. Interestingly, returns on capital increase with decreasing state share.[5]

Another branch of studies on capital allocation focuses on the relationship between provincial saving and investment. In their extensive analysis of capital mobility in China, Boyreau-Debray and Wei (2004) find that provinces with low capital productivity were the ones that experienced capital inflows between 1984 and 2001. As opposed to non-state and international investment, investment made through government budget and (state-owned) financial institutions seem to react negatively to an increase in the marginal productivity of capital. They suggest that the strongest determinant of capital allocation in China is the prominence of SOEs (state-owned enterprises) in local economies. Thus, it seems that the government systematically allocates capital away from more productive towards less productive regions. As shown in Chen and Yao (2011), there seems to be a crowding-out effect of government infrastructural investment on private consumption in Chinese regions. All in all, capital mobility in China is low and the degree of financial integration even seems to have decreased in the 1990s (Boyreau-Debray and Wei, 2004). In Li (2010), regional saving and investment have indeed been strongly related between 1978 and 2006. To our knowledge, it is one of the few papers where large discrepancies across provinces in external positions are noticed, albeit in a side remark. Xu (2008), Curtis and Mark (2010) and Ho et al. (2010) all find a low level of consumption risk sharing among Chinese provinces.

The capital allocation issue is obviously related to global imbalances. Gourinchas and Jeanne (2013) observe that developing countries whose productivity grew faster had capital outflows between 1980 and 2000. By introducing an investment and a saving wedge in an otherwise standard small open economy model, they identify saving wedges as the key driver of this pattern. The finding of Song et al. (2011) that regions with faster growth in private employment seem to

[4]In an environment of incomplete information, market failure and financial instability, keeping interest rate, credit allocation and capital account in check could well be welfare improving. In a later phase of development, the costs of distorted capital allocation, efficiency loss and insufficient access to international financial markets begin to rise (Huang and Wang, 2011).

[5]Collectively-owned firms are an exception: they have high returns in spite of large local government share. They argue that the fact that returns on capital are comparatively lower in West and South China suggests an efficiency loss, certainly due to inward investment channelling by authorities.

have larger external surpluses and higher GDP per capita growth rate suggests the existence of a similar pattern inside China. Dadush and Stancil (2011) argue that the capital flows conundrum could be explained by differences between industrialized and emerging markets pertaining to entry costs, real returns on investment, risk, availability of investment projects or assets and the predominant role of government in the accumulation of foreign reserves. Indeed, the Gourinchas and Jeanne puzzle is less marked once one accounts for public aid flows and reserve accumulation because current account typically is a poor proxy to adequately capture capital flows (Alfaro et al., 2011). In their study, private capital indeed seems to behave more in accordance with the standard model.

It has been suggested that Chinese current account data are not reliable. Large measurement errors driven by hidden capital inflows – underestimated returns on foreign investment and misreporting of exports/imports – are supected since the mid-2000s (Zhang, 2008). Against such a background, it should not come as a surprise that China is a natural suspect in explaining why the world has been running current account surpluses over the past few years (Economist, 2011).

An increasing number of studies using household sample surveys try to identify factors driving high savings in China. Aziz and Cui (2007) point to the progressive decline in household disposable income and their decreasing labor share in the economy. Chamon and Prasad (2010) rely on annual household survey data and find that savings rate increased in all demographic groups as a consequence of the *"breaking of the iron rice bowl"*.[6] In another study, Chamon et al. (2013) establish that the last decades saw an increase in income uncertainty and a decline in pension replacement rate.

These results contrast with household studies focusing exclusively on demographic factors. Life-cycle motives seem partly successful in explaining high household saving (e.g. Modigliani and Cao, 2004; Curtis et al., 2011). Interestingly, the life-cycle hypothesis does far better if one includes motivation to invest in housing (Chao et al., 2011). A strong effect of property prices on consumption spendings using city level data is found in Chen et al. (2011). Demographic gender structure seems to matter as well. The exogenous decline in fertility in the early 1970s enabled to estimate that urban households having had daughter at the time increased saving as they provide less elderly support (Banerjee et al., 2010). Du and Wei (2010) suggest that intensified competition in the marriage market due to the gender gap could be held responsible for up to half of recent current account imbalances.

If household saving have been under close scrutiny, corporate and government saving rate have contributed to imbalances as well (Ma and Yi, 2011; Yang et al., 2011). Kuijs (2006) points to the fact that corporate and government saving are indeed high relative to international standards. For Anderson (2009), the recent surge in Chinese current account in the 2000s is

[6] Shift of expenditures on education, housing and health care from state-owned enterprises to households during reforms.

linked to a massive capacity build-up of state-owned firms in resources and heavy industries (steel, chemicals, oil, construction), which took over domestic demand and even began to export.

The massive surpluses could be related to the trade and production structure of the Chinese economy. Jin (2012) shows that capital-intensity of a country's export and its production structure affect its demand for financial capital and determine the global allocation of capital.[7] Johansson and Wang (2011) suggest that the slow Chinese structural change from secondary to the tertiary sector and the large weight of industry and manufacturing in the Chinese economy may be a byproduct of financial repression. China's exports are part of a quite persistent chain of production: the importance of processing trade implies that the current account is expected to be robust against fast appreciation (Girardin and Owen, 2011).

Export-led growth policy is another potential explanation for the capital allocation puzzle. Aizenman and Lee (2010) propose a growth model based on exports supported by a mercantilist hoarding of reserves. The key element is the introduction of learning by doing externality in exports. The direct importance of exports in value-added terms seems to be modest on the national level (Anderson, 2007).[8] Still, the key potential role of technology transfers and positive externalities (e.g a higher competitivity in the domestic sector) are not considered in such figures. For some export-led growth cases in point like Japan, South Korea and China, Aizenman and Lee (2008) find evidence for financial mercantilism (i.e. financial repression and heavy state involvement in capital allocation) rather than monetary mercantilism (i.e. real exchange rate manipulation). In another study, using a large sample of emerging and developed economies, Aizenman and Lee (2007) show that exposure to potential financial crisis and sudden stops seems to be more decisive than exchange rate manipulation for explaining reserve accumulation. It somewhat rationalizes the strategy of systematic hoarding of foreign reserves adopted by the PBC[9] since the 1997 Asian Financial Crisis.

At last, the quality of institutions and the legal system could potentially play an important role too. Aguiar and Amador (2011) show that external surpluses and asset accumulation can be obtained as political economy outcome of a limited commitment toward non expropriation of international investment position and competition for political power.

[7] It depends on the trade-off between the composition effect (capital flows towards economies becoming more specialized in capital-intensive goods) and the standard convergence effect. As the real capital intensity of Chinese exports is low, savings are invested abroad in more capital-intensive economies.

[8] Using total exports to GDP ratio as an indicator is misleading. Anderson suggests that one has to substract imports and convert that domestic content share into value-added terms by subtracting input purchases from other domestic sectors. At around 10% of GDP in 2006, the value added from exports in China was half that of Taiwan or Thailand and only slightly higher than India's.

[9] People's Bank of China.

3.2 Model framework and data

3.2.1 Model set-up

The baseline version of the small open economy model used in this paper is similar to the one developed in Gourinchas and Jeanne (2013). Time is discrete and there is no uncertainty. A single homogeneous good is produced.

The production function is of the Cobb-Douglas type:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

Factor markets are competitive. The aggregate BC of the economy is

$$Y_t = C_t + I_t + R^* D_t - D_{t+1}$$

where R^* is the world gross interest rate and D the external debt.

Capital inflows (i.e. an increase in debt) correspond to the gap between investment and saving:

$$\begin{aligned} D_{t+1} - D_t &= C_t + I_t + R^* D_t - Y_t - D_t \\ &= I_t - \underbrace{(Y_t - C_t - D_t(R^* - 1))}_{S_t} \end{aligned}$$

The dynamics of capital over time is

$$I_t = K_{t+1} - (1-\delta)K_t$$

Gourinchas and Jeanne introduce an investment or capital wedge (τ_k) that impacts on gross return (R_t):

$$(1-\tau_k)R_t = R^*$$

The marginal product of capital net of depreciation is

$$R_t = \alpha(k_t/A_t)^{\alpha-1} + 1 - \delta$$

where k is capital per capita. Plug the former into the latter expression to find the capital stock per efficient unit of labor:

$$\tilde{k}_t = \frac{K}{AN} = \left(\frac{\alpha}{\frac{R^*}{1-\tau_k} + \delta - 1}\right)^{\frac{1}{1-\alpha}} = \tilde{k}^*$$

Countries have an exogenous productivity path bounded from above by the world productivity frontier, which grows at rate g^*:

$$A_t \leq A_t^* = A_0^* g^{*t}$$

For a finite period of time, a country's TFP could grow faster than world's TFP. The evolution over time of domestic relative to world productivity is captured by the technology catch-up parameter (i.e. a positive π means that a country catches up relative to the world):

$$\pi_t = \frac{A_t}{A_0 g^{*t}} - 1$$

Representative households maximize a *CRRA* utility function:

$$\begin{aligned} U_t &= \sum_{s=0}^{\infty} \beta^s N_{t+s} u(c_{t+s}) \\ u(c_t) &= \frac{c_t^{1-\gamma}}{1-\gamma} \end{aligned}$$

subject to the following budget constraint

$$N_t w_t + N_t z_t = C_t + K_{t+1} - (1-\tau_s) R^* K_t - D_{t+1} + (1-\tau_s) R^* D_t$$

Wages (w) are equal to the marginal product of labor. A saving wedge (τ_s) is introduced at the household level. It can be interpreted as a tax on capital income. Revenues generated by the wedges ($z_t = \tau_k R_t k_t + \tau_s R^*(k_t - d_t)$) are redistributed in a lump-sum fashion.

The Euler equation is

$$\begin{aligned} c_t^{-\gamma} &= \beta R^* (1-\tau_s) c_{t+1}^{-\gamma} \\ &= \beta (1-\tau_k) R_t (1-\tau_s) c_{t+1}^{-\gamma} \end{aligned}$$

It is assumed that the rest of the world is composed of steady-state advanced economies with the same preferences and no saving wedge:

$$R^* = \frac{g^{*\gamma}}{\beta}$$

3.2.2 General remarks

When not mentioned otherwise, data used in this chapter are from the *National Statistical Yearbooks* of the People's Republic of China and from the *Provincial Statistical Yearbooks* of the 22 provinces, 5 autonomous regions and 4 municipalities of Mainland China.[10] The *China Data*

[10] The autonomous regions are Tibet, Xinjiang, Guangxi, Inner Mongolia and Ningxia. The cities of Beijing, Tianjin, Shanghai as well as the region of Chongqing are municipalities. Thereafter, the term province will be used as general qualifier.

Center (CDC) of the University of Michigan provides electronic access to the yearbooks and made main statistics conveniently available.[11] For most provinces, our online access only covers regional statistical yearbooks in the 1990s and 2000s. Thus, it happens that the data are sometimes incomplete. We will primarily rely on data directly retrieved from recent online yearbooks and complete possible gaps with CDC sheets. This allows us to take account of revisions as much as possible.

The quality of provincial and aggregate Chinese *National Accounts* data is an important issue that we explore in the second chapter of the thesis, where we focus on some stylized facts and discuss the quality and aggregation properties of the data. This analysis revealed large discrepancies between aggregate statistics and the sum of provincial statistics. For example, the sum of province-level GDPs was about 11 percent higher than the officially published national value in 2010. The bulk of this large error stems from an excess of regional over national investment, which has been widening since the mid-1990s. Conversely, the discrepancy between cumulated provincial saving and national saving shows no clear trend over time. Still, the sum of province-level saving overestimated national values by around 7 percent of China's GDP in 2010. All things considered, it suggests that, since the mid-2000s, the sum of province-level net exports will generally be lower than the corresponding official aggregate statistics. Other authors have argued that China's current account surplus is overstated for a variety of reasons (see Zhang, 2008). Whether regional data are more affected than national ones is an open question (e.g. the *2004 Economic Census* validated provincial GDP data and invalidated national ones (Holz, 2008)).

But while there is considerable uncertainty concerning the levels of aggregate and regional statistics, our exploratory analysis also shows that that the sum of province-level aggregates is generally highly correlated with movements in national statistics. Even though yearly level data are noisy, our empirical analysis focuses on cross-sectional patterns over three decades of economic reforms. For that reason, we are reasonably confident that our province-level data capture important aspects of long run external balances in China's regions. Tibet is excluded for data availability reasons. Information on the computation of capital stock (Section A.1.2.1), productivity (Section A.1.2.2) and capital flows (Section A.1.2.3) is available in appendix.

3.3 Regional investment and saving wedges

3.3.1 Productivity catch-up

Regional abbreviations used in the graphics as well as employment growth rates and productivity catch-up parameters are available in Table 3.1 (main results). Regional TFP growth rates are in

[11] `http://chinadataonline.org/`. The CDC reports data values as soon as they are published in the corresponding yearbook. Although data have sometimes been subject to official revisions in later years, the CDC did not systematically adapt past values.

Table 3.6 (detailed results). The geographical distribution of catch-up parameters is best appreciated using a map where provinces are classified per quartile (Figure 3.1). Catch-up values range from -0.45 to 0.86. We observe a high heterogeneity in TFP with 18 regions having a negative value and 12 a positive one.

As expected, more open coastal regions – where preferential policies were implemented first – generally have a positive catch-up parameter relative to China's productivity growth.[12] In the *Pearl River Delta*, Guangdong is known to have benefited from foreign investments and a shift of productive capacities from Hong-Kong and Macau. It has the lead with a value of 0.86. Bordering Taiwan, Fujian recorded a high growth of productivity over the sample period as well (0.34). Likewise, the regions of the *Yangtze Delta* (e.g. Jiangsu (0.51) and Zhejiang (0.17)) have been catching up with the exception of a slight decrease for Shanghai (-0.17). The last cluster of emerging provinces locates around the *Bohai Sea*: Tianjin, Shandong and Hebei have values ranging between 0.12 and 0.34. Puzzlingly, Beijing has been falling behind (-0.26), possibly due to the presence of the non-productive state sector.

The northeastern "Chinese Rustbelt" (Manchuria), although known as being still relatively wealthy, benefited less from economic reforms with values between -0.30 and -0.08. In the South, with the exception of Chongqing (0.05), provinces fell behind relative to national productivity growth although already counting among the poorest regions at the time reforms started (e.g. Guizhou -0.45, Yunnan -0.26). Among western provinces, Xinjiang (0.06), Shaanxi (0.14) and Inner Mongolia (0.34) managed to improve their relative position while the rest experienced a deterioration (from -0.17 in Ningxia to -0.44 in Qinghai). In Central China the situation is heterogeneous as well with some provinces being roughly neutral (e.g. Henan (0.01) and Jiangxi (-0.04)) and others having negative value (Anhui (-0.33), Hubei (-0.26) and Hunan (-0.37)).

At this point, it is worth mentioning that all regions massively caught up compared to world TFP. To put it differently, the convergence of Chinese productivity to the world frontier is characterized by a concomitant internal divergence in regional TFP.

3.3.2 China's internal capital allocation puzzle

In Figure 3.2, we plot the catch-up parameters discussed in the preceding section and in appendix (A.1.2.2) against the final relative changes in capital flows (Section A.1.2.3). A clear pattern emerges: provinces that caught up relative to national TFP had capital outflows while those that fell behind had capital inflows. The relationship is highly significant. Thus, there seems to be a *capital allocation puzzle* at the regional level inside China. Remarkably Gourinchas and Jeanne found it to be a case in point of the puzzle: China happened to locate right on the international regression line (i.e. in the southeast quadrant). We suspect that this "Russian Doll-like" pattern could well repeat itself at lower administrative level.

[12]We discuss the choice of the reference TFP growth rate in Sections A.1.2.2 and A.1.3.1.

Starting from that empirical fact, this chapter focuses on the following set of issues:

1. What drives the internal *capital allocation puzzle*? Is it investment-driven, saving-driven or a combination of both? (rest of Section 3.3)

2. Are our estimated long run wedges related to usual suspects proposed by the literature on global imbalances? Is the regional cross-sectional variability of the wedges useful in shedding light on the general patterns of capital flows? (Section 3.4)

3. To which extent are the general results robust to alternative data (Section 3.5), parameter assumptions (Section A.1.3) and model extensions (Section A.1.4)?

3.3.3 The Investment Puzzle

Gourinchas and Jeanne introduce an investment wedge on gross return defined as

$$(1-\tau_k)R_t = R^*$$

where R corresponds to the marginal product of capital net of depreciation and $R^* = g^{*\gamma}/\beta$ by assumption.
The steady-state capital stock per efficiency unit of labor is

$$\widetilde{k}^* = \left(\frac{\alpha}{\frac{R^*}{(1-\tau_k)}+\delta-1}\right)^{\frac{1}{1-\alpha}}$$

Thus, as they assume common parameters among provinces, differences in $\widetilde{k}^*$ exclusively arise from different τ_ks. To identify the wedges, they propose a decomposition of average investment over GDP of the following form (see detailed derivation in Section A.2.1):

$$i \quad = \quad \underbrace{\frac{1}{T}\frac{\tilde{k}^*-\tilde{k}_0}{\tilde{k}_0^{\alpha}}}_{convergence} + \underbrace{g^*\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)}}_{catch-up} + \underbrace{(g^*n+\delta-1)\tilde{k}^{*(1-\alpha)}}_{trend}$$

Investment is deflated using the same index as for gross fixed capital formation (GFCF): we use the consumer prices index (CPI) and the price of investment in fixed assets (PIFA) as soon as available. GDP is deflated using CPI. By implementing a grid-search, the $\widetilde{k}^*$s (and related τ_ks) needed to match observed is are identified.[13] From the decomposition formula, one can easily see that their methodology implies that provinces with high relative investment are attributed a high capital per efficient unit of labor.[14] Differences in $\widetilde{k}^*$ drive most of the variations in the three channels and determine investment wedges. Note that a higher $\widetilde{k}^*$ implies a lower τ_k:

[13] As in the original paper, we assume $\gamma = 1$, $\delta = 0.06$, $\alpha = 0.30$ and $\beta = 0.96$. Other variables are estimated from the data.

[14] For example, the western region of Xinjiang with high investment rate has a bigger $\widetilde{k}^*$ than the more developed Guangdong (5.3 vs 2.1).

$$\tau_k = 1 - \frac{R^*}{\frac{\alpha}{\tilde{k}^{*1-\alpha}} - \delta + 1}$$

We are primarily interested in the wedge estimates rather than in the respective channels of investment (see the following footnote for a discussion of the decomposition).[15] Thereafter, we use the qualifier of implicit or wedge-adjusted returns as $(1-\tau_k)R_t$ by holding R constant for convenience. For example, a province with a highly negative wedge is said to have high implicit return and high investment relative to the model. In fact, it is an abuse of language: the wedge-adjusted returns should always correspond to the reference return R^* because perfect capital mobility is assumed. Thus, in the preceding example, the negative wedge means that this region has a lower ex-ante home return R and a friction makes it higher in order for it to correspond to R^*. Either way, it can be interpreted as an investment subsidy.

Results of average investment rates and investment wedges are in Table 3.1 (main results). See Table 3.6 (detailed results) for information on capital stock and the channels. Investment wedges are mostly negative and range from -9.10 to 0.15%. In Figure 3.5, we provide a map of the wedges to make the discussion more convenient. The geographic distribution follows a clear pattern: the West and the Metropolises have the highest implicit rates of return (more negative wedges or, to put it differently, the lowest ex-ante returns) while the Center and the East Coast have lower returns (less negative wedges or higher ex-ante returns). In fact, the spatial distribution resembles the one of average investment over GDP.[16]

In Figure 3.3, there is a noisy – but nevertheless positive – relationship between investment wedges and productivity catch-up: provinces with higher productivity growth have lower absolute distortions (less negative wedges), lower implicit returns and lower investment. In other words, regions growing faster implicitly subsidize less gross returns on capital. Thus, there seems to be an investment puzzle at the regional level in China (i.e. no negative relationship between investment wedges and productivity).[17]

At first sight, our results are somewhat counterintuitive. In Gourinchas and Jeanne, countries with negative catch-up parameter had lower average investment rate than richer countries. Thus,

[15] The convergence component is the initial investment necessary to reach the steady-state capital stock starting from the initial capital in efficiency units. Its distribution is broadly similar to the one of i (and $\tilde{k}^*$). On average, it accounts for a little more than one fifth of average investment. For its part, the catch-up or productivity channel is the investment required by falling behind or catching up compared to reference TFP. It closely follows the distribution of the productivity catch-up parameters: provinces that have been lagging behind have large negative values while we note a positive contribution in regions with high technology growth. At last, the trend channel captures the amount of investment needed to compensate for capital depreciation. It captures the bulk of i (around 80% on average) and is highly related to the spatial distribution of relative investment.

[16] Interestingly, the level of investment over GDP (i) is not related to productivity (π) *per se* (correlation of 0.01 vs 0.40 for the wedges). This is an important point: our approach seems to convey different information than raw investment rate data. As a results, it may enable us to identify the frictions that drive investment patterns away from the small open economy model.

[17] The positive slope coefficient of Figure 3.3 is significant at the 6% confidence level using jackknife standard errors. Note that it loses significancy once Guangdong (*GD*, East Coast) is excluded from the sample. Excluding the second outlier as well (Hunan, *HA*) would preserve significancy at the 5% level.

they were attributed a low $\widetilde{k}^*$ and a high τ_k, meaning that their implicit return was lower (i.e. their ex-ante return on domestic capital needed to be higher than the world interest rate). In our case, we find the opposite pattern. Some relatively poor provinces experienced high investment rates over the period. They were therefore attributed a high $\widetilde{k}^*$ and a low (more negative) τ_k.

In Table 3.2, we aggregate the frictions of the 30 regions into 6 larger areas. It confirms the broad geographical pattern that we identified: City-Provinces (Metropolises) and the West subsidize investment more while the East Coast and Manchuria have less negative values. We discuss the possible determinants of investment frictions in Section 3.4.1.

This result is a first blow to the baseline neoclassical framework and stands in sharp contrast with Gourinchas and Jeanne where investment wedges were negatively related to development in productivity, following the intuitive mechanism that countries with less frictions – à savoir less positive wedges and lower implicit taxes – catch up in terms of TFP. The positive relationship that we observe should attenuate the positive correlation of the catch-up parameter and capital inflows predicted by the neoclassical model and thus make the *capital allocation puzzle* less stringent.[18]

At first blush, one would naturally suspect the strongly negative investment wedge value of some regions (e.g. Xinjiang, Qinghai, Ningxia or Inner Mongolia) to be a consequence of *Xibu Kaifa* ("Develop the Great West"). It consists in massive investment programs in infrastructures (mainly transportation, natural resources extraction and power generating facilities).[19] Indeed, Brandt et al. (2012) found an increasing infrastructure share of capital stock in hinterland regions. According to them, the bulk of these investments ended up in (less productive) state-related enterprises. They argue that, even accounting for infrastructure investment, the increasing misallocation of capital made already high initial differences in TFP worse. Therefore, Brandt et al. convincingly made the point that *Xibu Kaifa* was the key driver of the increase in productivity distortions since the mid-1990s.

On the one side, this may explain why we find that regions that lose ground in terms of productivity seem to enjoy higher investment subsidies. On the other side, they find that internal capital misallocation (state vs non-state) – rather than the interregional one – contributed to the recent increase in distortions. As a matter of fact, they argue that interregional frictions were considerable but constant between 1985 and 2007, the lion's share originating in persistent labor rather than capital misallocation. That may explain why the cross-regional correlation of investment frictions and productivity is not that salient. Furthermore, investment and capital intensity of these economies could already have been high before *Xibu Kaifa* due to the strong presence of the state.[20] The large and negative investment wedge value for Beijing, Tianjin and Shanghai is

[18] We find that regions with high productivity tend to have higher investment wedges. Thus, they have lower investment rate, higher net exports and lower capital inflows (or more outflows).

[19] Officially launched in 2000, it actually encompasses the southern regions as well. In fact, all regions except the East Coast will be concerned: the CCP intended to follow a similar strategy for Manchuria (*Northeast Area Revitalization Plan*) and the Center (*Rise of Central China Plan*).

[20] In our subsample estimations of Section 3.5.1, we see that the positive correlation is strongest for the early period (1984-1997), before the "Develop the Great West" policy was implemented.

less surprising: they are geographically small, highly dynamic urban areas.

3.3.4 The Saving Puzzle

Gourinchas and Jeanne (2013) decomposed relative capital flows into four channels.[21] A step by step explanation of their methodology is available in Section A.2.2. We implement a grid-search procedure to identify the saving wedges required by the model to match the empirical capital flows (i.e. so that the sum of the four channels corresponds to cumulated relative flows data).[22]

Results of the capital flows decomposition are available in Table 3.6 (detailed results). As for investment wedges, we focus on frictions rather than discussing the importance of the channels.[23] Capital flows and saving wedges are available in Table 3.1 (main table). They range from -3.50 (Guangdong) to 2.58% (Guizhou). By focusing on the geographical distribution of the identified saving wedges, an obvious pattern emerges (map in Figure 3.6): from Tianjin to Guangdong, the entire coastal area has a highly negative saving wedge. Paradoxically, most of these regions have been catching up compared to China and the rest of the world. Thus, households should have been borrowing to raise their consumption. The negative net exports resulting from lower saving would then be interpreted as capital inflows. However, it is not the case: these provinces have massive capital outflows. In order to make the model consistent with data, a large implicit saving

[21] The convergence term captures the amount of capital necessary to reach the steady-state capital per efficiency unit of labor:

$$\frac{\Delta D^c}{Y_0} = \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{y}_0}(ng^*)^T$$

External borrowing needed to finance domestic investment is captured by the investment channel (i.e. a region that is catching up necessitates more capital inflows):

$$\frac{\Delta D^i}{Y_0} = \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T \pi$$

The next term gathers the cumulated debt inflows required to hold the relative debt ratio constant (trend growth):

$$\frac{\Delta D^t}{Y_0} = \frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \psi(\tau_s)\,[ng^*\phi(\tau_s)]^T\,\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}$$

At last, the saving term captures the intertemporal consumption decision of households (given a positive catch-up parameter, they will borrow on international markets to raise consumption):

$$\frac{\Delta D^s}{Y_0} = \frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0}\,\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T \sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t \left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right]$$

[22] As in Gourinchas and Jeanne, we assume log utility ($\gamma = 1$). Section A.1.3.2 investigates the impact of alternative CRRA values. The $\tilde{k}^*$ identified in the investment wedge computation are used. Furthermore, $\pi = \pi_T$ and a linear convergence to the steady-state catch-up is assumed ($\pi_t = f(t)\pi$ with $f(t) = min(\frac{t}{T}, 1) \leq 1$).

[23] The convergence and investment components remain similar independently of the type of flows used as they do not depend on the saving wedge (given the same assumed $g^*$and thus identical π and R^*). The convergence channel is correlated with steady-state capital stock (and thus average investment and investment wedges). The investment channel is strongly related to the distribution of catch-up parameters. The distribution of the trend channel closely follows initial external positions. With the convergence channel, the saving channel accounts for a large chunk of capital flows. It ends up driving most differences in the patterns of flows and is highly correlated with the final distribution of saving wedges.

subsidy (i.e. a negative saving wedge) is needed.

Some resource-abundant western provinces (e.g. Inner Mongolia, Xinjiang and Shaanxi) have a negative saving wedge as well. Central China, Manchuria, southern provinces as well as some western regions have roughly neutral or positive saving wedges (i.e. they tax savings). In Table 3.2, we provide results for average distortions over regional clusters. The East Coast provinces have the lowest saving wedges (GDP-weighted average of -2.73%), followed by the Metropolises (-0.93%). The Center and Manchuria have a low positive value. Savings in the West are implicitly subsidized on average (-0.85%) but there is no homogeneous geographical pattern. For instance, Qinghai has a positive value while Shaanxi is found to be clearly negative. The South has the highest saving wedges on average (0.49%), with Chongqing being the only region subsidizing savings.

In Figure 3.4, as in Gourinchas and Jeanne, the *capital allocation puzzle* manifests itself through a highly significant negative relationship between productivity catch-up and saving wedges. Provinces that have been catching up are the ones that implicitly subsidize savings more, causing a saving glut that translates into capital outflows. There seems to be a saving puzzle at the regional level in China. Standard theory predicts that provinces with high productivity should experience capital inflows (lower saving and positive saving wedge). Thus, all but four provinces are in the "wrong" quadrant.[24] The relationship between both variables is as marked as in Gourinchas and Jeanne, where China figured close to the international regression line in the southeast quadrant. The identified frictions are more correlated with productivity than cumulated capital flows (-0.94 vs -0.63). It makes us confident that these frictions are more useful – or at least convey different information – in explaining deviations from the baseline neoclassical model than raw capital flows data. We have a significant negative constant while Gourinchas and Jeanne had a near zero one. This is only a byproduct of our parametrization.

3.4 Beyond the wedges

We established the existence of an investment and a saving puzzle inside China. This *capital allocation puzzle* is reminiscent of the patterns found at the international level in Gourinchas and Jeanne. Importantly, investment wedges alone are not sufficient to generate the negative relationship between capital flows and productivity of Figure 3.2. As in the original paper, using our empirical estimates of investment wedges and switching off saving wedges – assuming them to be zero – leads to predicted capital flows being strongly positively correlated with TFP. Thus, as on the international level, saving frictions are the main driver of the puzzle.

In this part, we intend to investigate whether variables mentioned in the literature (Section 3.1.1) are related to the identified frictions. To start with, by regressing the wedges on a large

[24] Jilin (Manchuria), Jiangxi (Center), Shanxi (West) and Shanghai (Metropolise) have a negative catch-up and saving wedge. They are thus compatible with the prediction of the standard model that falling behind in terms of productivity implies capital outflows (higher saving).

number of "usual suspects", we aim at testing to which extent our estimates convey useful information. Moreover, their (limited) cross-sectional long run variability could help us identifying important explanatory variables. In a first step, we discuss univariate regressions of the wedges. Next, we allow for one additional control to at least partly alleviate the omitted variable issue (the level of economic development proxied by real GDP per capita). In a third step, we account for the variability of the wedges using combinations of selected factors. At last, we estimate saving wedges for three subsamples and construct a panel of frictions. A list of the available provincial characteristics is in appendix (Section A.1.1). At this point, it should be emphasized that this is only an exploratory, mainly descriptive step towards better understanding external imbalances inside China. The limitations of our approach are manifold.[25]

3.4.1 Potential explanatory factors of investment wedges

Before starting to explore how provincial characteristics correlate with the wedges, one may wonder how the frictions are linked to the level of economic development (i.e. the average of real GDP per capita relative to national value).[26] It is *per se* not significantly correlated with the wedges but once its squared value is added, its coefficients turn out to be highly significant. The relationship is concave: economic development initially increasingly makes a dent in investment but after a certain level (roughly 100% of national value), the relationship turns increasingly negative and investment is more and more subsidized relative to the baseline model.

Coefficients of potential explanatory factors of investment wedge normalized by their cross-sectional mean and jaccknife p-values are available in Table 3.3.[27] A summary of the factors is available in appendix in Section A.1.1. The ownership structure of investment in fixed assets (*SOInvFA*) seems to be highly negatively correlated with investment frictions. A larger share of state-owned investment in fixed assets has a strong negative effect on the wedges. The presence of the state thus seems to foster investment. The effect becomes larger once one controls for development level. In Figure 3.7, we provide a scatter plot of the negative relationship between *SOInvFA* and τ_k. On the graph, it is obvious that Beijing, Tianjin and Shanghai are outliers

[25] 1.The model only delivers one long run wedge for each province over the entire sample period. Subsamples estimates allow us to construct a panel but the noisiness of data impairs the estimation quality of short time-length and the model's framework makes less sense. 2.Most factors used on the right-hand-side are not available over the entire period. What is more, changes in definition or recording methodology are observed over time. We thus focus on the 1997-2009 period for all factors (2010 is the steady-state and does not enter in the investment and capital flows indicator). 3.Our measure of flow is a rough approximation of current account using only goods and services. 4.The conceptual mapping from frictions to wedges is not straightforward and is a convolution of many frictions. 5.What makes sense on the microeconomic level does not necessarily mean that the cross-provincial variability is informative. 6.We ignore endogeneity issues but control for economic development to at least alleviate the omitted variable issue. 7.Expectations are ignored as we develop no structural model to justify the impact of explanatory factors on the wedges.

[26] We construct population data from multiple censuses and sample surveys available in regional and national yearbooks. See the second chapter for more.

[27] We choose the HC3 heteroskedasticity correction for two reasons. First, in our sample, compared to other estimators of the variance-covariance matrix of the residuals, inference is more conservative. Second, it is a close approximation of the jackknife standard errors, a concept that is particularly appropriate in our framework.

(southwest quadrant): they seem to subsidize investment more although their share of state-owned firms is below average. The foreign and residual shares (*FOInvFA* and *REInvFA*) have the opposite effect.[28]

This pattern repeats itself once one looks at a broader indicator of the presence of the state such as the state-owned gross industrial output value share (*SOGIOV*). Interestingly, more marketized regions (*Market*) seem to have higher investment wedges (lower investment) even when controlling for differences in economic development. The presence of the state in the construction sector gross output value (*SOCGOV*) tends to magnify investment subsidies.

Indicators summarizing the economic structure of the regional economies seem to be related to frictions. The following factors are negative and highly significant: the share of the construction and tertiary sector relative to GDP (*SectorConst* and *SectorTert*), our indicator for sectoral economic concentration (*StructConc*) and the share of coal and oil extraction relative to GDP (*CoalOil*). All promote investment relative to the neoclassical model. In a similar fashion to private firms, the importance of the industrial sector relative to GDP (*SectorInd*) has a positive coefficient.

Indicators capturing the extent to which regions are integrated into the world economy do not seem to be systematically related to investment frictions. All have a positive sign. One of our indicators of financial development (*Loans*) is highly negatively correlated with the wedges. One may come to the conclusion that financially more developed regions seem to have higher investment (more negative investment wedge). However, interpreting this indicator as a proxy for financial development may be misleading. In our opinion, it primarily captures the presence of state-owned banks, which rationalizes the negative coefficient.[29]

In Table 3.7, more explanatory factors are available. Human capital indicators related to education (*TertiaryEduc*, *HighEduc*) or innovation (*Patents*) seem to foster investment, as is a higher social security coverage (*SOCSEC*). Most of these results are driven by Metropolises. Some demographic characteristics seem to matter for investment wedges: regions with high sex ratio imbalances (*SexRatio*) and high share of ethnic minorities (*EthnicShare*) have more negative wedges.

Even though the cross-sectional variability is limited, we try to include many variables in the same regression. In Table 3.4, we pick six factors that we consider may play a key role in

[28]Collective-owned enterprises seem to exhibit the same patterns as private firms. In the early reform period, de facto private firms were still registered as collective ones to benefit from various privileges (Brandt and Rawski, 2008, chapter 1). Consequently, we merge both categories (*REInvFA*). The collective share of total investment is negligible anyway.

[29]The negative coefficient could reflect credit policies implemented by SOBs. In fact, this finding is not surprising: a substantial share of aggregate deposits and loans in China figures on big state banks' balance sheets. Even smaller banks are mostly close to local governments. Banks are thus largely owned by the state and closely work hand in hand with local authorities and SOEs. Thus, a high loans over GDP ratio could rather be suggestive of a strong government-led investment policy that biases investment frictions in the region downward rather than broad financial development. The fact that loans – rather than deposits – are particularly strongly negatively associated with investment wedges gives weight to that argument.

investment frictions, one for each broad category of variables. The presence of the state in gross fixed capital formation (*SOInvFA*), the importance of loans in financial insitutions (*Loans*) and the size of the construction sector (*SectorConst*) are all strongly negatively related to the wedges and bias investment upward. These variables are mostly robust to the inclusion of other factors in the regression. In conclusion, it seems that the presence of state-owned firms and banks is strongly associated with investment frictions.

3.4.2 Potential explanatory factors of saving wedges

We follow the same exposition strategy as in the preceding section. Table 3.3 summarizes the effect of the factors on saving wedges. Economic development is associated with the wedges in a convex way: initially, saving frictions are increasingly negative as regions get more developed (i.e. they subsidize saving more and more). Then, at around 130% of national GDP, frictions start to increase and regions decrease implicit saving subsidies.

The independent variables are identical as in the preceding section and are described in appendix (Section A.1.1). Indicators related to investment ownership are correlated with saving frictions: the presence of the state (*SOInvFA*) seems to push up the wedges (lower saving) while private and collective firms (*REInvFA*) have the opposite effect. Foreign firms (*FOInvFA*) are not robust to the inclusion of development.

We expect broader indicators of the importance of the private economy to be highly correlated with saving frictions. It seems to be the case: state-owned firms in the industrial (*SOGIOV*) and construction sector (*SOCGOV*) make a dent in saving while more marketized regions (*Market*) seem to promote saving. The coefficient on private and self-employed employment share (*Empl Private*) has no stable sign over specifications. In Figure 3.8, we provide a scatter plot that illustrates the clear positive relationship between the presence of the state (*SOGIOV*) and saving frictions. On the graphic, the western provinces of Shaanxi and Xinjiang seem to be outliers: they considerably subsidize saving in spite of having a high share of state-owned firms.[30]

In the economic structure category, the importance of the industrial sector over GDP (*Sector Ind*) is strongly negatively related to saving frictions. This is consistent with the idea that industry-intensive provinces tend to export physical capital to the rest of the world and other regions. The tertiary sector (*SectorTert*) has an opposite effect. Higher housing price growth (*HousingPrice*) magnifies savings but is not robust.

All indicators of the integration of the economies with the world have a negative coefficient. More open regions in terms of international trade (*Openness*), provinces with higher share of multinational firms in international exports (*MNE*) or a bigger share of FDI compared to GDP (*FDI*) are all implicitly subsidizing saving. However, only *MNE* is robust to the inclusion of

[30] This result is possibly due to poor data. In Section 3.5.4, we find that they may have higher saving wedge than official data suggest.

development. We provide a graphical representation of the strong negative association between the presence of multinational firms (*MNE*) and saving frictions in Figure 3.9. Again, a few western provinces stand out as they have lower share of international firms than average but still negative saving wedge.

Financial development (*Deposits* and *Loans*) is positively related to the wedges (i.e. it lowers saving). Note that interpreting loans as an indicator for the presence of the state would lead to results consistent with earlier findings.

In Table 3.7, human capital indicators correlate positively with saving wedges but the relationship is mostly not significant. Interestingly, once one controls for economic development, social security coverage (*SOCSEC*) seems to reduce saving while its coefficient has an opposite sign as standalone. Of the demographic factors, only sex ratio (*SexRatio*) and urbanization rate (*UrbRate*) are significant. If anything, regions with more unbalanced male to female ratio and more urbanized regions seem to have lower saving relative to the neoclassical model.[31]

From results in Table 3.4, where we regress saving frictions on key factors summarizing regional characteristics, we observe that the presence of state-owned firms in the industrial sector (*SOGIOV*), the importance of the industrial sector (*SectorInd*) and the share of international firms in exports (*MNE*) seem to be particularly robust. Their coefficient is smaller than in the preceding specifications but of the same sign. In this and later regressions, we do not include human capital indicators, social security and other demographic factors (apart from urbanization).[32]

Saving frictions are the driver of the internal *capital allocation puzzle* in China. Making them time-varying could give us more power to disentangle the importance of the respective factors discussed so far. We estimate the frictions for three subsamples (1984-1992, 1993-2001 and 2002-2010). Unfortunately, there are only a few variables for which we have reliable results for all subsamples, which restricts the set of possible regressors. In Table 3.5, we provide a panel regression with six factors that are deemed representative of regional characteristics in a given province. We use two alternative variance-covariance matrices.[33]

As in preceding tables, the share of state-owned firms in the industry (*SOGIOV*) is strongly positively related to frictions. The share of private employment (*EmplPrivate*) has the opposite

[31]Wei and Zhang (2009) find a positive effect of sex imbalances on saving at the provincial level. However, there are many differences with respect to sample (theirs is 1980-2007), estimation (they use panel data and other controls) and definition of saving (they consider $Y-C$ while our net exports are $Y-C-G-I$) that make direct comparison hazardous.

[32]First, human capital indicators did not seem to be robust to the inclusion of other factors. Second, the literature provides no clear guidance regarding their potential effect. At last, there are highly endogenous. As for social security, its introduction and extension was politically-driven and started in targeted urban areas. The self-selection issue is too obvious. Moreover, we did not find comparable provincial data for earlier periods. While we do not deny that demographic factors could play an important role at the household level, our investigations led us to think that the cross-section of macroeconomic regional data is not very helpful in testing them.

[33]We provide heteroskedasticity and autocorrelation robust variance-covariance matrix estimates (Arellano) and clustered estimates on the regional level (Liang and Zeger). The Arellano standard errors are very similar to the uncorrected standard errors. In our sample, inference sometimes changes considerably by using clustered standard errors.

effect: it acts like a saving subsidy.[34] The available balance of funds in banks and financial institutions (the difference between deposits and loans normalized by provincial GDP) is not significant.[35] The size of the industrial sector (*SectorInd*) is another robust driver of saving frictions. Openness *per se* does not seem to have an impact on frictions but FDI does: regions that receive a higher share of FDI relative to their output (FDI) tend to promote saving more. Obviously, our results could be spurious (i.e. driven by the fact that there is a clear downward trend in saving wedges over the three time periods). We run the same specification but introduce time fixed-effects. The share of industry and FDI are still highly significant while the coefficients on the private/state sector become insignificant.[36]

3.4.3 Private and state net exports

If, as suggested by our former analysis, the presence of state-owned and international firms is essential in explaining the *capital allocation puzzle*, a decomposition of net exports for different regions should be informative. Indeed, there is a rich cross-regional variation in the level of state's presence and international integration. By using data on the composition of gross fixed asset investment and the share of state output from Brandt et al. (2012), we separate net exports into state and non-state component for the 1997-2012 period and normalize them by regional output.[37]

In Figure 3.10 (above), we see that more marketized regions with a strong presence of private and international firms (i.e. the East Coast and the City-Provinces) have a large non-state saving surplus while other regions have balanced net exports on average. From that picture, we infer that non-state net exports have either been neutral or positive in most regions and thus partly responsible for the large capital outflows observed at the national level. In fact, households and firms in high productivity East Coast regions should have imported capital and hinterland regions exported capital. Once more, this table tells us the opposite story.

But this is only one side of the *capital allocation puzzle* coin. From preceding sections, we know that many hinterland regions have experienced large capital inflows and implicitly tax

[34] The literature (notably Song et al. (2011)) has emphasized that the major distinction between private and state-owned firms is that the former are financially repressed whereas the latter have preferential access to bank credit. Our results are compatible with that hypothesis: the expanding private sector can only finance its growth from retained earnings. This may explain the negative coefficient.

[35] Another implication of Song et al. (2011) is that during the transition process, state firms shrink in favor of private enterprises. As the former's economic importance dwindles and investment opportunities dry out, regions with faster growing private sector have an increasing surplus of deposits compared to loans as the largely state-owned financial sector does not redirect funds to the emerging private sector.

[36] It is not surprising: the massive decrease in the share of state-owned firms in industry and the growing private employment share are typical stylized facts of the Chinese economy that have been observed in all regions. With such a low number of time periods (three), removing the trend inevitably makes these variables less informative.

[37] We use data on investment in fixed assets by ownership and attribute the state- and collective-owned shares to the state. The rest is the non-state sector. The nominal (broad) state output share of Brandt et al. (2012) (non-agricultural sector) is applied to our GDP data assuming the 2000-2007 decline rate to be constant for 2008-2012. State net exports are constructed as $NX^S = Y^S - G - I^S$ and non-state net exports as $NX^P = Y^P - C - I^P$. All data are nominal.

savings, while they have been falling behind in terms of productivity. In Figure 3.10 (below), it seems obvious that the state sector is the key driving force behind such a regional pattern: large state net exports deficits largely overturn small positive non-state ones, particularly in the West and in the South. Interestingly, increasing capital imports by the state sector annihilate rising large non-state savings in Metropolises. It explains their – initially surprising – neutral net export position observed in the second chapter.

In conclusion, this simple decomposition provides us with additional evidence that both the state and the non-state sectors play an essential role in the emergence of the *capital allocation puzzle*.

3.5 Data robustness checks

3.5.1 Subsamples

Although the adopted model has to be thought of as a long run one, an estimation of subsamples may shed light on the variations in capital flows over decades and enable to better track the emergence of the *capital allocation puzzle* observed in the preceding sections.

For the period leading up to the Asian Financial Crisis (1984-1997), our flow indicator is far lower (between -1.3 and 3.1) than for the entire sample (-20.4 to 20.7). Catch-up parameters are in a similar range.[38] While there is little change in the negative relationship between flows and productivity, the positive link between investment wedges and TFP is more marked and turns out to be highly significant. The strong negative pattern of saving wedge and productivity is similar.

Things change in the sample embodying the accelerated integration of China into the world economy (1998-2010). Due to the higher reference GDP level, flows are lower than for the entire sample (-4.1 to 7.1) but clearly larger than for the initial subsample. The flows-productivity relationship is not significant anymore. Thus, it seems that the *capital allocation puzzle* has disappeared. Remarkably, the change in pattern is due to some inner provinces such as Shaanxi, Inner Mongolia, Chongqing and Shanxi that have flipped from negative catch-up in the first subsample to a positive one in the second one while continuing importing physical capital. On the other side, while mostly still being capital exporters, some eastern regions started to fall behind relative to national values in terms of productivity (e.g. Zhejiang and Guangdong). Investment wedges are now unrelated to catch-up parameters. While our regression results for investment frictions are relatively similar, the strong positive effect of state-owned firms presence on saving frictions disappears. Indicators of international integration are less informative as well. The importance of the industrial and tertiary sector, housing prices and natural resources seems to play a bigger role in explaining saving frictions.

[38]Cumulated provincial instead of national values are used for the computation of the reference productivity growth rate. The reason is that due to data aggregation issues, large differences between national and provincial TFP may arise in subsamples.

By comparing the geographical distribution of saving wedges between both subsamples, we observe that high saving subsidies have shifted inland. On the map in Figure 3.11 (1998-2010 sample) compared to Figure 3.6 (1984-2010 sample), Inner Mongolia, Shanxi, Shaanxi and Chongqing now have among the most negative saving wedge. To a lesser extent, central provinces exhibit the same pattern. The distribution for the earlier subsample (1984-1997) rather corresponds to the one of the full sample. Even though major inner regions' saving wedges have turned negative, they continue to massively import capital while they catch up in terms of productivity.

As a result, the *capital allocation puzzle* and the investment puzzle may be slowly disappearing. Still, the rising inner regions are no exception to the rule in terms of the saving puzzle: they replaced East Coast provinces in the sense that they subsidize saving and catch up relative to China. The crucial difference is that, even by doing so, they are the ones that end up importing physical capital. First 2011/2012 net exports figures confirm that trend. By contrast, the East Coast experienced capital outflows all along. On the one side, the much advocated rebalancing towards domestic consumption and inner provinces development may already have started. On the other side, there are some reasons to remain sceptical about the inland take-off in productivity.[39]

3.5.2 A simple error correction mechanism

The quality and properties of Chinese data have been intensively discussed in the literature. Some issues concerning macroeconomic provincial data are discussed in the second chapter. At this stage, we want to tackle three major issues that potentially put our results at risk.

First, any factor systematically biasing investment statistics could make the investment puzzle disappear. As we have seen in the empirical part, SOEs' presence in the economy seems to massively influence the patterns of investment rate. One may argue that large public gross fixed capital formation is better captured by the statistical system than (smaller) private projects. Furthermore, state-owned firms possibly have incentives to overreport investment (it is a key variable for monitoring). This would lead to a too low investment in regions where marketization is more advanced (typically the East Coast) and too high figures in the hinterland compared to reality. We capture this possibility by adopting the following investment error correction mechanism:

$$Z_t^{i,synth} = \left(1 - \left[\left(\frac{X_t^i}{median_t\left(X_t^i\right)} - 1\right) \times w\right]\right) Z_t^{i,data}$$

[39]It seems worth mentioning that this productivity reversal may happen at high cost. Brandt et al. (2012) find that western regions particularly suffered from a decrease in realized TFP due to an increasing inefficiency of capital allocation between the state and non-state sector. In Section 3.4.3, we show that state net exports deficits and low non-state net exports drive capital inflows in these regions. Moreover, our findings may partly be driven by idiosyncratic data issues: light data of chapter 2 suggest that GDP growth could have been grossly overestimated (e.g. in Inner Mongolia). Finally, alternative data from Brandt et al. (2012) in Section 3.5.4 suggest that some may have substantially lower productivity growth (e.g Xinjiang, Inner Mongolia and Shaanxi).

where Z would correspond to I and X to the share of state-owned firms in total investment in fixed assets. A province (i) with relatively high share of state-owned investment in fixed assets will have a $X/median(X)$ ratio higher than one. Given an adjustment weight w, synthetic investment will be lower than in official data.[40]

Second, relative saving of more developed regions is far higher than in the rest of the country. Any factor biasing consumption of those areas downward would artificially reinforce the *capital allocation puzzle*. It is often argued that the NBS underestimates private consumption across the board.[41] For urban and richer areas where an emerging middle-class has been triggering near double digit real consumption growth over the last decade, this could particularly bias net external balance upward. To account for that, we apply the same formula as for investment with Z being S and X the urbanization rate.

At last, we tackle aggregation issues. Provincial saving and investment figures do not add up to national values. Over the last decades, investment – and to a lesser extent saving – have been historically too high on the regional level compared to national data. We take that into account by multiplying them separately by a common adjustment ratio after the error correction to get a perfect match between national and cumulated data.

We apply our correction method to aggregate investment and saving before deflating. New net exports and cumulated relative capital flows are derived from these "synthetic" figures. Thereafter, we discuss implications using $w = 0.40$. We compare the synthetic time series to the raw data (both adjusted to aggregate to national values). The average investment rate of the new time series is between 10 percentage points lower (Qinghai) and 4 percentage points higher (Jiangsu). For saving rate, average adjustments between -8pp (Liaoning) and 11pp (Henan) are recorded. The effects on external positions ($NX = S - I$) are large: relative average net exports are between 8pp lower (Liaoning) and 14pp higher (Gansu).[42] By way of example, in Figure 3.12, one can observe net exports for the data and the synthetic series for two regions: Guangdong (East Coast) and Shaanxi (West). The correction is massive: both provinces end up having the same surplus in 2010 although being cases in point of a surplus and deficit region.

Interestingly, even such large biases would not flip the relationship found between capital flows, investment wedges, saving wedges and catch-up parameters. In Figure 3.13, we compare raw data to the corrected ones and illustrate how the relationship between capital flows and

[40] For example, in 2010, Shaanxi exceeds the median by 45%. Given a weight of 0.40, the correction factor is of 0.18. Only 82% of investment is considered and this region has a substantial decrease in its investment rate from 0.68 to 0.55. Note that the 1997 values for X had to be used for the entire 1984-1997 period due to data availability issues.

[41] Key issues are a too low inputed housing consumption, the report of fringe benefits paid by companies as investment and the lack of representativity of the household survey (Jun and Tian, 2013). How they influence the relative distribution of saving is debatable: the first and third arguments suggest the more developed East Coast saving to be lower while the second one would rather decrease saving and investment in inner provinces as SOEs tend to give more privileges to employees.

[42] Values for City-Provinces are exceptional: due to their high urbanization rate, they register a decrease of between 19 and 25pp in net exports.

productivity catch-up flattens out but still stays negative. Main results for saving wedges are robust to the discussed modifications but less so for investment wedges.

3.5.3 International capital flows

We focused on external balances at the regional boundary. An important issue is to apprehend whether our results hold for (exclusively) international capital flows. As showed in the literature (e.g. Alder et al., 2013), the integration of the Chinese economy into the world supply chain and the creation of special economic zones was a pivotal development step. One would expect provinces more active in international trade to have positive growth and TFP impulses. In fact, as discussed in the literature review, export-led-growth is one of the hypotheses having the potential of rationalizing the *capital allocation puzzle*.[43]

Custom data for international exports and imports in dollars are provided by the *China Data Center* (from 1984 to 1991). We complete them with data on trade by place of destination/origin from the *Provincial Statistical Yearbooks* (available from 1992 to 2010). They are transformed into RMB using the implicit exchange rate used in national statistics. We refer to the second chapter (Section 2.4) for more information on these data.

In Figure 3.14, we see that the *capital allocation puzzle* is still present. Some provinces are outlier due to huge capital outflows and relatively moderate catch-up (e.g. Fujian and Zhejiang). Strikingly, most regions are not heavily involved in international trade and only a few eastern provinces seem to drive the general pattern.[44] The link between frictions and catch-up is similar as are most of the qualitative regression results.

3.5.4 Alternative data

We explore to which extent our results are robust to alternative (and better) data. In their paper on factor market distortions inside China, Brandt et al. (2012) carefully revised and assembled macroeconomic provincial time series.[45] We are particularly concerned about large systematic biases in official employment data, mainly because of the *Hukou* registration system and problems of primary and state-sector employment reporting (Brandt et al., 2012). Furthermore, we relied on official CPI while they constructed sectoral GDP deflators. A central difference is that they exclude the agricultural sector from their analysis.

[43]Empirical work at our Chair has showed that evidence for cointegration and Granger causality between GDP, exports and imports is weak and limited to some coastal provinces (Herzog, 2013). Still, we expect evidence to be far stronger at the local level (prefectures, counties and townships). The adoption of a broader view than trade, say the inclusion of associated gross fixed capital formation, technology transfer and other positive externalities (e.g. increased competitivity of the domestic sector) could make the relationship between growth and exports more potent, even on the provincial level.

[44]Without the high value of Hainan (South), Beijing (Metro), Zhejiang, Fujian, Guangdong and Jiangsu (East Coast), the negative relationship would disappear.

[45]We gratefully thank the authors for providing the data.

Their sample ends in 2007. To fill the gap, we assume the growth rate of their data to be similar to ours for capital, employment and real GDP. The national time series are obtained by aggregating regional data. This should deliver sensible results as they already took care of aggregation properties during the data preparation process. Some provinces are excluded from their sample.[46] The national reference TFP growth rate (8.7%) is slightly higher than in our baseline version (7.07%). Their final capital to output ratio is lower than ours. Obviously, much of our capital increase has been absorbed in higher productivity growth in their data.

Their provincial TFP over the period is correlated with ours (0.64) and the general pattern is comparable although some regions experience a switch in catch-up parameter.[47] We relate the capital flow figures of this study to the new catch-up parameters and find the strong negative pattern to be robust (Figure 3.15). In fact, compared to the baseline graphic (Figure 3.2), data points are more equally distributed along the catch-up axis. The positive relationship between investment wedges and TFP is still positive but not significant anymore, while the pattern for saving wedges is preserved. Investment wedges from our baseline version are highly correlated with the alternative ones (0.95) but it is less the case for saving wedges (0.70). In Figure 3.16, where we compare the baseline to the new figures, one sees that there are some substantial shifts in saving wedge.[48]

There are some noteworthy changes at the supraregional level as well. In Table 3.2 (last column), we provide regional statistics using the new saving wedge (real GDP-weighted). The Metropolises now subsidize saving more than the East Coast. Central regions and Manchuria's frictions are now clearly negative while they were slightly positive before. Importantly, the West does not seem to subsidize saving anymore. The positive wedge for the South is much bigger.

In spite of these changes, the econometric patterns discussed in the main sections are comparable. The effect of the presence of the state/private, international enterprises and economic structure on the frictions even seems larger.

3.6 Conclusion

3.6.1 Summary

This paper presents a first systematic analysis of external imbalances inside China. We estimate regional total factor productivity growth over three decades of economic reforms (1984-2010). By plotting productivity against the final relative change in capital flows approximated by net

[46] Tibet, Hainan and Hunan have been excluded. Only Hunan is sizeable in terms of GDP. Another difference is that they merged Chongqing into Sichuan. We compare it to actual Sichuan and exclude Chongqing from the sample.

[47] The TFP pattern changes considerably for seven provinces. Shanxi, Liaoning, Jilin, Shanghai, Anhui and Hubei switch from negative to near zero or positive catch-up using their data. We found Xinjiang to be slightly positive and it turns out to be clearly negative according to theirs.

[48] Among substantial negative shifts in saving wedge, Beijing changes from to 0.54 to -0.61%, Liaoning from 0.04 to -0.75%, Anhui from 1.05 to -0.87% and Hubei from 0.15 to -1.75%. Conversely, Jiangxi turns from -0.69 to 0.62 and Xinjiang from -1.28 to 2.03%.

exports, we observe that provinces that caught up relative to national productivity had capital outflows (i.e. positive net exports). Thus, there seems to be a *capital allocation puzzle* inside China reminiscent of the findings of Gourinchas and Jeanne (2013) at the international level.

Starting from that empirical finding, we follow up by identifying the drivers of that pattern. The methodology developed in Gourinchas and Jeanne is adopted to identify frictions affecting investment and saving in Chinese regions. A small-open economy model is augmented with two wedges. The first one (the investment wedge) affects gross return on aggregate capital. It is identified in matching an empirical with a theoretical decomposition of investment rates. By relating investment wedges to productivity, we find an investment puzzle: regions that caught up relative to the rest of China seem to have higher wedge (lower investment rate), while provinces that fell behind implicitly subsidized investment. This is a first blow to the baseline neoclassical framework and stands in sharp contrast with international patterns.

The second friction (the saving wedge) is comparable to a tax on capital income of households. It is identified in matching an empirical with a theoretical decomposition of cumulated relative capital flows (i.e. net exports). As in Gourinchas and Jeanne at the international level, we find a saving puzzle: the relationship between productivity catch-up and saving wedges is negative and very significant. Provinces that caught up are the ones that implicitly subsidized saving, causing a saving glut that translates into capital outflows. This is a second blow to the neoclassical model as standard theory predicts that provinces with high productivity should experience capital inflows (positive saving wedge and lower saving). As opposed to investment wedges, saving frictions are the main driver of the *capital allocation puzzle*.

We relate the wedges to a large number of "usual suspects" typically suggested by the literature. The cross-regional long run variation in frictions suggests some robust explanatory variables. Characteristics related to the investment structure of the economy robustly account for a high part of the cross-regional variation in investment wedges: a high share of the state in investment in fixed assets or in construction gross output value robustly acts as an investment subsidy (i.e. it lowers investment wedges). The share of state-owned firms in gross industrial output value magnifies investment as well. In terms of economic structure, a higher share of the construction sector relative to GDP raises investment rate. A marked presence of the formal, state-near financial sector – loans in financial institutions – seems to foster investment as well.

Saving wedges are correlated with a large number of variables. There seems to be an ubiquitous positive effect on saving wedge of the state's involvement in the economy (i.e. it lowers saving), independently of whether one considers investment in fixed assets or gross industrial output value. On the other side, a greater importance of foreign- and privately-owned enterprises increases saving compared to the neoclassical model. Among particularities linked to economic structure, the share of the industrial sector has a similar effect. Integration into the world supply chain is another important factor: FDI and the presence of multinational enterprises impact negatively on saving wedges (i.e. it implicitly subsidizes saving). Financial development – deposits and loans in financial institutions – seems to make a dent in saving.

The *capital allocation puzzle* is driven by both the visible hand (the state) and the private sector. By constructing non-state sector net exports, we show that more marketized regions with a strong presence of private and international firms (i.e. the East Coast and the City-Provinces) have a large non-state saving surplus while other regions have balanced non-state net exports in average. In fact, massive state net exports deficits are largely responsible for large capital imports (negative saving - investment balances) in the Chinese hinterland.

Being aware of the noisiness of Chinese statistics, we discuss the effects on our general results of alternative data. In terms of subsamples, the general patterns seem to be more pronounced for the initial reform period (1984-1997). Due to a surge of productivity growth in some hinterland regions, the *capital allocation puzzle* disappears in the more recent period (1998-2010). Then, we propose a simple error correction mechanism by acting on three dimensions: account for the fact that public gross fixed capital formation is potentially better captured by the statistical system than (smaller) private projects, embed the assumption that consumption has been underestimated (particularly in urban and richer areas) and finally adjust regional data so that they aggregate to national official figures. Even assuming substantial errors would not invalidate our results. What is more, our patterns are robust to the use of (exclusively) international physical capital flows (i.e. trade balances). We use improved regional macroeconomic time series from Brandt et al. (2012) and find our results to be robust as well, even though productivity and saving wedges strongly differ for some regions.

3.6.2 Implications for global imbalances

In this section, we put our results in a broader perspective. As discussed in the second chapter, large surpluses driving global imbalances are generated by a few more developed provinces on the East Coast specialized in international trade. The majority of hinterland regions have had near neutral international trade balances since the mid-1990s. The drivers of international Chinese capital outflows are thus heavily concentrated. However, including interregional physical capital flows in the analysis – switching to the provincial difference between saving and investment – revealed a radically different picture: most hinterland provinces have been running large net exports deficit and have positive saving wedge.

While most of these regions are relatively small taken individually, their cumulated potential impact is growing. In 2010, the 15 hinterland regions typically having external deficit accounted for 27% of real cumulated GDP and nearly 38% of population. In terms of economic weight, a back-of-the-envelope calculation suggests that emerging China has already overtaken Germany and amounts to nearly three quarters of Japan's GDP in real PPP terms. These regions are on the brink of moving up in the world. By 2020, even under conservative assumptions, they should be comparable with top-tier developing countries. For instance, in terms of GDP, Inner Mongolia is predicted to approach South Africa, while Xinjiang will correspond to Israel and Anhui to Poland (HSBC, 2010).

Some of the reforms discussed in chapter one could ease off external imbalances in inner China. In the first place, as we have showed in this chapter, the visible hand is an important driver of capital imports. Eventually, the slowdown of state-led investment in infrastructure should contribute to a less negative net exports position. In addition to that, as opposed to coastal regions, households and local SOEs would benefit from more market-based energy and raw material prices. Lastly, inner regions may develop against a backdrop of financial repression easing and improving social security coverage.

Other factors may play a key role as well but are less dependent on whether the CCP will endorse its rebalancing agenda. Inevitably, the hinterland is going to integrate into the world supply chain and may take over a large part of international exports from the East Coast. This may not necessarily generate large surpluses as the catch-up potential in terms of infrastructure and urbanization is still substantial. A part of the ailing state investment could be counterbalanced by international and private projects. What is more, the return of large waves of migrants to the hinterland should promote consumption. At last, these regions will experience a later and slower demographic transition than the East Coast, lowering incentives for large immediate saving accumulation.

In conclusion, many of the points made before suggest that the rise of inner China has the potential to alleviate the issue of massive national external surpluses. Its development may not trigger such a large rise in savings as on the East Coast in the 2000s. As did the celebrated strategic inland retreat of the Communists in the 1930s, capital's long march west has the potential to change the underlying characteristics of the Chinese economy over the next decades.

Figure 3.1: Technology catch-up, 1984-2010

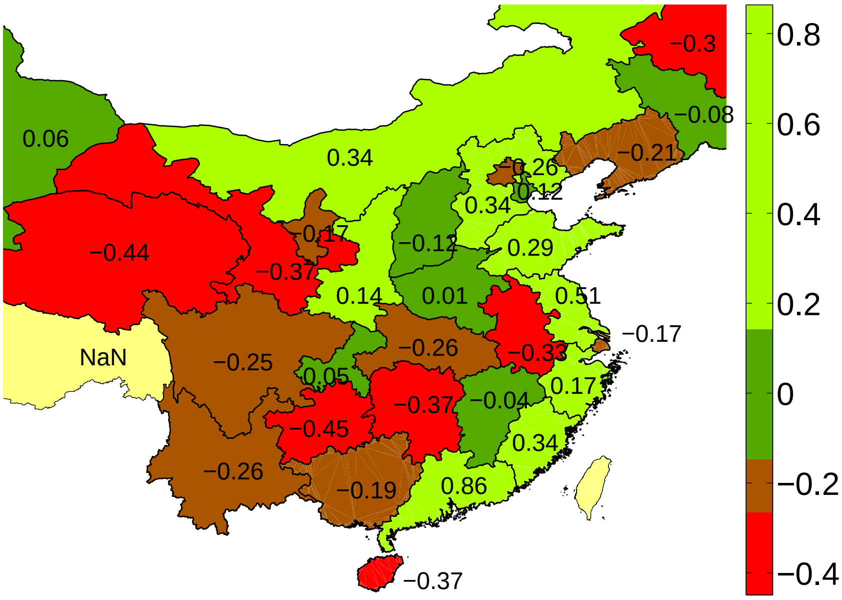


Table 3.1: Main results, 1984-2010

ID	Name	Employment growth (%)	TFP catch-up	Investment over GDP	Investment wedge (%)	Capital flows over GDP	Saving wedge (%)
BJ	Beijing	3.30	-0.26	0.58	-7.95	2.89	0.54
TJ	Tianjin	1.26	0.12	0.55	-6.75	-1.62	-2.40
HB	Hebei	1.56	0.34	0.39	-2.64	-7.51	-2.40
SXI	Shanxi	1.55	-0.12	0.49	-6.20	1.30	-0.63
IM	Inner Mong.	1.39	0.34	0.54	-5.61	8.43	-2.51
LN	Liaoning	1.11	-0.21	0.36	-3.07	-1.80	0.04
JL	Jilin	1.41	-0.08	0.42	-3.64	6.37	-0.41
HG	Heilongjiang	1.29	-0.3	0.38	-4.32	-2.47	0.59
SG	Shanghai	0.76	-0.17	0.54	-7.95	-3.22	-1.24
JG	Jiangsu	1.57	0.51	0.45	-3.93	-9.26	-3.21
ZJ	Zhejiang	2.23	0.17	0.43	-3.6	-8.73	-2.00
AH	Anhui	1.98	-0.33	0.37	-3.37	0.11	1.05
FJ	Fujian	2.66	0.34	0.42	-2.22	-4.89	-2.23
JX	Jiangxi	1.62	-0.04	0.40	-3.68	0.66	-0.69
SD	Shandong	1.79	0.29	0.45	-4.25	-5.77	-2.44
HE	Henan	2.30	0.01	0.43	-3.65	0.25	-0.98
HBI	Hubei	1.35	-0.26	0.45	-5.3	-0.06	0.15
HA	Hunan	1.57	-0.37	0.32	-0.59	0.49	1.29
GD	Guangdong	3.06	0.86	0.36	0.15	-20.35	-3.50
GX	Guangxi	1.96	-0.19	0.39	-2.50	5.84	0.24
HN	Hainan	2.11	-0.37	0.52	-7.07	2.24	0.99
CQ	Chongqing	1.22	0.05	0.41	-4.23	6.72	-0.73
SA	Sichuan	1.22	-0.25	0.40	-4.47	1.98	0.41
GZ	Guizhou	2.43	-0.45	0.42	-5.05	11.47	2.58
YN	Yunnan	2.15	-0.26	0.40	-4.22	8.12	0.97
TB	Tibet	-	-	-	-	-	-
SAI	Shaanxi	1.47	0.14	0.44	-4.59	5.82	-1.33
GS	Gansu	1.76	-0.37	0.43	-5.52	3.99	1.51
QH	Qinghai	1.96	-0.44	0.63	-9.10	12.52	2.04
NG	Ningxia	2.54	-0.17	0.61	-8.17	20.69	0.47
XJ	Xinjiang	1.64	0.06	0.55	-7.97	3.34	-1.28

Catch-up parameter computed relative to national TFP. Investment, GDP and capital flows are adjusted for initial price level using an expenditure basket by Brandt and Holz (2006). Investment is deflated using consumer price index (CPI) and the price of investment in fixed assets (PIFA) since 1992. GDP is deflated by CPI. Investment wedges are obtained by matching empirical average investment rate. Capital flows are the sum of external provincial surplus/deficit ($NX = S - I$) normalized by initial real output and deflated by the last period price level as in Gourinchas and Jeanne (2013). The deflator of capital flows is CPI and, since 1997, the producer price index of manufactured goods (PPI). Saving wedges are obtained by matching empirical relative capital flows.

Table 3.2: Investment and saving wedge for larger regions, 1984-2010

	Real GDP	Investment wedge (%)		Saving wedge (%)		
	share (2000)	unweighted	weighted	unweighted	weighted	section 3.5.4
Metropolises	0.07	-7.55	-7.70	-1.03	-0.93	-2.05
East Coast	0.43	-2.75	-2.77	-2.63	-2.73	-1.84
Center	0.19	-3.32	-3.40	0.16	0.06	-0.95
Manchuria	0.10	-3.68	-3.59	0.07	0.13	-0.32
West	0.08	-6.74	-6.11	-0.25	-0.85	-0.01
South	0.12	-4.59	-4.18	0.74	0.49	3.07

Average values over six larger macroeconomic regions. Weighted version with 2000 real GDP. Metropolises: Beijing, Tianjin and Shanghai. East Coast: Hebei, Shandong, Jiangsu, Zhejiang, Fujian and Guangdong. Center: Henan, Hubei, Hunan, Anhui and Jiangxi. Manchuria: Liaoning, Jilin and Heilongjiang. West: Shanxi, Inner Mongolia, Shaanxi, Ningxia, Gansu, Qinghai and Xinjiang. South: Chongqing, Sichuan, Yunnan, Guangxi, Guizhou and Hainan.

Figure 3.2: Capital flows vs productivity catch-up, 1984-2010

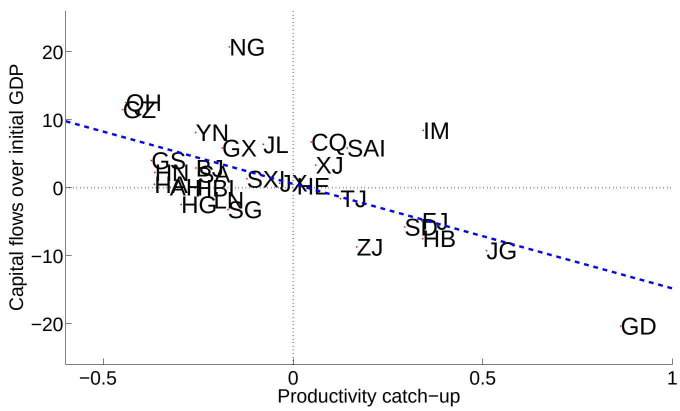


Figure 3.3: Investment wedge vs productivity catch-up, 1984-2010

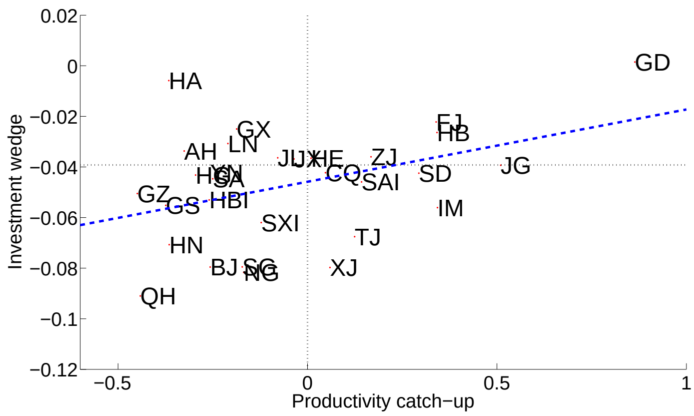


Table 3.3: Factor regressions of investment and saving wedge (I)

	Investment wedge				Saving wedge			
	Univariate		Ec.Dvpt control		Univariate		Ec. Dvpt control	
	Coeff.	P-value	Coeff.	P-value	Coeff.	P-value	Coeff.	P-value
Investment Str.								
SOInvFA	-5.2	0.01	-8.3	0.00	4.7	0.00	3.4	0.03
FOInvFA	0.5	0.55	1.5	0.14	-1.0	0.01	-0.4	0.50
REInvFA	7.3	0.00	7.4	0.00	-3.9	0.04	-3.6	0.03
State vs Private								
SOGIOV	-1.4	0.09	-2.6	0.01	1.8	0.00	1.4	0.01
EmplPrivate	-1.3	0.12	-3.1	0.16	-1.3	0.14	1.7	0.09
Market	2.6	0.13	7.7	0.00	-3.2	0.00	-2.5	0.08
SOCGOV	-1.8	0.07	-3.0	0.01	2.4	0.00	1.6	0.05
Economic Str.								
SectorPrim	0.8	0.52	0.7	0.79	1.6	0.01	0.1	0.94
SectorInd	3.3	0.08	4.8	0.00	-5.1	0.00	-3.9	0.02
SectorConst	-3.5	0.16	-6.3	0.01	2.6	0.03	0.6	0.76
SectorTert	-5.7	0.02	-6.5	0.03	1.2	0.35	5.8	0.00
StructConc	-0.6	0.00	-0.6	0.00	0.2	0.07	0.4	0.07
HousingPrice	-0.1	0.98	0.6	0.80	-2.4	0.03	-1.1	0.41
CoalOil	-0.7	0.07	-0.8	0.05	0.1	0.55	-0.1	0.61
International								
Openness	0.1	0.90	0.8	0.70	-0.7	0.00	-0.4	0.70
MNE	0.7	0.42	2.3	0.01	-1.6	0.00	-1.4	0.03
FDI	0.1	0.84	0.7	0.63	-0.9	0.03	-0.3	0.64
Financial Dvpt								
Deposits	-2.2	0.23	-2.4	0.20	0.4	0.47	1.7	0.00
Loans	-4.5	0.02	-4.8	0.01	1.2	0.12	2.8	0.00

Cross-sectional OLS regressions with 30 observations. First regression with constant and factor, second with control for level of economic development (average of real GDP per capita relative to national values). Coefficients in %. Factors normalized by their cross-sectional mean. P-values based on heteroskedasticity-robust standard errors HC3 (jackknife approximation). The dependent variable is the investment/saving wedge estimated for the 1984-2010 sample. The independent variables are the mean over 1997-2009 of the respective factor. See Section A.1.1 in appendix for a description of the factors.

Table 3.4: Factor regressions of investment and saving wedge (II)

	Investment wedge				Saving wedge		
	Coeff.	P-value			Coeff.	P-value	
	%	HC2	HC3		%	HC2	HC3
SOInvFA	-4.4	0.04	0.07	SOGIOV	0.9	0.07	0.12
Loans	-2.2	0.07	0.14	EmplPrivate	-0.7	0.40	0.48
HousingPrice	-2.0	0.31	0.40	HousingPrice	0.0	0.97	0.97
SectorConst	-3.8	0.05	0.11	SectorInd	-2.7	0.00	0.00
FDI	0.2	0.77	0.82	MNE	-0.9	0.02	0.03
UrbRate	-2.5	0.15	0.24	UrbRate	1.3	0.15	0.21

Cross-sectional OLS regressions with 30 observations. Coefficients in %. P-values based on heteroskedasticity-robust standard errors HC2 and HC3 (jackknife approximation). The dependent variable is the investment/saving wedge estimated for the 1984-2010 sample. The independent variables are the mean over 1997-2009 of the respective factor. See Section A.1.1 in appendix for a description of the factors.

Table 3.5: Panel factor regression of saving wedge for three subsamples

	No time fixed-effects			With time fixed-effects		
	Coeff.	P-value		Coeff.	P-value	
	%	AR	LZ	%	AR	LZ
SOGIOV	6.3	0.02	0.00	-0.9	0.82	0.43
EmplPrivate	-12.3	0.01	0.00	-4.1	0.45	0.01
DepMinLoans	0.9	0.67	0.17	1.6	0.50	0.03
SectorInd	-21.1	0.00	0.00	-11.4	0.07	0.00
Openness	1.0	0.63	0.12	0.6	0.80	0.40
FDI	-35.2	0.01	0.00	-41.1	0.03	0.00

Pooled OLS estimates with province fixed-effects. Coefficients in %, not comparable with cross-sectional regressions (i.e. not normalized by cross-sectional mean). P-values based on Arellano's heteroskedasticity and autocorrelation robust standard errors (AR) and clustered by province based on Liang and Zeger (LZ). The dependent variable is saving wedge estimated for three subsamples (1984-1992, 1993-2001 and 2002-2010). Independent variables are the mean over 1984-1991, 1993-2000 and 2002-2009 of the respective factor. EmplPrivate is in year level (1992, 2001 and 2010) because no value is available before 1992. See Section A.1.1 in appendix for a description of the factors.

Table 3.6: Detailed results, 1984-2010

Name	TFP growth (%)	Capital eff.units	Investment decomposition Conv.	Catch-up	Trend	Capital flows decomposition Conv.	Catch-up	Trend	Saving
Beijing	5.9	5.3	0.08	-0.04	0.53	29.0	0.4	-14.2	-12.3
Tianjin	7.6	4.5	0.12	0.01	0.41	26.2	-1.6	4.4	-30.6
Hebei	8.3	2.8	0.06	0.03	0.30	14.1	-1.5	8.1	-28.1
Shanxi	6.5	4.2	0.10	-0.01	0.40	23.3	-0.5	-4.1	-17.4
Inner Mong.	8.3	3.9	0.13	0.04	0.38	27.7	-1.3	12.0	-30.1
Liaoning	6.1	2.9	0.08	-0.02	0.30	17.0	0.0	-5.0	-13.8
Jilin	6.7	3.1	0.11	-0.01	0.32	23.4	-0.2	-2.3	-14.6
Heilongjiang	5.6	3.3	0.07	-0.03	0.34	15.4	0.5	-7.5	-10.9
Shanghai	6.3	5.3	0.12	-0.02	0.45	22.2	-1.1	-5.6	-18.7
Jiangsu	8.8	3.2	0.07	0.05	0.33	16.2	-2.1	13.5	-36.9
Zhejiang	7.7	3.1	0.08	0.02	0.34	22.0	-1.1	5.4	-35.2
Anhui	5.5	3.0	0.07	-0.03	0.33	18.4	0.7	-9.4	-9.6
Fujian	8.3	2.7	0.07	0.03	0.32	22.1	-0.9	11.1	-37.1
Jiangxi	6.9	3.1	0.08	0.00	0.33	17.6	-0.5	-1.0	-15.6
Shandong	8.1	3.3	0.08	0.03	0.35	19.1	-1.6	8.7	-32.0
Henan	7.1	3.1	0.09	0.00	0.34	23.6	-0.5	0.3	-23.2
Hubei	5.8	3.7	0.11	-0.03	0.37	23.9	0.1	-8.3	-15.8
Hunan	5.2	2.3	0.09	-0.03	0.26	20.3	0.5	-9.3	-11.0
Guangdong	9.7	2.1	0.02	0.06	0.28	7.9	-1.6	21.3	-47.9
Guangxi	6.2	2.7	0.10	-0.02	0.31	25.6	0.1	-6.0	-13.9
Hainan	5.2	4.7	0.11	-0.05	0.45	29.5	0.8	-15.6	-12.5
Chongqing	7.3	3.3	0.07	0.00	0.33	15.2	-0.6	1.2	-9.1
Sichuan	5.9	3.4	0.09	-0.02	0.34	19.1	0.3	-6.8	-10.6
Guizhou	4.6	3.6	0.08	-0.05	0.39	23.5	2.0	-16.6	2.5
Yunnan	5.9	3.3	0.07	-0.02	0.35	19.7	0.7	-8.1	-4.2
Tibet	-	-	-	-	-	-	-	-	-
Shaanxi	7.6	3.4	0.08	0.01	0.35	17.9	-0.9	4.0	-15.2
Gansu	5.2	3.8	0.08	-0.04	0.38	20.1	1.3	-12.0	-5.4
Qinghai	4.7	6.3	0.14	-0.07	0.55	36.7	2.0	-22.8	-3.4
Ningxia	6.3	5.5	0.11	-0.02	0.52	33.1	0.4	-8.6	-4.2
Xinjiang	7.3	5.3	0.07	0.01	0.48	15.7	-1.7	2.1	-12.8

Steady-state capital in efficiency unit matches average investment rate and depends on investment wedge.

Investment decomposition: Conv: initial investment needed to reach the steady-state capital stock. Catch-up: investment required by falling behind or catching up compared to reference TFP. Trend: amount of investment needed to compensate for capital depreciation.

Capital flows decomposition: Conv: amount of capital necessary to reach the steady-state capital stock. Catch-up: external borrowing needed to finance domestic investment. Trend: cumulated debt inflows required to hold the relative debt ratio constant. Saving: intertemporal consumption decision of households.

Table 3.7: Factor regressions of investment and saving wedge (III)

	Investment wedge				Saving wedge			
	Univariate		Ec. Dvpt control		Univariate		Ec. Dvpt control	
	Coeff.	P-value	Coeff.	P-value	Coeff.	P-value	Coeff.	P-value
Human Capital								
TertiaryEduc	-1.1	0.01	-1.9	0.01	-0.4	0.67	1.7	0.01
HighEduc	-1.2	0.11	-1.8	0.13	-0.7	0.52	1.6	0.19
Patents	-0.2	0.61	-0.1	0.94	-0.4	0.22	0.9	0.20
Social Security								
SOCSEC	-1.3	0.06	-3.0	0.02	-1.1	0.09	1.6	0.08
Demographics								
OldDepRatio	3.1	0.37	5.2	0.13	-1.8	0.32	0.9	0.66
SexRatio	-1.4	0.14	-1.6	0.07	1.5	0.04	1.0	0.14
UrbRate	-1.6	0.19	-4.5	0.10	-1.6	0.12	4.3	0.01
EthnicShare	-0.7	0.05	-0.9	0.01	0.5	0.15	0.3	0.38

Cross-sectional OLS regressions with 30 observations. First regression with constant and factor, second with control for level of economic development (average of real GDP per capita relative to national values). Coefficients in %. Factors normalized by their cross-sectional mean. P-values based on heteroskedasticity-robust standard errors HC3 (jackknife approximation). The dependent variable is the investment/saving wedge estimated for the 1984-2010 sample. The independent variables are the mean over 1997-2009 of the respective factor. See Section A.1.1 in appendix for a description of the factors.

Figure 3.4: Saving wedge vs productivity catch-up, 1984-2010

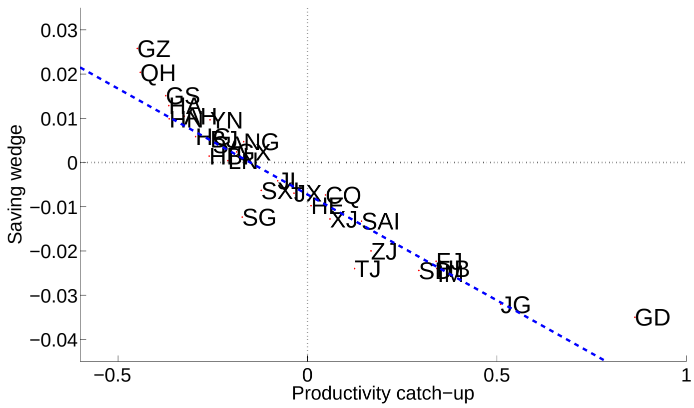


Figure 3.5: Investment wedges (τ_k), 1984-2010 (in %)

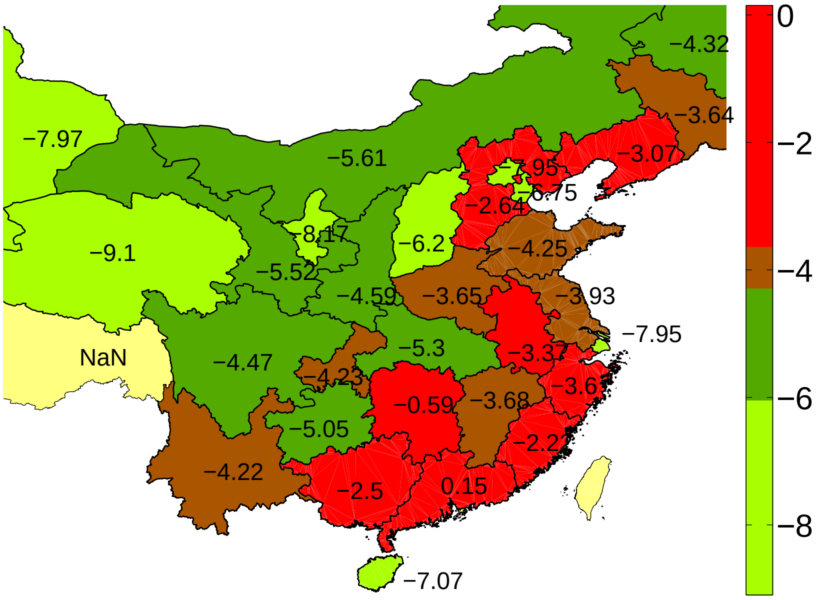


Figure 3.6: Saving wedges (τ_s), 1984-2010 (in %)

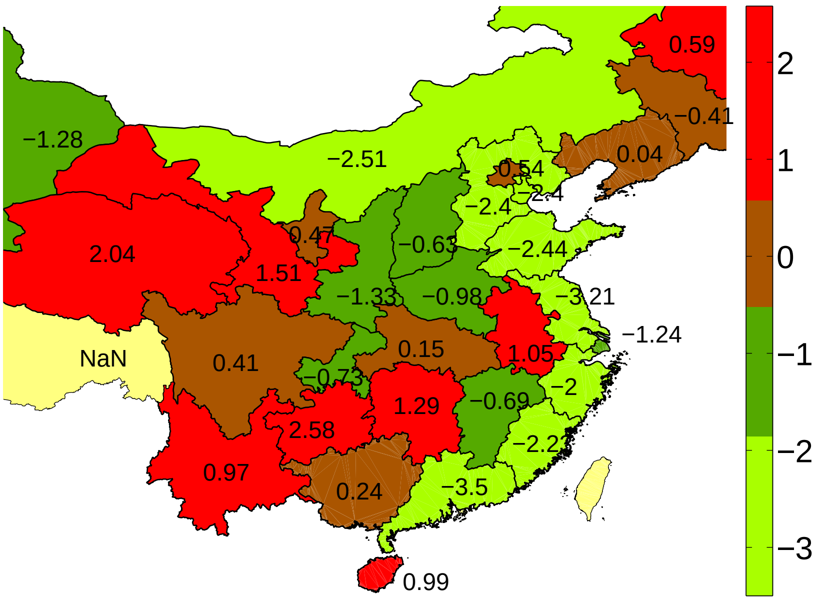


Figure 3.7: Investment wedge vs SOInvFA

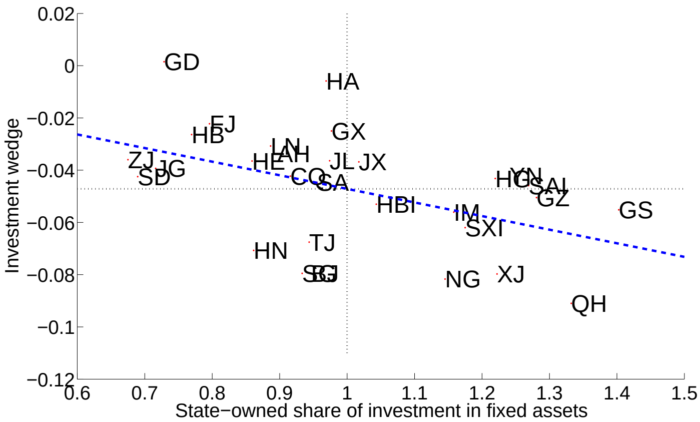


Figure 3.8: Saving wedge vs SOGIOV

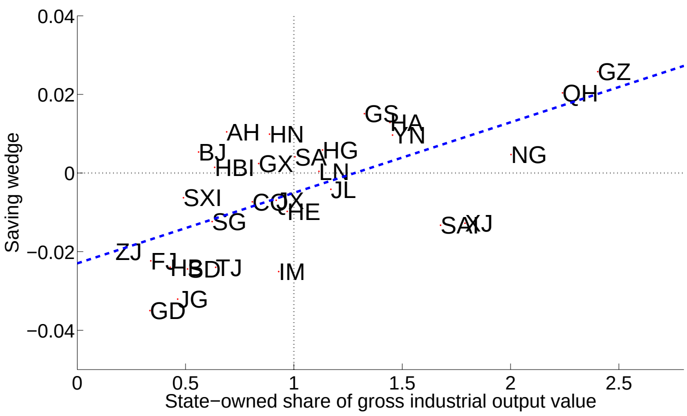


Figure 3.9: Saving wedge vs MNE

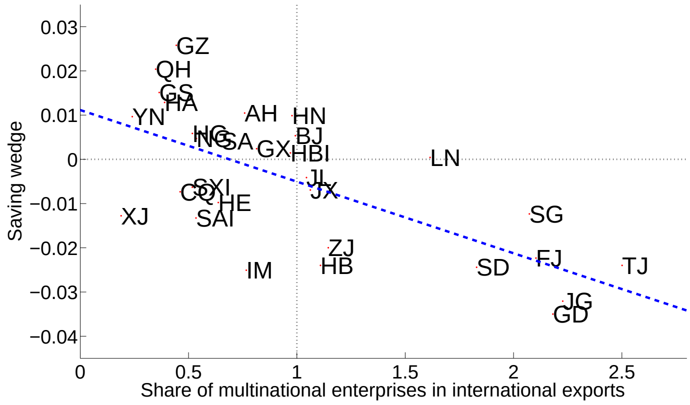


Figure 3.10: Non-state vs state net exports over total GDP for larger regions, 1997-2012

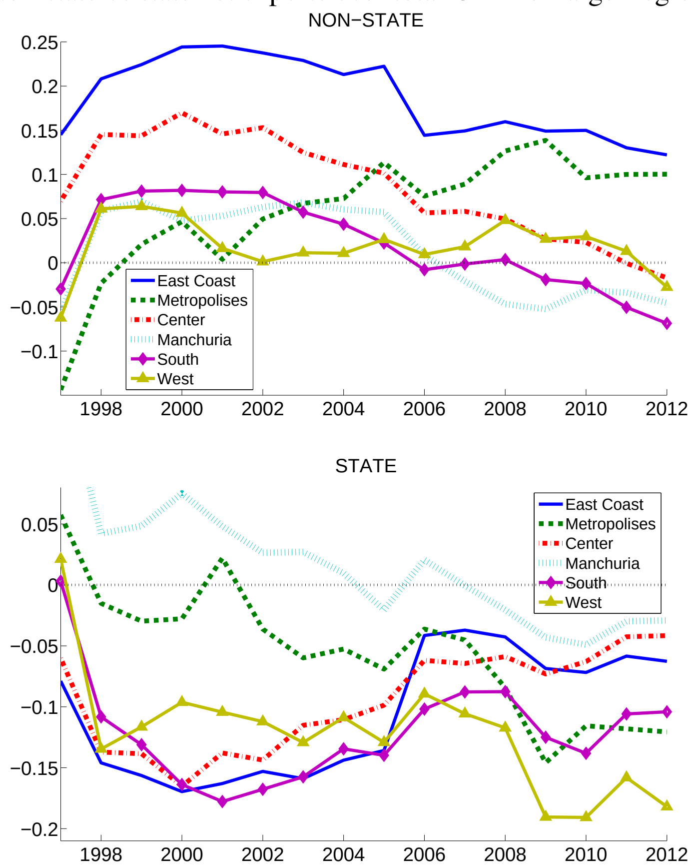


Figure 3.11: Saving wedges, 1998-2010 (in %)

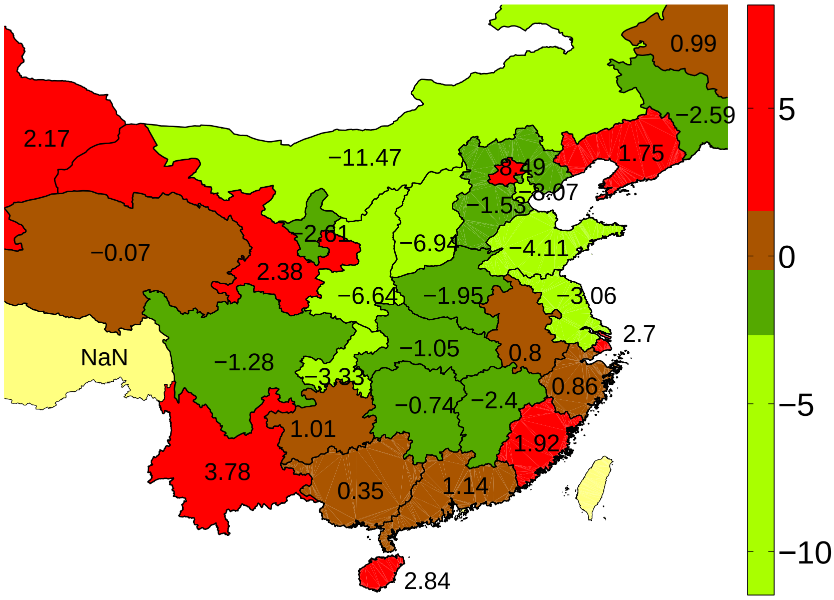
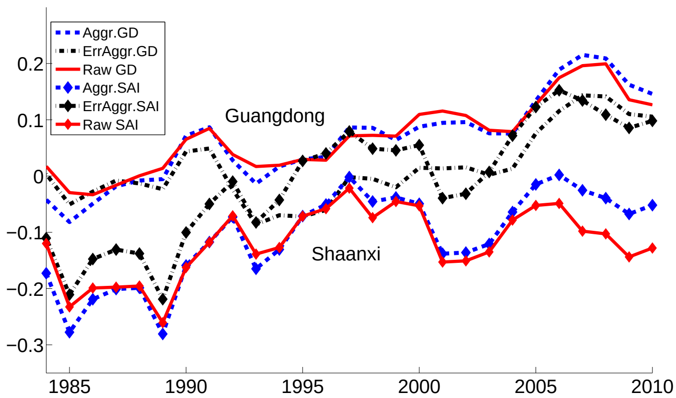


Figure 3.12: Net exports over GDP: aggregation and error correction



Aggr. is raw data adjusted to match national values. *ErrAggr.* is corrected for aggregation and investment/saving errors as explained in Section 3.5.2.

Figure 3.13: Productivity catch-up vs original (dashed) and error-corrected (red) capital flows

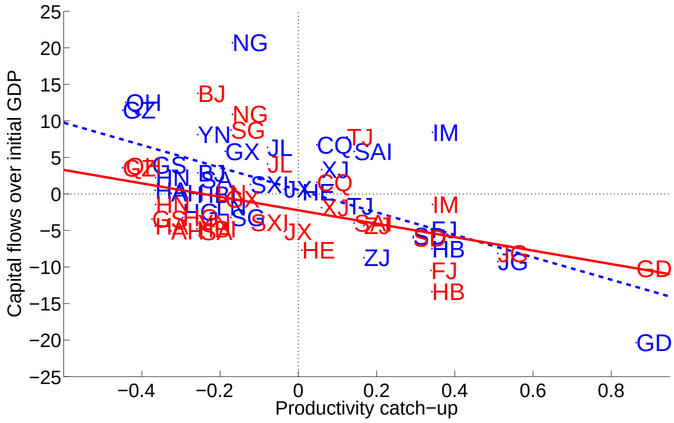


Figure 3.14: International trade flows vs productivity catch-up, 1984-2010

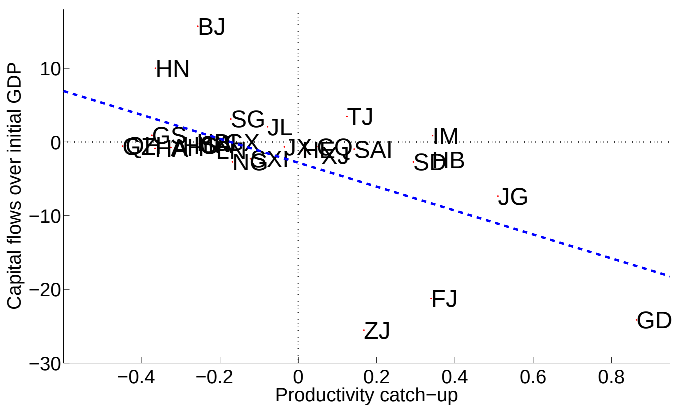


Figure 3.15: Capital flows vs productivity catch-up (alternative data from Brandt et al. (2012))

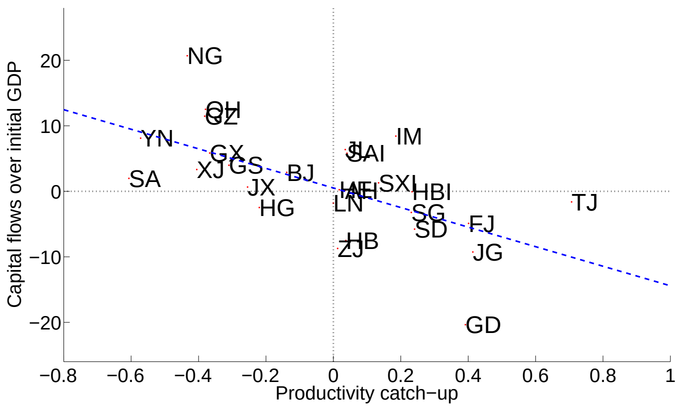


Figure 3.16: Saving wedge: baseline vs alternative data (Brandt et al. (2012))

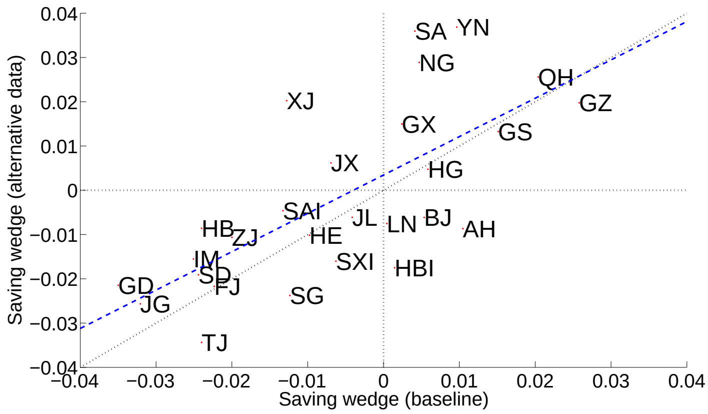


Figure 3.17: Sensitivity of wedges to alternative reference TFP (g^*)

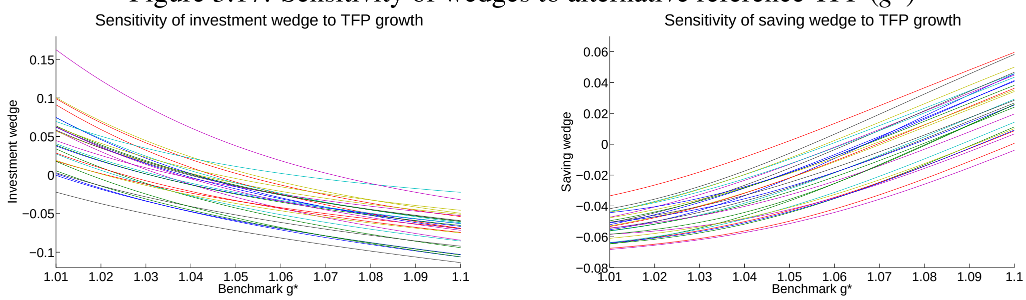


Figure 3.18: Sensitivity of wedges to alternative coefficient of relative risk aversion (γ)

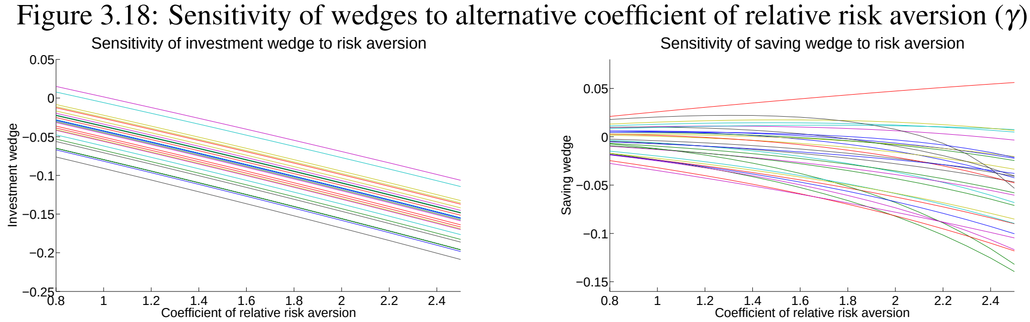


Figure 3.19: Initial external position over 1984 GDP (debt +, assets -)

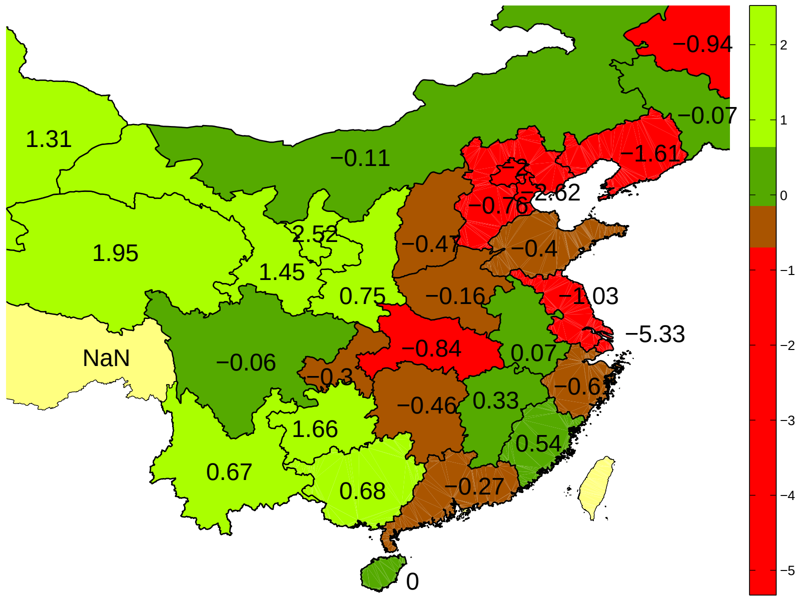


Figure 3.20: Sensitivity of saving wedge to weights on initial external position ($[0-1]$)

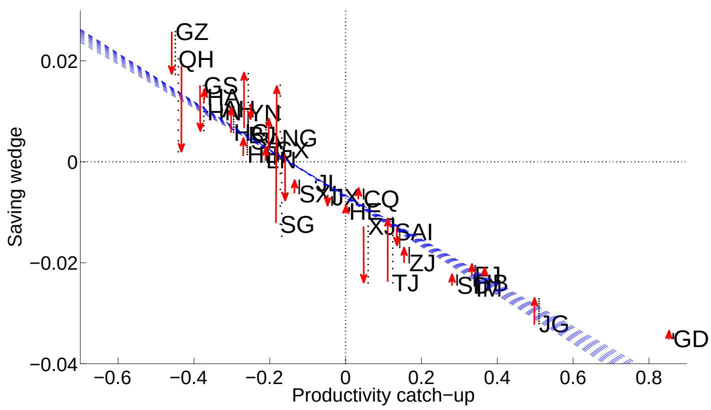
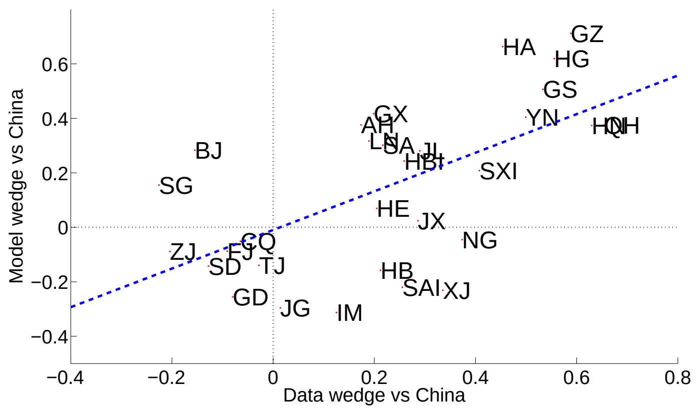


Figure 3.21: Implied real wage wedge relative to China: model vs data (- means high wage vs China)



Chapter 4

A Provincial View of Global Imbalances: Regional Capital Flows in China[1]

[1]This chapter is joint work with Prof. Dr. Mathias Hoffmann. It is currently under submission at the *International Monetary Fund Economic Review*.

4.1 Introduction

The empirical fact that international capital tends to flow uphill – from big emerging economies such as China to highly developed countries such as the US – has been an issue of intensive academic and policy debate over the last decade. This pattern is a theoretical challenge to neoclassical growth models and therefore has rightfully been dubbed a puzzle (Gourinchas and Jeanne, 2013). It is also often seen as the main symptom of a perceived imbalance in international capital flows that could distort exchange rates, interest rates and asset prices at a global level (Bernanke, 2007). Considerable research effort has therefore been given to explaining these patterns theoretically (Caballero et al., 2008; Mendoza et al., 2009; Song et al., 2011; Aguiar and Amador, 2011).[2] However, so far, we have relatively little evidence about the patterns of intranational capital flows in the country that – with its persistent surpluses over the last decade – best exemplifies this capital allocation puzzle: China.

We attempt to fill that gap in this paper. We study empirically the dynamics and determinants of net exports at the level of Chinese provinces. Understanding this "cross-section" of China's net exports provides a useful disaggregated perspective on global imbalances and their origins. Specifically, we model province-level net exports using a stylized intertemporal model of capital flows in which we allow for a simple form of financial frictions in the form of a "savings wedge" in the mould of Gourinchas and Jeanne (2013). Our framework builds on Hoffmann (2013) and nests two broad channels of external adjustment in interprovincial capital flows: the first is variation in intertemporal prices, which can further be disaggregated into variation in national real interest rate, the excess return on international assets over the domestic interest rate, and real exchange rate (i.e. the relative price of tradable and non-tradable goods). The second is intertemporal variation in quantities – cash flows of output, investment and government spending. As we show, our simple model can account for up to 85 percent of the net exports variation in a panel of 30 provinces over the 1985-2010 period. Variation in cash flow explains on average 70 percent of external adjustment and intertemporal prices, on average, account for the remaining 15 percent.

However, these numbers mask considerable cross-provincial heterogeneity in the importance of adjustment channels. First, as documented in the third chapter, China's provinces are characterized by an internal capital allocation puzzle, with some of the most quickly growing provinces displaying the most persistent surpluses. Second, our decomposition of provincial net exports puts us into a position to correlate province-level patterns of external adjustment with a host of regional characteristics: i) the relative role of private and state-owned enterprises (SOEs) in the provincial economy and their differential access to finance, ii) measures of openness to international FDI and trade, iii) the sectoral composition of local economies and iv) demographic factors. All of these characteristics have been identified in the empirical and theoretical lit-

[2] For more, we refer to the third chapter (Section 3.1.1) where we provide a review of the main theories and findings.

eratures as potential drivers of China's external surplus. However, a taxonomy that allows to assess the relative importance of these factors is lacking to date. By looking at the cross-section of China's province-level net exports, we can provide such a taxonomy by identifying through which channels these characteristics impact interprovincial and international capital flows.

Our results suggest that the relative role of private and state-owned enterprises (SOEs) has a particularly strong impact on the patterns of external adjustment at the province-level: variation in domestic (as opposed to international) interest rate and expected variation in future investment (i.e. net output) are much more important drivers of capital flows in provinces with high share of private enterprises in the economy. Conversely, in provinces with a strong presence of SOEs, we see that variation in international interest rate and relative price of non-tradables is more important. This pattern is consistent with a view of the Chinese economy (Song et al., 2011) in which private enterprises and households are subject to considerable financial repression, whereas state-owned enterprises have preferential access to international finance through the state-owned banking system. As a result, saving decisions by private households and firms should be driven by variation in the domestic (financially repressed) interest rate and firm savings should predict future private investment because such investment has to be financed from internal funds. This is what our empirical findings suggest. We also find a significant impact on capital flows from internal price adjustment (i.e. in the relative price of non-tradables such as a housing) in less developed regions. This seems consistent with the view that housing may serve as a savings vehicle where other investible assets are hard to come by.

The paper is structured as follows. Section 4.2 introduces our theoretical and empirical framework. Then, Section 4.3 discusses the data. In a next step, we present our main results in Section 4.4 and briefly discuss some alternative specifications in Section 4.5. At last, Section 4.6 concludes.

4.2 The framework

4.2.1 Model

Our analysis follows the tradition of the intertemporal approach to the current account (Sachs et al., 1981; Bergin and Sheffrin, 2000; Kano, 2008; Hoffmann, 2013). However, to our knowledge, we are the first to extend and apply the empirical framework used in these studies to intranational data and, in particular, to data from Chinese regions. Specifically, our setup extends Hoffmann (2013) to allow us to study a cross-section of regional economies. It is based on rather minimal identifying assumptions since it builds on the log-linearized version of an intertemporal budget constraint, similar to Lettau and Ludvigson (2001) and Gourinchas and Rey (2007).

Our starting point is the law of motion of a province's claims on the rest of the world (including other provinces and other countries), here expressed in tradable goods as

$$B_t^k = (1 + r_t^{T,k})B_{t-1}^k + Y_t^k - I_t^k - G_t^k - C_t^k$$

where B_t^k is the stock of out-of-region assets and Y_t^k, I_t^k, G_t^k and C_t^k denote the province-level (k) values of real output, investment, government consumption and private consumption respectively. The term $r_t^{T,k}$ denotes the interest rate (expressed in terms of tradable goods) that the province obtains on its (end-of-last-period) holdings of out-of-province assets, B_{t-1}^k. We can then define the provincial net exports balance as

$$NX_t^k = \Delta B_t^k - r_t^{T,k} B_{t-1}^k = NO_t^k - C_t^k$$

where we use the notation $NO_t^k = Y_t^k - I_t^k - G_t^k$ to denote net output (i.e. the cash flow available for consumption to the province's residents).

China has a closed capital account. As has been widely documented, most of its foreign assets are in the the hands of the public sector or of state owned enterprises, while private or politically non-connected firms and households are subject to a considerable degree of financial repression (see Aguiar and Amador, 2011; Song et al., 2011). Another justification for introducing a savings friction is that, in the long run framework of the third chapter, they have been identified as the key driver of provincial external balances (as opposed to investment wedges). Following Gourinchas and Jeanne (2013), we capture these frictions in a reduced form as a wedge between domestic and world real interest rates. Specifically, we model the *de facto* real interest rate faced by residents of province k as

$$r_t^{T,k} = (1 - \delta^k)(i_t^N - E_t(\pi_{t+1})) + \delta^k(i_t^W - \Delta s_{t+1} - E_t(\pi_{t+1}))$$

where i_t^N and i_t^W are the Chinese and the world (US) nominal interest rate respectively and Δs_{t+1} the percentage change in the nominal effective Renminbi exchange rate. Finally, π_{t+1} denotes Chinese tradables inflation. The coefficient δ^k captures differences across provinces in the degree of financial integration with world capital markets. We rewrite the preceding equation as

$$r_t^{T,k} = r_t^N + \delta^k \tau_t$$

where $r_t^N = i_t^N - E_t(\pi_{t+1})$ is the national (domestic) real interest rate and $\tau_t = i_t^W - \Delta s_{t+1} - i_t^N$ is the excess return of investing into the foreign bond while borrowing in Chinese currency. Here, $\delta^k \tau_t$ can be interpreted as a measure of the province-level savings wedge.[3] This decomposition

[3] To see the formal similarity with a savings wedge in the Gourinchas-Jeanne setup, write $(1 + r_t^{T,k}) = (1 + i_t^W)(1 - \tau_t^k)/(1 + \pi_t^k)$, where τ_t^k is a province-specific wedge and i_t^W is the nominal world rate of interest. In our setup, we assume $\tau_t^k = \delta^k \tau_t$ (i.e. the province-level wedge is the product of a province-level degree of financial integration and a China-wide wedge vis-à-vis the rest of the world). Taking logs then gives the representation above. The assumption implicit in this formulation is that time variation in savings wedges is common across provinces, whereas the relative degree of access of provinces to the global capital market is unchanged over time. Since province-level interest rates

of the regional real interest rate has an intuitive interpretation. The first term (r_t^N) corresponds to saving incentives arising from the domestic real interest rate. The second term reflects variation in the excess returns on the international bond (τ_t). Note that the impact of τ_t is allowed to vary across provinces according to the loading parameter $\delta^k \in [0:1]$. A weight δ^k of one means that the province has full access to international markets, so that $r_t^{T,k} = i_t^W - \Delta s_{t+1} - E_t(\pi_{t+1})$, the real return on the foreign bond. A weight of zero indicates that the region is financially repressed, so that households and firms are forced to invest into national assets at rate $r_t^{T,k} = r_t^N$. For example, we would expect that more urbanized provinces with more international trade or regions with a stronger presence of state-owned enterprises – which have preferential access to international finance – would have a relatively higher level of δ^k (lower financial repression). By contrast, less open and financially developed provinces may be characterized by a lower value of δ^k (more financial repression). We will corroborate this conjecture in our empirical analysis.

Imposing the usual transversality constraint, the above law of motion can be solved forward, to yield the non-linear intertemporal budget constraint:

$$B_{t-1}^k = \sum_{l=0}^{\infty} E_t \left\{ R_{t+k}^{T,k} \left[C_{t+l}^k - NO_{t+l}^k \right] \right\}$$

where $R_{t+l}^{T,k} = \left[\prod_{i=0}^{l} (1 + r_{t+i}^{T,k}) \right]^{-1}$. We build on Kano (2008) and log-linearize this expression to obtain a formula for the net exports / net output ratio:[4]

$$\widetilde{\frac{NX}{NO_t}}^k = c \sum_{l=1}^{\infty} \kappa^l E_t \left\{ \Delta\tilde{c}_{t+l}^k - \tilde{r}_{t+l}^{T,k} \right\} + \sum_{l=1}^{\infty} \kappa^l E_t \left\{ \tilde{r}_{t+l}^{T,k} - \Delta\widetilde{no}_{t+l}^k \right\} \tag{4.1}$$

We provide a detailed derivation of a version with savings wedge in appendix (Section B.2.2). Here, Δno and Δc are the growth rates of net output and consumption expenditure respectively and the tilde denotes deviations from the unconditional mean. The parameters b and c are the long-term means of B/NO and C/NO. The discount parameter takes the form $\kappa = exp\left[E(\Delta no_t^k) - E(r_t^{T,k}) \right]$. In the derivation, we have assumed that $E(\Delta no_t^k) = E(\Delta c_t^k)$. Note that the approximation above follows directly from the intertemporal budget constraint and that we have, so far, not imposed any restrictions on technology or preferences.

In what follows, we restrict this setup using some theory. Specifically, we posit that each province's representative agent has lifetime CRRA utility over a consumption bundle composed

are not directly observable, this approach allows us to calibrate τ_t directly from observables while estimating δ^k as a parameter of the model.

[4] Kano (2008) obtained an expression for the CA/NO ratio. As no income flows data among regions are available, we use the approximation $\widetilde{\frac{NX}{NO_t}}^k = \widetilde{\frac{CA}{NO_t}}^k - b\tilde{r}_t^{T,k}$, where b is the steady-state value of foreign assets.

of a tradable and non-tradable good:

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[\frac{X\left(C_t^{N,k}, C_t^{T,k}\right)^{1-\gamma}}{1-\gamma} \right]$$

where

$$X_t^k = X\left(C_t^{T,k}, C_t^{N,k}\right) = C_t^{T,k\,\alpha} \times C_t^{N,k\,1-\alpha}$$

A detailed derivation of the model is available in appendix (Section B.2.1.1). In this setting, it is well known that the intertemporal consumption allocation can be solved for independently from the intratemporal allocation between tradable and non-tradable goods. Specifically, we can define the price index of aggregate consumption by recognizing that, for any such index P_t^{*k}, it must be true that $P_t^{*k} X_t^k = C_t^{T,k} + P^k C_t^{N,k} = C_t^k$ for all P_t^k. Then replacing C_t^k with $P_t^{k*} X_t^k$ in the budget constraint, one obtains the Euler equation

$$E_t \left(\beta \frac{P_t^{*k}}{P_{t+1}^{*k}} \left(\frac{X_t^k}{X_{t+1}^k} \right)^{\gamma} \left(1 + r_{t+1}^{T,k}\right) \right) = 1$$

which can be rewritten in terms of aggregate consumption expenditure as

$$E_t \left(\beta \left(\frac{C_t^k}{C_{t+1}^k} \right)^{\gamma} \left(\frac{P_t^{*k}}{P_{t+1}^{*k}} \right)^{1-\gamma} \left(1 + r_{t+1}^{T,k}\right) \right) = 1 \tag{4.2}$$

The aggregate price index for consumption is given by $P_{t+1}^*/P_t^* = (P_{t+1}/P_t)^{1-\alpha}$. Hence, (4.2) links aggregate consumption expenditure growth to the consumption-based real interest rate, which is the national real interest rate corrected for the savings wedge and real exchange rate changes (defined as the change in the relative price of the non-traded good relative to the tradable good). Assuming that consumption growth, the real exchange rate, and the real interest rate are jointly log-normal, Bergin and Sheffrin (2000) show that this condition can be log-linearized to obtain

$$E_t(\Delta c_{t+1}^k) = \frac{1}{\gamma} E_t \left(r_{t+1}^k \right) + constant \tag{4.3}$$

where r_t^k is the consumption-based real interest rate of province k,

$$r_{t+1}^k = r_{t+1}^{T,k} + (1-\alpha)(\gamma - 1)\Delta p_{t+1}^k$$

and where Δp_{t+1}^k reflects the change in relative non-tradable prices. We provide an interpretation of this equation at the end of Section B.2.1.1.

We now substitute for consumption growth and the real interest rate term on the right-hand side of the log-linearized budget constraint.[5] Plugging in for $r_{t+1}^k/\gamma = E(\Delta c_{t+1}^k)$, and using the

[5] This follows Bergin and Sheffrin (2000) and Bouakez and Kano (2009). However, these models do not feature a

decomposition $r_t^{T,k} = r_t^N + \delta^k \tau_t$ from above, we obtain the following expression for net exports

$$\frac{\widetilde{NX_t}^k}{NO_t} = -\sum_{l=1}^{\infty} \kappa^l E_t \Delta\widetilde{no}_{t+l}^k + \phi \sum_{l=1}^{\infty} \kappa^l E_t \widetilde{\Delta q}_{t+l}^k + [1-\phi] \sum_{l=1}^{\infty} \kappa^l E_t \widetilde{r^N}_{t+l} + \delta^k \left[[1-\phi] \sum_{l=1}^{\infty} \kappa^l E_t \widetilde{\tau}_{t+l} \right] \tag{4.4}$$

where we have introduced additional notation so that $\phi = c\left(1 - \frac{1}{\gamma}\right)$ and $\Delta q_{t+1} = (1-\alpha)\Delta p_{t+1}$ is the change in the provincial real exchange rate (i.e. the inflation differential in the relative price of non-tradables and tradables). A detailed derivation of this result is provided in Section B.2.3.1.

This equation suggests four channels of net exports adjustment.[6] The first term reflects the intertemporal consumption smoothing channel that is emphasized by basic versions of the neoclassical model (see e.g. Obstfeld and Rogoff, chapter 2). If output is below (above) trend, so that the sum of its expected changes is positive (negative), the country should run a deficit (surplus) *ceteris paribus*. It is the intuition underlying this channel that has contributed to the conventional perception of China's persistent surpluses as an empirical puzzle: according to this intuition, an emerging economy with high future expected GDP growth rates should run a deficit.[7]

The second to fourth terms all capture how expected variation in prices and interest rates impacts on capital flows. These channels can therefore potentially help explain departures from the simplest neoclassical benchmark model of net exports behavior. The second term is the effect on intertemporal substitution of expected changes in the local price of non-tradables (i.e. intratemporal substitution). If the price of the provincial consumption bundle relative to tradable goods is expected to rise in the future, there is an incentive to save more. In analogy to Hoffmann (2013), we refer to this channel as "internal tilting" since it is driven by relative variation in expected prices of only regionally consumed (non-tradable) to both internationally and domestically consumed (tradable) goods. For example, we would expect that anticipated rises in the local price of housing, schooling or medical care could be important determinants of saving decisions.

The third and fourth terms capture how variation in the (China-wide) real rate of interest and in the impact of the excess return on the foreign bond respectively affect province-level capital flows. If province-level interest rates are temporarily high (because national interest rates or the savings wedge are high), so that the sum of future interest-rate deviations from the long-term mean interest rate is positive, consumers will want to defer consumption and save more. We call the first term the domestic interest rate channel since national – as opposed to global – interest rate variation should matter only in repressed financial markets. We refer to the second term

savings wedge.

[6] Thereafter, we assume $0 < \phi < 1$, which is fulfilled for values of risk aversion (γ) higher than one and most empirical values of the consumption ratio (c).

[7] However, as we discuss in detail below, even in quickly growing economies this channel can also be consistent with surpluses. If financial frictions require firms to finance investment from retained earnings , these saving surpluses will predict increases in investment that can at least temporarily exceed output growth.

as the world interest rate channel. Clearly, both channels become stronger as the intertemporal elasticity of substitution $(1/\gamma)$ – and $(1-\phi)$ – increases.

4.2.2 Empirical implementation

A detailed explanation of the following steps is provided in appendix (Section B.2.3.1). Equation (4.4) is the focus of our empirical analysis of province-level net exports. For each region in our sample, we proxy the expectations on the right hand side of (4.4) using a vector autoregressive model (VAR):

$$X_t^k = \sum_{l=1}^{p} A_l(k) X_{t-l}^k + \varepsilon_t^k$$

where $X_t^k = \begin{bmatrix} \Delta no_t^k & \Delta q_t^k & r_t^N & \tau_t & (NX/NO)_t^k \end{bmatrix}'$ is the vector of endogenous variables, the $A_l(k)$ are 5×5 coefficient matrices of the p-th order VAR and ε_t^k is the vector of reduced-form residuals. Stacking $Z_t^k = \begin{bmatrix} X_t^k, & X_{t-1}^k, & \ldots & ,X_{t-p+1}^k \end{bmatrix}'$, one can write the VAR companion form as VAR(1) so that

$$Z_t^k = A_{\{k\}} Z_{t-1}^k + U_t^k \tag{4.5}$$

where $A_{\{k\}}$ is the companion matrix of the VAR estimated on province k data and $U_t^k = \begin{bmatrix} \varepsilon_t^k, & 0, & \ldots & ,0 \end{bmatrix}'$ the associated vector of residuals. Then, once the VAR-parameters have been estimated, the expectation terms are easily backed out as

$$\sum_{l=1}^{\infty} \kappa^l E_t X_{t+l}^k = e_x' \kappa A_{\{k\}} \left[I - \kappa A_{\{k\}}\right]^{-1} Z_t^k$$

where X_t stands, in turn, for Δno_t^k, Δq_t^k, r_t^N, τ_t, $\frac{NX}{NO}{}_t^k$ and e_x is the unit vector associated with the position of x in the vector Z_t^k (i.e. the first unit vector for Δno, the second for Δq_t etc.). Plugging this representation of the expectation terms into (4.4) above, one gets the NX/NO ratio predicted by the model for each province:

$$\frac{\widehat{NX}}{NO}{}_t^k = \left[-e_{\Delta no}' + \phi e_{\Delta q}' + (1-\phi)(e_r' + \delta^k e_\tau')\right] \kappa A_{\{k\}} \left[I - \kappa A_{\{k\}}\right]^{-1} Z_t^k \tag{4.6}$$

where again $\phi = c\left(1 - \frac{1}{\gamma}\right)$ and where we denote the predicted value of NX/NO from the model with a hat.

For each province and for any known set of parameter values $1/\gamma$, κ, c and δ, the predicted net exports can now be compared to the actual net exports. This can be done either through an informal comparison of the predicted net exports with the data (in terms of correlation and variance) or formally, based on a Wald test.[8] Note that in the above setup, we let the VAR-

[8]Rewriting equation (4.6) for a given companion matrix A as $e_{nx}' = \left[-e_{\Delta no}' + \phi e_{\Delta q}' + (1-\phi)(e_r' + \delta^k e_\tau')\right] \kappa A \left[I - \kappa A\right]^{-1}$ and denoting the right-hand side of this restriction with

parameters vary at the provincial level, allowing for potentially very different dynamics in outputs and prices across regions.

One decision we have to take at this junction is to what extent we want to allow the parameters of the theoretical model like c (the long-term consumption ratio) and in particular $1/\gamma$ (the intertemporal elasticity of substitution) to differ across regions. In principle, c can be recovered from the data. *Prima facie*, it would seem natural to restrict the preference parameter γ to be the same across regions. However, we would expect that the technologies available for intertemporal substitution – and, thus, measured elasticities – vary widely across provinces, e.g. with the level of development. Whether we would also expect this to be the case with respect to the extent to which provinces have access to international markets is an open question. On the one hand, more open or developed provinces may benefit from a more developed financial system and may have access to finance from international banks or firms. On the other hand, state-owned firms may have a privileged access to international markets. We therefore estimate $1/\gamma$, κ and δ using a GMM-procedure for each province separately. We discuss the details of this estimation in Section 4.4.1.

4.2.3 Channels of province-level external adjustment

Once the parameters κ, c, γ and δ have been determined, we can use (4.6) to decompose the variance of each province's net exports as follows. Write the component that is unexplained by the model as $res^k = NX^k/NO^k - \widehat{NX/NO}^k$, take the variance on both sides and plug in for $\widehat{NX/NO}^k$ from (4.6). Then, dividing by $var(NX^k/NO^k)$, one gets

$$1 = \beta^k_{\Delta no} + \beta^k_{\Delta q} + \beta^k_r + \beta^k_\tau + \beta^k_{res} \tag{4.7}$$

$\Psi(A)$, the Wald-statistics $[e'_{nx} - \Psi(A)] \frac{\partial \Psi(A)}{\partial A} var(A)^{-1} \frac{\partial \Psi(A)}{\partial A}' [e'_{nx} - \Psi(A)]'$ is asymptotically distributed as a χ^2 with m degrees of freedom where m is the dimension of the companion matrix A.

where

$$\beta^k_{\Delta no} = \frac{cov\left(-e'_{\Delta no}\kappa A_{\{k\}}\left[I-\kappa A_{\{k\}}\right]^{-1}Z^k_t, NX^k/NO^k\right)}{var(NX^k/NO^k)}$$

$$\beta^k_{\Delta q} = \frac{cov\left(\phi e'_{\Delta q}\kappa A_{\{k\}}\left[I-\kappa A_{\{k\}}\right]^{-1}Z^k_t, NX^k/NO^k\right)}{var(NX^k/NO^k)}$$

$$\beta^k_{r} = \frac{cov\left((1-\phi)e'_{r}\kappa A_{\{k\}}\left[I-\kappa A_{\{k\}}\right]^{-1}Z^k_t, NX^k/NO^k\right)}{var(NX^k/NO^k)}$$

$$\beta^k_{\tau} = \frac{cov\left(\delta^k(1-\phi)e'_{\tau}\kappa A_{\{k\}}\left[I-\kappa A_{\{k\}}\right]^{-1}Z^k_t, NX^k/NO^k\right)}{var(NX^k/NO^k)}$$

$$\beta^k_{res} = \frac{cov(res^k, NX^k/NO^k)}{var(NX^k/NO^k)}$$

where again $\phi = c\left(1-\frac{1}{\gamma}\right)$. Here, $\beta^k_{\Delta no}$ is the contribution of net output variation (consumption smoothing or net output channel), $\beta^k_{\Delta q}$ is the contribution of expected changes in relative price of non-tradables (internal price channel), β^k_r is the contribution of (expected) variation in the national real rate of interest (domestic channel) and β^k_τ the variation arising from changes in excess returns (international channel). Note that τ is always used in the VAR predictions but is not necessarily present in (4.6) because we allow for a weight (δ) of zero in the grid-search. The coefficient β^k_{res} is the fraction of the variance of province k's net exports that remains unexplained by the model.

For notational compactness, we collect the various β^k_xs into the vector

$$\beta^k = \left[\begin{array}{ccccc} \beta^k_{\Delta no} & \beta^k_{\Delta q} & \beta^k_r & \beta^k_\tau & \beta^k_{res} \end{array}\right]'$$

and we call β^k the pattern of external adjustment of province k. In what follows, we also allow for the possibility that the elements of β^k vary over time.[9]

At the level of each province, the elements of β^k could easily be estimated from time series OLS regressions of the expected present values of $\widetilde{\Delta no^k}$, $\widetilde{\Delta q^k}$, $\widetilde{r^N}$ and $\widetilde{\tau}$ on NX^k/NO^k respectively. However, our main interest in this paper is also to analyze to what extent province-level characteristics (such as financial and economic development, industrial structure, demography, etc...) affect the patterns of external adjustment and, potentially, also to allow for time-variation in these variables. We therefore posit that the external adjustment in province k through a given channel is an affine-linear function of a vector z^k_t of province-level characteristics so that

$$\beta^k_x(t) = \beta_x + \gamma'_x z^k_t \tag{4.8}$$

[9]Note that these β^k_xs are not to be confused with the discount parameter of the utility function (β).

where x denotes the respective channel. The coefficient β_x measures the average (across-provinces) importance of channel x and the vector γ_x describes the sensitivity of the respective external adjustment channel to variation in characteristics (z) across provinces. This assumption on $\beta_x^k(t)$ allows us to analyze the cross-provincial variation in external adjustment patterns using a panel set-up. Specifically, we estimate $\beta_x^k(t)$ from the following relationship

$$x_t^k = \alpha + \tau_t + \mu^k + \beta_x^k(t) \times \left[\frac{NX}{NO}\right]_t^k + \psi' \times z_t^k + \nu_t^k \tag{4.9}$$

where x_t^k stands in turn for the VAR-implied expectations of the corresponding channel. On the right hand side of (4.9), α is a constant and τ_t and μ^k are time- and province fixed-effects. The vector z_t^k stacks characteristics as before. For each channel, equation (4.9) can be estimated as a panel regression once we plug in from (4.8) above:

$$x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left[\frac{NX}{NO}\right]_t^k + \gamma_x' \times z_t^k \times \left[\frac{NX}{NO}\right]_t^k + \psi' \times z_t^k + \nu_t^k \tag{4.10}$$

The coefficient on $\left[\frac{NX}{NO}\right]_t^k$ then measures the average importance of the channel x across provinces (β_x), whereas the coefficients on the interaction terms of the regional net exports with the province-level characteristics (γ_x) capture the sensitivity of the respective channel to variation in characteristics over provinces and time.

4.3 Data

4.3.1 General remarks

When not mentioned otherwise, data used in this chapter are from the *National Statistical Yearbooks* of the People's Republic of China and from the *Provincial Statistical Yearbooks* of the 22 provinces, 5 autonomous regions and 4 municipalities of Mainland China.[10] The *China Data Center* (CDC) of the University of Michigan provides electronic access to the yearbooks and made main statistics conveniently available.[11] For most provinces, our online access only covers regional statistical yearbooks in the 1990s and 2000s. Thus, it happens that the data are sometimes incomplete. We will primarily rely on data directly retrieved from recent online yearbooks and complete possible gaps with CDC sheets. This allows us to take account of revisions as much as possible.

The quality of provincial and aggregate Chinese *National Accounts* data is an important issue

[10] The autonomous regions are Tibet, Xinjiang, Guangxi, Inner Mongolia and Ningxia. The cities of Beijing, Tianjin, Shanghai as well as the region of Chongqing are municipalities. Thereafter, the term province will be used as general qualifier.

[11] `http://chinadataonline.org/`. The CDC reports values as soon as they are published in the corresponding yearbook. Although data have sometimes been subject to official revisions in later years, the CDC did not systematically adapt past values.

that we explore in the second chapter of the thesis, where we focus on some stylized facts and discuss the quality and aggregation properties of the data. This analysis revealed large discrepancies between aggregate statistics and the sum of provincial statistics. For example, the sum of province-level GDPs was about 11 percent higher than the officially published national value in 2010. The bulk of this large error stems from an excess of regional over national investment, which has been widening since the mid-1990s. Conversely, the discrepancy between cumulated provincial saving and national saving shows no clear trend over time. Still, the sum of province-level saving overestimated national values by around 7 percent of China's GDP in 2010. All things considered, it suggests that, since the mid-2000s, the sum of province-level net exports will generally be lower than the corresponding official aggregate statistics. Other authors have argued that China's current account surplus is overstated for a variety of reasons (see Zhang, 2008). Whether regional data are more affected than national ones is an open question (e.g. the *2004 Economic Census* validated provincial GDP data and invalidated national ones (Holz, 2008)).

While there is some uncertainty concerning the levels of aggregate and regional statistics, our exploratory analysis also showed that the sum of province-level GDP, investment and saving data is generally highly correlated with movements in aggregate statistics.[12] Since our empirical analysis focuses on a log-linearized model that emphasizes the movements in these variables over time rather than their specific levels, we are reasonably confident that our province-level data capture important aspects of external adjustment among China's provinces. In the appendix (Section B.1), we provide a description of the data and indicators used in our analysis. Apart from Tibet – for which data are incomplete – we are able to estimate the model for all 30 provinces.

4.4 Results

4.4.1 Fitting the model to province-level net exports

We estimate the province-level VAR with one or two lags.[13] This allows us to back up the VAR-implied expectations on the right hand side of the present-value relation (4.4). For each province, we then estimate the parameters of the model – $1/\gamma$, κ and δ – based on a three-dimensional grid-search procedure that minimizes the squared deviation between the right hand side of (4.4) and the respective province's observed net export / net output ratio. In the grid-search procedure, we let the coefficient of relative risk aversion (γ) vary between 0.2 and 5, the discount parameter (κ) between 0.900 and 0.995 and the world market integration parameter (δ) between 0 and 1.

Table 4.1 summarizes the estimated parameter values, the general fit of the model in terms of correlation and the relative variance of predicted to actual net exports. In order to better

[12]Over 1985-2010, the correlation of the first difference of national net exports with cumulated net exports is 0.80. It rises to 0.87 for 2000-2010, the period in which global imbalances arose.

[13]Additional lags generally do not improve the fit.

appreciate the economic importance of provinces, we provide their relative share of cumulated 2000 real GDP and their rank. The ten largest provinces – accounting for more than 60% of output – are in bold type. A geographical representation of provinces' economic size classified by quartile and normalized by the largest region is available in Figure 4.2. We refer to Table 4.4 for more details about the specification (sample length, number of lags, net output deflator, consumption ratio and ϕ parameter).[14] For most provinces, our simple model provides a good fit: the mean correlation between actual and predicted net exports is 0.96 while the lowest value is 0.77. The model also matches the standard deviation of actual province-level net exports quite closely: the average relative standard deviation is 1.06.

While large cross-provincial variations exist, on average, we find a plausible value of γ of 2.59 (i.e. an average elasticity of substitution of 0.39). This coefficient is comparable with values conventionally used in the literature and somewhat lower than estimates of Hoffmann (2013) for China as a whole (0.71 for 1982-2010 with updated IFS data). For the discount factor, we find a value of 0.95 on average. In general terms, the GDP-weighted means are close to the non-weighted parameter values. Consequently, results of small – and potentially "noisier" – provinces do not affect the parameters and the fit massively. Note in particular that the model seems to perform especially well when applied to relatively large and more developed provinces. Figure 4.1 provides a graphical representation of the predicted and real net exports of the three largest provinces (Shandong, Guangdong and Jiangsu) and seven other provinces representative of the geographical and structural diversity of China.

4.4.2 Channels of adjustment

We turn to the decomposition of the variance of province-level net exports into four channels as in equation (4.7). For each province, the results of this decomposition are given in Table 4.2. By way of example, Figure 4.3 provides a plot of actual and model-implied net exports along with a breakdown into the four channels for four provinces: Liaoning, Shanghai, Guangdong and Yunnan.

With a little more than three quarters on average, expected variation in regional cash flow – net output (NO) – accounts for the bulk of variation in province-level capital flows, with variation in intertemporal prices – internal price (IP), domestic (DR) and international interest rate (IR) – accounting for the rest. However, these numbers mask considerable variation across provinces. We provide a map of the variance decomposition of net output in Figure 4.4, where provinces are classified by quartile. Variation in cash flow available for consumption seems to be important

[14] The model is estimated on the 1985-2010 period with the exception of five regions that have a reduced sample. Guangxi, Yunnan (South) and Shaanxi (West) experienced huge decline in net exports toward the end of the sample. Shanxi and Ningxia (West) had very high volatility in their relative net exports in the initial reform years. The consumption ratio is estimated over the sample length using the same deflator as for output and government consumption. Some smaller, less developed provinces happen to have a value higher than one but the GDP-weighted value is 0.92. Apart from a few exceptions, the ϕ parameter is lower than one. Thus, even if large variations in parameters exist, the GDP-weighted mean of all parameters is in an economically plausible value range.

in some coastal regions as well as central China. By contrast, in Figure 4.5, the dynamics of the relative price of non-tradables seem to play a bigger role in less developed regions (i.e. the South and part of the Center and Manchuria) while its contribution is negative for most eastern provinces.

In the City-Provinces such as Shanghai and Beijing, variation in world interest rate is an important driver of variations in net exports. This is clearly in line with the notion that these major cities are more open to foreign trade and capital, which we would expect to facilitate access to finance from abroad. Conversely, in the East Coast provinces, with their many private SMEs that do not have access to formal finance, financial repression – expected variation in domestic real interest rate – is a key driver of capital flows. A geographical representation of the interest rate channel (both domestic and international) is available in Figure 4.6.

4.4.3 Regional external adjustment: panel analysis

In the preceding section, the contribution of the different factors has been estimated for each region separately. In order to gain an overview of the general patterns of regional external balance, we now turn to estimating the patterns of external adjustment in a panel framework as discussed in Section 4.2.3. We start with a general characterization of external adjustment in the average province. Equation (4.10) is estimated without any province-level characteristics (i.e. without the z_t^k), which gives us the specification

$$x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left(\frac{NX}{NO}\right)_t^k + v_t^k$$

where the x stands for one of the VAR-implied expectations of the four channels from equation (4.4). Results of this exercise (i.e. the β_xs) are given in the first panel of Table 4.3, where bold type indicates significancy at the 10% confidence level. The panel-based estimates reveal again that the bulk of capital flows among Chinese provinces is driven by expected variation in quantities: net output fluctuations explain around 70 percent of the variation in NX/NO for the average province. Variation in the expected relative price of non-tradables and expected domestic and international interest rates each drive around 5 percent. Though these numbers seem small at first sight, they are all significant.

The second row of the table shows the result of a panel estimate in which we weigh the channels and net exports by the provincial real GDP share.[15] A geographical representation of the relative weights is available in Figure 4.2. While variation in net output remains the main driver of NX/NO (the point estimate now increases to 0.82), we now also find a much bigger role for the domestic interest rate channel than in the unweighted panel (11 percent of the variation in NX/NO vs 5 percent). At first sight, this suggests that financial repression may be an important driver of saving dynamics in the more developed and larger provinces with their many private

[15] We use 2000 real GDP as in Table 4.1 normalized by the largest province (Shandong=1).

firms, most of which do not have access to formal finance. It seems that the significancy of the relative price of non-tradables and the international interest rate channels is driven to a large extent by smaller regions as it turns out to disappear in this configuration. The residual share is at its lowest (3 percent).

The third row presents results from an alternative weighting scheme that is based on the variance of the unexplained part of province-level net exports. In this weighting procedure, regions that are better explained by our model get a higher weight.[16] Estimates are qualitatively similar to the non-weighted and GDP-weighted versions, with all channels – with the exception of the domestic interest rate (DR) – being significant.

In the second and third panels of Table 4.3, we estimate the external adjustment patterns for geographic subgroups of provinces using GDP-weighting.[17] For provinces constituted of a major metropolitan areas (such as Beijing and Shanghai), we again find a big role for variation in international interest rate. To a lesser extent, the same seems true of southern provinces and Manchuria, two regions where the presence of large state-owned firms could have facilitated access to international financial markets. Conversely, in the East Coast and central regions, the domestic interest rate channel is found to be relatively important, consistent with the view that these provinces seem to have much less access to international finance.[18]

Interestingly, variations in domestic prices in non-tradables have a strong negative influence in eastern regions while the coefficient is small and positive in two neighboring geographic areas (the Center and Manchuria). Apart from idiosyncratic data issues, a potential explanation for the lack of significancy in the channels of western China is its heterogeneity and the high number of provinces (7). What is more, the model performs less well for those regions. In the last row of the second panel (*NoMetro&EC*), we gather inner provinces (regions not being part of the more developed East Coast and not being Metropolises) in the same group. The difference between East Coast regions and inner regions does not seem to arise from the net output channel (0.86 vs 0.83) but rather from the relative importance of the internal price channel (-0.18 vs 0.05) and, again, the domestic channel (0.28 vs 0.07). The unexplained part is higher in hinterland provinces (0.04 vs 0.00).

The third panel of Table 4.3 also provides an indication that the pattern of adjustment is strongly affected by the degree to which a provincial economy can be characterized as either market-based or centrally planned. To this end, we use an index of marketization initially devel-

[16]This weighting procedure uses the absolute residual share of the variance decomposition ($[max(abs(RES)) - abs(RES^k)]^2$). As for real GDP, we normalize by the highest weight (i.e. Hainan=1 has the best fit).

[17]The definition of regional clusters is as in preceding chapters. Metropolises: Beijing, Tianjin and Shanghai. East Coast: Hebei, Shandong, Jiangsu, Zhejiang, Fujian and Guangdong. Manchuria: Liaoning, Jilin and Heilongjiang. Center: Henan, Hubei, Hunan, Anhui and Jiangxi. West: Shanxi, Inner Mongolia, Shaanxi, Ningxia, Gansu, Qinghai and Xinjiang. South: Chongqing, Sichuan, Yunnan, Guangxi, Guizhou and Hainan.

[18]The fact that Manchuria has a large domestic and international channel is not necessarily a contradiction. While this cluster historically had large state-owned firms, particularly in the heavy industry and energy-related sector, FDI, foreign and private firms started to thrive in the later reform period, particularly in Liaoning.

oped by Fan et al. (2001) and split provinces into two equal subgroups of high and low marketization.[19] Again, net output fluctuations remain the predominant force of external adjustment in both groups. But there is a marked distinction in the way in which prices and interest rates influence capital flows in the two categories of provinces: in the high-marketization group of provinces, variation in domestic and world interest rate respectively explains 15 and 13 percent of the variation in province-level net exports. In the low-marketization group, variation in world interest rate does not matter at all, while financial repression – variation in domestic interest rate – explains a much smaller share. Finally, in the low-marketization group of provinces, there is also a small but significant role for expected non-tradable price changes in explaining NX/NO while this figure is slightly negative and not significant for more advanced regions.

4.4.4 External adjustment and characteristics (standalone)

Our ultimate goal is to assess the relative empirical merit of different theoretical explanations that have been put forward for China's current account surplus. To this end, we now characterize the heterogeneity in external adjustment patterns across provinces more sharply by correlating them with province-level characteristics. We categorize these characteristics into four groups, each of which corresponds roughly to a broad set of theoretical explanations that have been put forward for China's big saving surplus: i) indicators of the relative role of state-owned and private enterprises in the local economy and financial development, ii) indicators of integration into the world economy, iii) indicators of sectoral composition and iv) demographic indicators.

To quantify the impact of these characteristics on province-level external adjustment, we use the panel setup with interaction terms (4.10).[20] Specifically, we estimate:

$$x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left(\frac{NX}{NO}\right)_t^k + \gamma_x{}' \times z_t^k \times \left(\frac{NX}{NO}\right)_t^k + \psi' \times z_t^k + \varepsilon_t^k$$

where z_t^k is our vector of regional characteristics. To gain an impression of the link between external adjustment and particular provincial factors, we start by considering z_t^k one by one (one individual variable at a time). To at least partly alleviate the omitted variable issue, we systematically control for the mean level of development of a region ($\overline{Dvpt}$), measured here as the average of real GDP per capita relative to national values over the sample period. As in the second and third panels of Table 4.3, channels and net exports are weighted by provincial real GDP share.[21]

Table 4.5 (left panel) gathers the results that we obtain for indicators related to the importance of the state/private sector in the regional economies. By looking at the interaction term of

[19] Rank based on the mean of the index over 1997-2005. Factors: government and market, ownership structure, goods market development, factor market development and legal framework.

[20] For a similar specification applied to international shock transmission during the interwar gold standard, see Hoffmann and Woitek (2011).

[21] There are two reasons why we favor this weighting scheme. First, small provinces have typically noisier data and very large variations in net exports. Second, essentially, we want to shed light on the drivers of global imbalances.

economic development and relative net exports in the following regressions, we generally find a bigger role for global variation in international interest rate in more developed provinces, and a more mitigated role for the internal terms of trade. This pattern is consistent with the view that housing prices are a less important margin of adjustment in more developed parts of the country, where *de facto* access to world financial markets is slightly better.

The results from Table 4.5 also clearly suggest that a higher share of SOEs (as measured by the share of state-owned firms in gross industrial output value, *SOGIOV*) in the provincial economy leads to a smaller contribution of the net output channel to external adjustment as well as a larger internal price and world channel. Alternative indicators of the presence of the state in the economy would lead to similar patterns.[22] Complementary evidence is borne out once we condition on a general indicator of the extent to which a province can be characterized as a market economy: higher levels of marketization (*Market*) coincide with a significantly bigger role of the domestic (financially repressed) interest rate and a smaller role of the internal price channel.

The literature – notably Song et al. (2011) – has emphasized that the major distinction between private and state-owned firms is that the former are financially repressed whereas the latter have preferential access to bank credit and therefore – indirectly – to international financial markets. Our results provide strong cross-provincial evidence that supports this view. In particular, if the expanding private sector can only finance its growth from retained earnings, we will see that province-level surpluses predict increases in investment and, *ceteris paribus*, declines in net output. This is perfectly in line with our finding that the role of net output as a channel of external adjustment increases with the importance of the private sector. In the same mould, we would expect saving and investment decisions by financially repressed private households and firms to be more dependent on future domestic interest rates than on international ones, whereas the opposite should be true for state-owned enterprises with (implicit) access to international financial markets. This is exactly the pattern that we find in the data.

A last factor that we focus on is the balance of funds available in banks and financial institutions (the difference between deposits and loans normalized by provincial GDP). Another implication of Song et al. (2011) is that during the transition process, state firms shrink in favor of private enterprises. As the former's economic importance dwindles and their investment opportunities dry out, regions with faster growing private sector have an increasing surplus of deposits compared to loans as the largely state-owned financial sector does not redirect funds to the emerging private sector. At first sight, our results are compatible with that explanation as they suggest that larger deposit surpluses ($(Deposits - Loans)/GDP$) make the domestic interest rate

[22] As the bulk of bank loans mainly goes to state-owned enterprises, we could use loans in financial institutions over GDP as an alternative indicator for the presence of the state in the economy. The sum of deposits and loans relative to GDP is usually interpreted as a rough index of financial development. This indicator would behave qualitatively in a similar way as loans, suggesting that financially more developed provinces have lower net output channel and higher international interest rate channel.

channel more important for explaining net exports adjustment.

In the second part of Table 4.5 (right panel), we discuss some demographic factors. Urbanization (*Urbanization*) is related to a decrease in the importance of the net output channel and an increase in the contribution of other channels to net exports adjustment, particularly international interest rate. Thus, it seems that a higher level of urbanization – and the associated increase in economic complexity – of a region gives rise to alternative smoothing possibilities compared to the classical textbook adjustment. Unbalanced sex ratios (*SexRatio*) have been proposed as one of the main driver of Chinese imbalances (Du and Wei, 2010). In fact, it seems that internal price of non-tradables relative to tradables play a more significant role in regions with higher male to female ratio.[23] We observe a similar pattern for our human capital indicator: student enrollment in higher education (*HighEduc*) seems to drive up the importance of that channel.

In the first part of Table 4.6 (left panel), we discuss factors related to the integration of provinces into the world economy. As we would expect, openness (*Openness*) generally increases the role of world financial markets and therefore of the world interest rate for external adjustment. It also lowers the role of internal price adjustment of non-tradables, again consistent with the view that housing price adjustments are less important where alternative investible assets are available – which is likely to be the case in more open provinces. However, we generally also see a bigger role for the domestic (financially repressed) interest rate in more open provinces. This could be the case because these regions are also the ones where private enterprises tend to grow most quickly. Furthermore, it could be that our identification strategy does not always sharply separate domestic from international interest rate dynamics.

As discussed in the literature (e.g. Alder et al., 2013), special economic zones (SEZs) have played a pivotal role in China's development over the last decades. In our sample, FDI (*FDI*) seems to be significantly associated with an increasing contribution to international interest rates variability and a decreasing contribution to net output variability. Thus, regions where firms increasingly get access to foreign financing seem to experience a decrease in the classical channel of adjustment: private enterpreneurs are less dependent on their own savings to finance investment projects and supply working capital but increasingly have access to international markets via foreign firms.[24] Conclusions are very similar by using the relative importance of foreign investment in fixed asset (*FOInvFA*) as an alternative proxy.

In the second part of Table 4.6 (right panel), we focus on key indicators of economic structure. In our sample period (1986-2010), for most regions, the massive transformation of the Chinese economy is known to have primarily been driven by the structural change from a largely agricultural and resource-based economy to an industrial one. Private and foreign firms rapidly expanded at the cost of state-owned firms, particularly in the manufacturing sector. A larger

[23] Note that one would need reliable regional yearly time-variation in this indicator in order to truly test this theory in our framework.

[24] Note that FDI is not robust to the inclusion of development. They are highly correlated.

share of the industrial sector relative to GDP (*Industry*) seems to be associated with a significant increase in the contribution of the net output channel and a decrease in other channels. It could indirectly corroborate earlier findings (i.e. if a rising share in industry was associated with a decreasing state sector).[25]

The share of the construction sector in the economy (*Construction*) is strongly correlated with the internal price channel, which seems to make sense (i.e. this channel plays a bigger adjustment role in regions with increasing importance of housing/infrastructure in output). At last, the domestic interest rate seems to play an important role in regions with an increasing patent density (*Patents*), even when controlling for large differences in economic development. If innovation stems from financially repressed private firms, this could be compatible with the preceding findings. The – initially surprising – negative coefficient on net output is driven by a few well-integrated provinces (e.g. Beijing and Shanghai) that already started the transition from an industrial to a service economy towards the end of the sample.[26]

4.4.5 External adjustment and characteristics (multivariate)

In Table 4.7, we provide a multivariate regression with six main explanatory factors. Our goal is to shed light on the drivers of more recent variations in regional external balances. For that reason, we focus on the second half of the sample (1997-2010). Historically speaking, it captures the acceleration of the integration of coastal regions into the world economy and the general spread of marketization into the Chinese hinterland. An added benefit of limiting our sample period in this way is that all province-level characteristics with the exception of the demographic one (*SexRatio*) are available on an annual basis. This allows us to treat them as time-varying in our interaction regressions and increases the data variation that we can use for identification. As a broad and representative vector of provincial characteristics is used, we do not control for the level of economic development anymore. Factors are cross-sectionally demeaned.

The patterns of sex imbalances (*SexRatio*) are unchanged. The relative importance of the state in gross industrial output value (*SOGIOV*) is not significant anymore. Instead, the share of private employment (*EmplPrivate*) seems to play a key role in net exports adjustment, particularly for the net output channel (positive level-effect) and the international interest rate channel (positive interaction term). Interestingly, surpluses in the financial sector ($(Deposits - Loans)/GDP$) are strongly correlated with an increase in the importance of the interest rate channel, particularly with the domestic one. At this point, it seems that the accumulation of large surpluses of deposits over loans is potentially linked to the emergence of private firms that largely rely on self-financing and, ultimately, on domestic interest rate as suggested by Song et al. (2011).

[25] With some exceptions (e.g. Beijing or Shanghai), it seems true that high share in industry has been concomitant with a low share of state employment. However, it is not the case for relative changes over time.

[26] As already shown in Table 4.3, Metropolises tend to have a small net output channel.

As before, FDI (FDI) seems to increase the share of the international channel in net exports adjustment. Importantly, it has an even stronger opposite effect on the domestic channel. Regions with larger FDI share have better access to international markets and become less sensitive to domestic interest rate (i.e. are less financially repressed). The relative size of the industrial sector *per se* ($Industry$) does not seem to impact on net exports adjustment anymore. Still, it has a positive level effect on the internal price channel and the international channel as a standalone while it lowers the net output channel.

In conclusion, while it seems that price channels are robust and still informative, variations in quantities are more difficult to disentangle in such a multivariate framework. This is not surprising given the fact that net output itself is a linear combination of three variables that potentially react very differently to our explanatory factors.

4.4.6 Implications for China's aggregate surplus

In this section, we examine the implications of our province-level analysis for China's aggregate surplus. Figure 4.7 plots the sum of province-level net exports over our sample period against the sum of fitted values from our province-level models.[27] The two lines comove almost perfectly. In particular, the China-wide aggregate of our fitted province level net exports clearly replicates the run-up in China's net exports from the late 1990s until 2007/08 and the subsequent sharp decline. This suggests that our model has substantial power for understanding the province-level sources of global imbalances before 2007/08 and of their subsequent – partial – correction.

To shed light on this issue, we add up the regional model-based decompositions from each province to obtain China-wide aggregates of our four channels (Figure 4.8). As was the case for most provinces, the bulk of variations in aggregate net exports and also most of the run-up over our sample period are driven by intertemporal variation in national cash flow (net output). This channel also accounts for most of the correction of China's surplus. The negative 2009 shock stems from a large number of regions, independently of their characteristics. It is corroborated by official current account statistics at the national level.

Variation in the world interest rate plays only a relatively minor role overall, consistent with the view that China's economy as a whole is relatively closed so that variation in global interest rate plays only a limited role for the saving decisions of private households and firms and, eventually, for aggregate external surpluses. This is consistent with the view that, over most of the first decade of the 2000s, China's external balance was to a large extent reflected in official reserve accumulation, which, in turn, was driven by the need to counteract appreciation pressure on the Renminbi.

While the role of expected variation in the domestic interest rate – our measure of financial repression – appears limited overall, it makes a persistent and positive contribution to China's

[27]The *raw* line corresponds to the data for all regions over 1985-2010.

surplus. In fact, from the mid-1990s to 2000, it was the main driver of national net export dynamics. It started to rise again in the second half of the 2000s against the (possible) backdrop of an increasing discrimination in private firms' access to finance. More than half of the 2008 negative shock in that channel stemmed from three large and highly marketized provinces (Jiangsu, Shandong and Guangdong).

Variations in the internal terms of trade had a stabilizing effect on net exports in the period after the turn of the millenium, which was characterized by high and increasing surpluses. This suggests that internal price pressure on housing, medical expenses and schooling had a major dampening effect on China's burgeoning external surplus during this period. Non-tradable inflation therefore contributed substantially to the required internal revaluation of the Renminbi that could not occur externally in a system of largely fixed nominal exchange rates.

4.5 Robustness checks

4.5.1 Alternative specification

We construct an alternative panel with important adjustments in the net output deflators discussed in Section B.1.2 and in the number of lags (1 or 2). We choose an alternative specification. Of the 30 provinces, 26 have a change in specification (10 in the number of lags, 12 in the deflator and 4 in both). We were not able to find an alternative for 4 provinces.[28]

Mean deep parameters are roughly similar to the baseline case. The general fit is only marginally worse and the model still does a remarkable job in explaining regional net exports. In the panel setting, the net output channel rises in importance while most price channels become smaller and lose significancy. With real GDP-weighting, two price categories are still significant: the internal price channel is more negative (-0.13 vs -0.02) and the domestic one nearly doubles in size (0.21 vs 0.11). The main regional results discussed in Section 4.4.3 are maintained. The East Coast and the Center have an even larger financial repression channel. Net output adjustments are now clearly more important in more marketized provinces than in less marketized ones.

It seems that the increase in the size of the net output and domestic interest rate channels comes at the cost of less significancy and a lower coefficient in our factor regressions. The presence of the state in the economy (e.g. *SOGIOV*) now seems to significantly negatively impact on the financial repression channel instead of net output, which is still compatible with our story. Interestingly, in a shorter sample (1997-2010), the coefficient on the share of private employment interacted with net exports has a positive and significant sign for the net output channel. On top of that, it is robust to the inclusion of other factors.[29] It corroborates earlier findings.

[28] By using alternative deflators, Jilin experiences a huge shock in net exports. Jiangxi has highly asymmetric channels. Shandong and Gansu have a miserable fit. These provinces are neither related geographically nor are they similar economically speaking. Thus, they should not invalidate our test.

[29] Private employment is only time-varying for 1997-2010. That is why we did not use it in the full sample part.

Most importantly, our conclusions concerning the impact of the respective channels on global imbalances are similar: the aggregate net output and domestic interest rate channels have similar dynamics. The world interest rate does not contribute to aggregate imbalances anymore. The internal price channel is still negative over the period with the exception of 2009/2010.

4.5.2 Region-specific interest rate

As explained in Section B.1.4, we used national RPI (retail price index) inflation as a proxy for inflation in tradable good in our proxy for domestic interest rate and used regional RPI in the internal price index of Section B.1.6. Considering differences in inflation across provinces (i.e. make the domestic interest rate region-specific) could influence our patterns substantially. In this section, we consider provincial instead of national RPI inflation in the domestic interest rate channel.

Changes in parameters and general fit are minor. In the panel regressions, the financial repression channel is slightly smaller but still strongly significant. Other channels are not greatly affected. In the standalone factor regressions (with development as control), patterns are similar. If at all, coefficients on the presence of the state are even higher (e.g. $SOGIOV$, $Market$ and $(Deposits - Loans)/GDP$). Aggregate patterns are similar.

4.5.3 CES case

In the baseline model, the intratemporal elasticity of substitution was unity. An increase in the relative price of the tradable good of 1% thus implied a corresponding decrease in its relative consumption. A priori, as for intertemporal elasticities, large discrepancies in the level of development across regions could imply large differences in substitution technology. In that respect, a natural extension would be to allow for a flexible intratemporal elasticity as well. First, it could influence the size of the relative price channel in some provinces. Second, it is a theoretical extension of our model that may be useful for future applications.

As a theoretical contribution, we provide three methodologies to implement flexible elasticity in our framework. In Section B.2.1.4, we linearize around a constant steady-state drawing on an alternative derivation of Bergin and Sheffrin in Section B.2.1.3.[30] It is the version that we estimate in this section. As an alternative, we propose a linearization around a steady-state growth path assuming different functional growth forms for relative prices (Section B.2.1.5 and B.2.1.6).

Instead of $\phi \sum_{l=1}^{\infty} \kappa^l E_t \widetilde{\Delta q}^k_{t+l}$, the new internal price channel is

$$\phi \sum_{l=1}^{\infty} \kappa^l \frac{\bar{Q}^{1-\theta}}{(1-\alpha)+\alpha\bar{Q}^{1-\theta}} E_t \widetilde{\Delta q}^k_{t+l}$$

[30] We are grateful to Alexander Rathke for suggesting us that approach.

where $Q_t = P_N/P_T$ and θ is the intratemporal elasticity of substitution. We provide the new form of our empirical implementation in Section B.2.3.2. For $\bar{Q} = 1$, one obtains the baseline expression. The implementation is as follows: the variable $\bar{Q}^{1-\theta}$ is integrated in the grid-search that becomes four-dimensional. Its value is restricted between 0.2 and 5, which allows for a rich variation among regions in both deep parameters.[31]

The GDP-weighted average value of $\bar{Q}^{1-\theta}$ across provinces is of 1.55, larger than the value of 1 implicitly assumed in the baseline version. There are only minor changes in general fit and internal price channel size among regions. With regard to the factor regressions, coefficient of variables related to the importance of the state/private sector and sectoral composition are larger and still significant. However, it seems that the excess return channel is less informative: it is not strongly associated with variables of international integration anymore. Still, the same patterns arise in the 1997-2010 regression (multivariate). On the aggregate level, the contribution of internal prices to stabilizing Chinese net exports is higher (more negative value) while world interest rates are less volatile (i.e. they contribute less to the recent increase). Importantly, the contribution of both the net exports and financial repression channels is similar.

4.6 Conclusion

We have proposed a simple, theory-based framework to analyze capital flows among Chinese provinces. Our framework nests two broad channels of external adjustment in interprovincial capital flows. The first is variation in intertemporal prices, which we further disaggregate into variation in the domestic real interest rate, the excess return on international assets over the domestic rate, and variability in real exchange rate (i.e. the relative price of tradable and non-tradable goods). The second is intertemporal variation in quantities (cash flows of output, investment and government spending). As we show, our simple model can account for 85 percent of the variation in a panel of 30 province-level net exports over the 1985-2010 period.

More importantly, modelling province-level net exports allows us to identify how the patterns of external adjustment depend on province-level characteristics. We have focused on four groups of characteristics that the literature has emphasized as potentially important in explaining China's persistent surpluses since the mid-1990s: i) the relative importance of private and state-owned enterprises (SOE) and the differential access of these types of firms to finance, ii) a province's degree of integration into the world economy in terms of openness to FDI or trade, iii) sectoral composition and iv) demographics.

We find that there are major differences in the patterns of adjustment across provinces. In particular, the relative importance of SOEs and private enterprises in the local economy has a major bearing on the pattern of external adjustment. Intertemporal variation in net output – GDP less investment and government spending — is particularly important as a driver of capital flows

[31] For $\theta = 0.5$ or $\theta = 2$ this would allow for values of $\bar{Q}$ between 0.2 and 5.

in provinces with a strong presence of private firms, as is the domestic interest rate. This pattern is consistent with theories that see financial repression as a major source of China's persistent surpluses: under financial repression, private firms do not have access to bank finance and therefore have to finance investment from retained earnings. As a result, surpluses are better at predicting decreases in net output (via increases in investment) in provinces with a large share of private firms. The absence of access to international finance also means that the domestic (financially repressed) interest rate is the relevant driver of saving decisions of households and private firms.

Furthermore, we show that a higher integration into the world economy – international openness and FDI – is strongly related to a rising importance of international interest rate and to a decrease in intertemporal variation in quantities (net output). Foreign participation thus possibly alleviates financing constraints of the private sector. We also find that variation in non-tradable prices (e.g. housing) is an important driver of net export variation in less developed regions, suggesting that housing is particularly important as a savings vehicle when there is a lack of investible assets.

Our framework allows us to reconstruct Chinese net exports from the inside. We find that most of the 2000s run-up and the successive adjustment is driven by intertemporal variation in net output. During this period, the domestic interest rate channel (financial repression) makes a persistent and positive contribution to China's surplus. Variation in the world interest rate plays only a relatively minor (but increasing) role overall while internal price pressure on non-tradable goods has a major dampening effect on China's burgeoning external surplus.

Figure 4.1: NX/NO: data (solid) versus predicted (dashed), 1986-2010

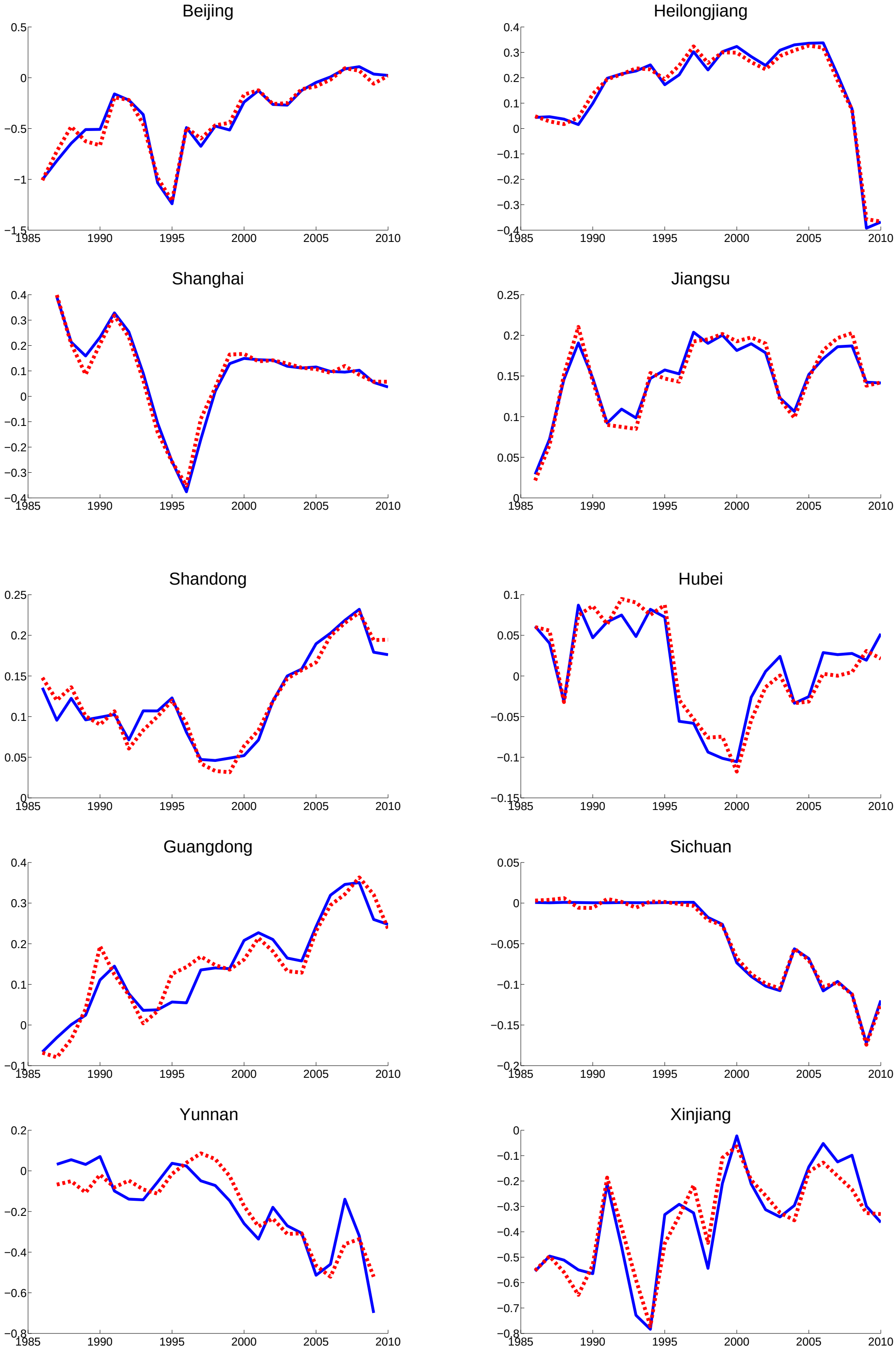


Figure 4.2: Real GDP weights (2000, largest province=1)

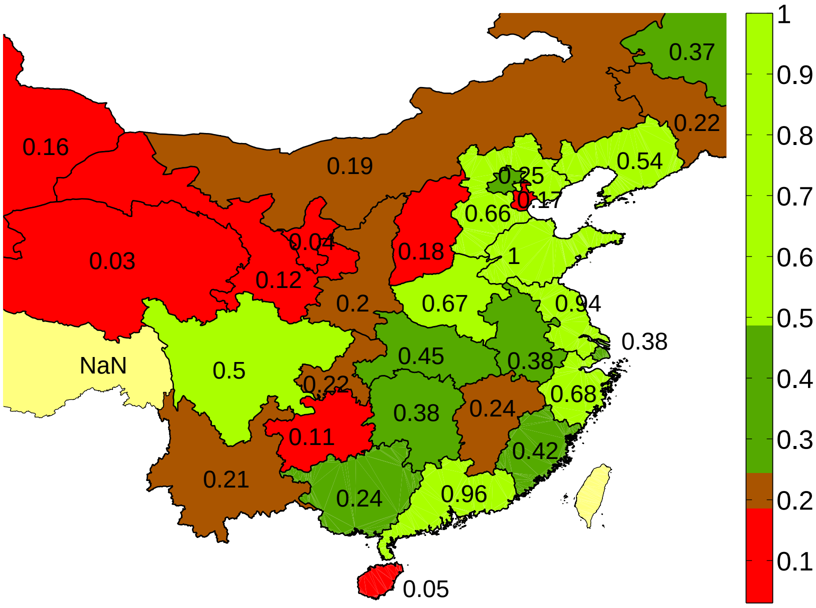


Table 4.1: Specification, grid-search results and basic fit measures

	γ	κ	δ	$\rho(\hat{x},x)$	$\sigma(\hat{x})/\sigma(x)$	*rGDP*	*rank*
Beijing	0.80	0.995	0.95	0.98	0.95	2.3%	15
Tianjin	5.00	0.995	0.90	0.99	0.88	1.5%	24
Hebei	**4.80**	**0.960**	**0.00**	**0.96**	**1.00**	**6.0%**	**6**
Shanxi	2.80	0.900	1.00	0.93	1.04	1.6%	23
Inner Mong.	0.70	0.900	0.00	0.98	1.06	1.7%	22
Liaoning	**1.60**	**0.940**	**0.30**	**0.99**	**1.03**	**4.9%**	**7**
Jilin	2.50	0.985	0.45	0.99	0.97	2.0%	18
Heilongjiang	1.50	0.935	0.00	0.99	0.96	3.4%	14
Shanghai	1.90	0.995	1.00	0.99	0.96	3.5%	12
Jiangsu	**1.10**	**0.900**	**0.00**	**0.99**	**1.15**	**8.6%**	**3**
Zhejiang	**1.70**	**0.925**	**0.00**	**0.97**	**0.98**	**6.2%**	**4**
Anhui	5.00	0.900	0.00	0.87	2.74	3.4%	13
Fujian	**5.00**	**0.900**	**0.95**	**0.99**	**1.18**	**3.8%**	**10**
Jiangxi	3.20	0.955	0.00	0.97	1.01	2.2%	17
Shandong	**2.60**	**0.925**	**0.15**	**0.97**	**1.03**	**9.1%**	**1**
Henan	**3.40**	**0.905**	**0.35**	**1.00**	**0.99**	**6.1%**	**5**
Hubei	**5.00**	**0.940**	**0.00**	**0.94**	**1.03**	**4.1%**	**9**
Hunan	5.00	0.900	0.00	0.94	1.88	3.5%	11
Guangdong	**2.30**	**0.900**	**1.00**	**0.94**	**1.03**	**8.8%**	**2**
Guangxi	1.10	0.995	0.00	0.98	0.99	2.2%	16
Hainan	1.10	0.985	0.00	1.00	1.00	0.4%	28
Chongqing	1.50	0.995	1.00	0.99	0.89	2.0%	19
Sichuan	**3.30**	**0.900**	**0.10**	**1.00**	**0.99**	**4.6%**	**8**
Guizhou	2.40	0.995	0.00	1.00	0.98	1.0%	27
Yunnan	3.30	0.945	1.00	0.87	0.92	1.9%	20
Tibet							
Shaanxi	0.60	0.995	0.00	0.77	0.72	1.8%	21
Gansu	0.80	0.995	0.00	0.86	0.97	1.1%	26
Qinghai	5.00	0.995	1.00	0.99	0.74	0.3%	30
Ningxia	2.00	0.995	1.00	0.98	0.87	0.3%	29
Xinjiang	0.70	0.995	0.00	0.93	0.92	1.4%	25
Median	2.35	0.95	0.05	0.98	0.99	2.2%	
Mean	2.59	0.95	0.37	0.96	1.06	3.3%	
Mean (rGDP)	2.69	0.93	0.32	0.96	1.10		

Results obtained from a three dimensional grid-search for γ, κ and δ by minimizing the squared distance between the real and estimated NX/NO. Tibet is excluded because of data issues. The ten largest provinces in terms of 2000 real GDP are in bold type and represent around 62% of cumulated output. The last row is the (2000) real GDP-weighted mean.

Table 4.2: Channels of external adjustment: variance decomposition

	NO	*IP*	*DR*	*IR*	*RES*
Beijing	0.51	0.00	0.17	0.25	0.06
Tianjin	0.78	0.16	-0.10	0.03	0.13
Hebei	0.96	-0.26	0.25	0.00	0.05
Shanxi	1.23	0.01	-0.03	-0.24	0.03
Inner Mong.	1.22	-0.05	-0.13	0.00	-0.04
Liaoning	0.50	0.14	0.14	0.24	-0.02
Jilin	1.07	0.11	-0.22	0.00	0.04
Heilongjiang	0.71	-0.01	0.26	0.00	0.04
Shanghai	0.94	-0.18	-0.15	0.33	0.06
Jiangsu	0.55	0.02	0.57	0.00	-0.13
Zhejiang	0.37	0.10	0.48	0.00	0.05
Anhui	2.05	0.30	0.03	0.00	-1.38
Fujian	1.05	-0.03	0.19	-0.05	-0.17
Jiangxi	0.59	-0.13	0.52	0.00	0.02
Shandong	1.88	-0.66	-0.01	-0.21	-0.01
Henan	0.93	0.03	0.06	-0.03	0.01
Hubei	0.95	0.21	-0.20	0.00	0.03
Hunan	1.79	-0.24	0.22	0.00	-0.77
Guangdong	0.70	-0.16	0.32	0.10	0.03
Guangxi	0.08	0.01	0.88	0.00	0.03
Hainan	0.74	-0.01	0.28	0.00	0.00
Chongqing	0.62	0.03	0.07	0.15	0.13
Sichuan	0.76	0.55	-0.35	0.03	0.01
Guizhou	1.13	0.08	-0.23	0.00	0.02
Yunnan	0.12	0.50	-0.24	0.41	0.20
Tibet					
Shaanxi	0.18	0.01	0.36	0.00	0.45
Gansu	-0.34	0.03	1.15	0.00	0.16
Qinghai	0.58	0.12	0.02	0.01	0.27
Ningxia	0.79	0.10	-0.01	-0.04	0.15
Xinjiang	0.42	0.01	0.43	0.00	0.14
Median	0.74	0.01	0.07	0.00	0.03
Mean	0.77	0.03	0.15	0.03	-0.01
Mean (rGDP)	0.89	-0.03	0.18	0.02	-0.05

The table presents estimates of the variance decomposition coefficients. NO: net output, IP: internal price, DR: domestic rate, IR: international rate. RES: unexplained part. The last row is the (2000) real GDP-weighted mean.

Table 4.3: Panel analysis of external adjustment

	NO	*IP*	*DR*	*IR*	*RES*
Weighting					
Same	**0.70**	**0.06**	**0.05**	**0.05**	**0.14**
	(11.17)	(3.12)	(1.69)	(1.73	(3.24)
Real GDP	**0.82**	-0.02	**0.11**	0.05	**0.03**
	(11.50)	(-0.55)	(3.28)	(1.26	(2.14)
Residual	**0.75**	**0.05**	0.05	**0.06**	**0.10**
	(10.78)	(2.71)	(1.31)	(1.77)	(2.92)
	NO	*IP*	*DR*	*IR*	*RES*
Regions					
Metro	**0.62**	-0.01	0.05	**0.24**	**0.10**
	(5.93)	(-0.20)	(0.62)	(9.43)	(40.83)
East Coast	**0.86**	**$-$0.18**	**0.28**	0.05	0.00
	(4.35)	(-3.09)	(5.08)	(0.77)	(-0.10)
Manchuria	**0.64**	**0.07**	**0.15**	**0.12**	**0.02**
	(11.13)	(1.91)	(2.40)	(2.08)	(1.90)
Center	**0.92**	**0.04**	**0.06**	**$-$0.03**	**0.02**
	(117.51)	(6.20)	(9.61)	(-25.34)	(2.30)
West	**0.71**	0.00	0.12	-0.01	0.18
	(2.85)	(-0.08)	(0.88)	(-0.77)	(1.55)
South	**0.58**	0.10	0.05	**0.14**	**0.13**
	(5.27)	(1.15)	(0.70)	(3.46)	(2.36)
No Metro & EC	**0.83**	**0.05**	**0.07**	0.01	**0.04**
	(10.97)	(2.67)	(3.53)	(0.33)	(1.79)
	NO	*IP*	*DR*	*IR*	*RES*
Dvpt Level					
Market High	**0.75**	-0.06	**0.15**	**0.13**	0.03
	(8.19)	(-1.18)	(3.17)	(3.42)	(1.55)
Market Low	**0.88**	**0.04**	**0.06**	-0.01	0.14
	(14.84)	(2.72)	(3.00)	(-0.70)	(1.46)

Panel estimates (β_x) of the respective channels from the regression $x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left(\frac{NX}{NO}\right)_t^k + \varepsilon_t^k$, where x_t^k stands for the VAR-implied expectations of the channels. T-statistics clustered by regions based on Liang and Zeger (1986) in parentheses. Bold type indicates significancy at the 10% confidence level. The first weighting procedure (*Real GDP*) uses 2000 real GDP computed using provincial CPI. The second weighting procedure (*Residual*) uses the absolute residual share of the variance decomposition $[max(abs(RES)) - abs(RES^k)]^2$. Both are normalized by the highest regional value. In the second and third panels, relative net exports and channels are GDP-weighted.

Table 4.4: Specification, data and grid-search results

	Sample	*Lag*	*NO Defl*	*c*	ϕ
Beijing	85-10	1	RPI	1.38	-0.34
Tianjin	85-10	2	RPI PIFA	0.77	0.61
Hebei	85-10	1	GDP PIFA	0.79	0.63
Shanxi	87-10	1	RPI PIFA	0.88	0.56
Inner Mong.	85-10	1	RPI PIFA	1.04	-0.45
Liaoning	85-10	1	GDP PIFA	0.78	0.29
Jilin	85-10	1	RPI PIFA	0.91	0.55
Heilongjiang	85-10	1	GDP PIFA	0.87	0.29
Shanghai	85-10	2	RPI PIFA	0.76	0.36
Jiangsu	85-10	1	RPI PIFA	0.68	0.06
Zhejiang	85-10	1	GDP	0.85	0.35
Anhui	85-10	1	RPI	1.00	0.80
Fujian	85-10	1	GDP	0.98	0.79
Jiangxi	85-10	1	GDP PIFA	1.00	0.69
Shandong	85-10	1	GDP PIFA	0.80	0.50
Henan	85-10	1	GDP PIFA	1.02	0.72
Hubei	85-10	1	RPI	0.99	0.79
Hunan	85-10	2	GDP PIFA	0.94	0.75
Guangdong	85-10	1	GDP	0.86	0.49
Guangxi	85-08	1	RPI PIFA	1.02	0.09
Hainan	85-10	1	RPI	1.19	0.11
Chongqing	85-10	1	GDP PIFA	1.20	0.40
Sichuan	85-10	1	GDP PIFA	1.06	0.74
Guizhou	85-10	1	RPI PIFA	1.21	0.70
Yunnan	85-09	2	GDP PIFA	1.04	0.72
Tibet					
Shaanxi	85-09	2	GDP PIFA	1.11	-0.74
Gansu	85-10	2	GDP PIFA	1.13	-0.28
Qinghai	85-10	1	RPI PIFA	1.67	1.34
Ningxia	89-10	2	RPI PIFA	1.40	0.70
Xinjiang	85-10	1	RPI PIFA	1.07	-0.46
Median				1.00	0.52
Mean				1.01	0.39
Mean (rGDP)				0.92	0.43

Specification and choice of net output deflator as in main text. Consumption ratio (c) estimated from the data. Implied $\phi = c \times (1 - \frac{1}{\gamma})$ from grid-search (γ). Tibet is excluded because of data issues.

Table 4.5: Channels of external adjustment and province-level characteristics (I), 1986-2010

STATE	*Net Output*	*Internal Price*	*Interest Rate* *Domestic*	*Interest Rate* *Intern.*	*Interest Rate* *Total*
NX/NO	**0.98**	0.04	0.10	**−0.14**	−0.04
	(8.05)	(0.74)	(1.36)	(−3.67)	(−0.49)
$SOGIOV$	−0.02	0.00	0.01	0.01	0.02
	(−0.82)	(−0.05)	(0.42)	(1.32)	(1.11)
$\overline{Dvpt} \times NX/NO$	−0.09	**−0.08**	0.01	**0.16**	**0.17**
	(−0.78)	(−1.72)	(0.18)	(6.26)	(2.40)
$SOGIOV \times NX/NO$	**−0.29**	**0.17**	−0.02	**0.09**	0.08
	(−2.31)	(2.05)	(−0.29)	(2.10)	(0.97)
NX/NO	**1.02**	**0.17**	−0.11	**−0.16**	**−0.27**
	(6.60)	(3.71)	(−1.20)	(−3.19)	(−2.56)
$\overline{Dvpt} \times NX/NO$	−0.12	0.01	−0.09	**0.17**	0.08
	(−0.93)	(0.28)	(−1.14)	(5.63)	(0.93)
$\overline{Market} \times NX/NO$	−0.01	**−0.04**	**0.06**	0.01	**0.06**
	(−0.34)	(−3.87)	(3.09)	(0.65)	(2.74)
NX/NO	**1.01**	0.04	0.08	**−0.15**	−0.07
	(9.71)	(0.85)	(1.32)	(−3.81)	(−0.94)
$\frac{Deposits-Loans}{GDP}$	**−0.03**	0.01	0.00	**0.01**	**0.02**
	(−2.41)	(1.65)	(0.72)	(2.09)	(2.26)
$\overline{Dvpt} \times NX/NO$	**−0.12**	−0.05	−0.01	**0.18**	**0.17**
	(−1.72)	(−1.48)	(−0.10)	(5.64)	(3.14)
$\frac{Deposits-Loans}{GDP} \times NX/NO$	−0.15	−0.02	**0.10**	0.02	**0.13**
	(−1.61)	(−0.30)	(2.39)	(0.78)	(2.22)

DEMOGRAPHICS	*Net Output*	*Internal Price*	*Interest Rate* *Domestic*	*Interest Rate* *Intern.*	*Interest Rate* *Total*
NX/NO	**0.83**	−0.01	**0.11**	**0.04**	**0.15**
	(20.63)	(−0.34)	(4.08)	(2.74)	(4.71)
$Urbanization$	0.03	0.00	**0.05**	**−0.02**	0.02
	(0.91)	(0.09)	(1.70)	(−1.65)	(0.71)
$\overline{Dvpt} \times NX/NO$	0.38	−0.14	−0.08	0.01	−0.06
	(1.45)	(−0.82)	(−0.38)	(0.15)	(−0.35)
$Urbanization \times NX/NO$	**−1.54**	0.25	0.26	**0.50**	0.76
	(−2.16)	(0.51)	(0.52)	(2.67)	(1.51)
NX/NO	6.30	**−4.32**	2.61	−2.02	0.60
	(1.62)	(−2.48)	(1.44)	(−1.49)	(0.23)
$\overline{Dvpt} \times NX/NO$	**−0.15**	−0.04	0.01	**0.18**	**0.19**
	(−1.66)	(−1.55)	(0.08)	(6.84)	(2.66)
$\overline{SexRatio} \times NX/NO$	−5.00	**4.10**	−2.37	1.77	−0.60
	(−1.37)	(2.51)	(−1.40)	(1.38)	(−0.25)
NX/NO	**0.96**	0.05	0.09	**−0.14**	0.05
	(9.22)	(1.39)	(1.14)	(−3.51)	(−0.51)
$\overline{Dvpt} \times NX/NO$	0.08	**−0.23**	0.06	**0.16**	0.23
	(0.44)	(−3.72)	(0.36)	(3.33)	(1.21)
$\overline{HighEduc} \times NX/NO$	−2.07	**1.70**	−0.51	0.16	−0.34
	(−1.53)	(3.48)	(−0.44)	(0.45)	(−0.25)

The table reports the results of panel regressions of the form $x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left(\frac{NX}{NO}\right)_t^k + \gamma_x' \times \left(\frac{NX}{NO}\right)_t^k \times z_t^k + \psi' \times z_t^k + \varepsilon_t^k$, where x_t^k stands in turn for the VAR-implied expectations of the respective channel. The vector z_t^k stands for the different potential explanatory variables. T-statistics clustered by regions based on Liang and Zeger (1986) in parentheses. Bold type indicates significancy at the 10% confidence level. The last column (Total) results from merging the domestic with the international interest rate channel.

$\overline{Dvpt}$: mean real GDP per capita relative to national values over 1986-2010. $SOGIOV$: share of state-owned gross industrial output value. $(Deposit - Loans)/GDP$: deposits minus loans in banks and financial institutions normalized by GDP. $\overline{Market}$: marketization index developed by Fan et al. (2001), 1997-2005 average. $Urbanization$: population share living in urban area (cross-sectionally demeaned). $\overline{SexRatio}$: male to female ratio (2000 Census). $\overline{HighEduc}$: student enrollment in institutions of higher education relative to population, 1997-2010 average.

Table 4.6: Channels of external adjustment and province-level characteristics (II), 1986-2010

	Net	Internal	Interest Rate				Net	Internal	Interest Rate		
INTERNATIONAL	*Output*	*Price*	*Domestic*	*Intern.*	*Total*	**STRUCTURE**	*Output*	*Price*	*Domestic*	*Intern.*	*Total*
NX/NO	**0.97** (7.90)	0.03 (0.67)	**0.10** (1.67)	**−0.14** (−3.69)	−0.03 (−0.47)	NX/NO	0.27 (1.23)	**0.35** (2.53)	0.18 (1.12)	0.06 (0.52)	**0.24** (1.82)
$Openness$	**0.02** (2.44)	0.01 (1.50)	**−0.01** (−1.96)	0.00 (−0.82)	**−0.01** (−2.10)	$Industry$	−0.03 (−0.61)	−0.02 (−0.78)	0.03 (1.35)	0.00 (0.08)	0.04 (1.09)
$\overline{Dvpt} \times NX/NO$	−0.11 (−0.94)	−0.02 (−0.44)	−0.03 (−0.43)	**0.17** (6.12)	**0.14** (1.90)	$\overline{Dvpt} \times NX/NO$	**−0.15** (−2.33)	−0.05 (−1.51)	0.01 (0.20)	**0.18** (8.39)	**0.20** (3.38)
$Openness \times NX/NO$	−0.07 (−1.25)	**−0.08** (−3.99)	**0.11** (2.15)	**0.03** (1.73)	**0.13** (2.28)	$Industry \times NX/NO$	**1.55** (3.70)	**−0.67** (−2.10)	−0.20 (−0.78)	**−0.43** (−2.19)	**−0.63** (−2.96)
NX/NO	**0.87** (11.74)	−0.01 (−0.26)	**0.10** (3.19)	0.02 (0.37)	**0.11** (1.74)	NX/NO	**1.14** (**7.14**)	−0.06 (−1.04)	0.11 (0.90)	**−0.17** (−4.28)	−0.06 (−0.38)
FDI	0.06 (0.84)	0.00 (0.01)	0.01 (0.37)	−0.05 (−1.02)	−0.04 (−0.93)	$Construction$	0.23 (1.13)	0.00 (0.00)	−0.04 (−0.33)	−0.14 (−1.64)	−0.18 (−1.19)
$FDI \times NX/NO$	**−1.34** (1.73)	−0.34 (−0.80)	0.38 (0.57)	**1.30** (3.37)	**1.68** (2.71)	$\overline{Dvpt} \times NX/NO$	−0.12 (−1.52)	−0.06 (−1.63)	0.01 (0.13)	**0.17** (6.67)	**0.18** (2.88)
						$Construction \times NX/NO$	−2.99 (−1.21)	**1.84** (2.16)	−0.25 (−0.13)	0.44 (0.66)	0.19 (0.08)
NX/NO	**0.98** (8.31)	0.04 (0.72)	**0.10** (1.76)	**−0.14** (−3.87)	−0.04 (−0.63)	NX/NO	**0.71** (3.75)	0.05 (0.41)	**0.26** (2.46)	−0.08 (−1.13)	0.19 (1.41)
$\overline{Dvpt} \times NX/NO$	−0.07 (−0.51)	−0.01 (−0.15)	−0.10 (−1.33)	**0.15** (5.06)	0.05 (0.70)	$\overline{Dvpt} \times NX/NO$	0.33 (1.04)	−0.07 (−0.35)	**−0.28** (−1.77)	0.07 (0.67)	−0.21 (−1.02)
$\overline{FOInvFA} \times NX/NO$	−0.93 (−1.49)	−0.50 (−1.13)	**1.29** (3.54)	**0.40** (2.52)	**1.69** (4.28)	$\overline{Patents} \times NX/NO$	**−1.09** (−1.70)	0.03 (0.08)	**0.67** (2.19)	0.26 (1.24)	**0.93** (2.26)

The table reports the results of panel regressions of the form $x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left(\frac{NX}{NO}\right)_t^k + \gamma_x' \times \left(\frac{NX}{NO}\right)_t^k \times z_t^k + \psi' \times z_t^k + \varepsilon_t^k$, where x_t^k stands in turn for the VAR-implied expectations of the respective channel. The vector z_t^k stands for the different potential explanatory variables. T-statistics clustered by regions based on Liang and Zeger (1986) in parentheses. Bold type indicates significancy at the 10% confidence level. The last column (Total) results from merging the domestic with the international interest rate channel.

$\overline{Dvpt}$: mean real GDP per capita relative to national values over 1986-2010. $Openness$: international exports and imports over GDP. FDI: used FDI over GDP. $\overline{FOInvFA}$: foreign-owned share in investment in fixed assets (including Hong-Kong and Macau). $Industry$: industry sector share of GDP. $Construction$: construction sector share of GDP. $\overline{Patents}$: granted patents over population, 1997-2010 average.

Table 4.7: Channels of external adjustment and province-level characteristics (III), 1997-2010

	Net	*Internal*	*Interest Rate*		
	Output	*Price*	*Domestic*	*Intern.*	*Total*
NX/NO	**0.73** (13.62)	**0.06** (1.85)	0.06 (1.24)	**0.09** (3.73)	**0.16** (2.99)
$\overline{SexRatio} \times NX/NO$	−5.90 (−1.03)	**5.96** (1.94)	−2.99 (−0.90)	**2.06** (2.05)	−0.92 (−0.26)
$SOGIOV$	−0.01 (−0.34)	0.01 (0.63)	−0.01 (−0.63)	0.01 (1.38)	0.00 (0.11)
$SOGIOV \times NX/NO$	−0.59 (−1.00)	0.23 (1.20)	−0.04 (−0.13)	0.19 (1.17)	0.15 (0.38)
$EmplPrivate$	**0.08** (1.94)	**−0.03** (−1.90)	**−0.04** (−1.69)	−0.01 (−0.37)	**−0.04** (−1.66)
$EmplPrivate \times NX/NO$	−0.30 (−0.44)	0.36 (1.34)	−0.32 (−0.62)	**0.36** (1.97)	0.04 (0.07)
$\frac{Deposits-Loans}{GDP}$	**−0.03** (−1.97)	0.01 (1.49)	0.00 (0.42)	**0.01** (1.84)	0.01 (1.15)
$\frac{Deposits-Loans}{GDP} \times NX/NO$	**−0.47** (−3.07)	−0.02 (−0.35)	**0.31** (3.01)	**0.09** (2.42)	**0.40** (3.73)
FDI	0.16 (1.24)	−0.09 (−1.63	−0.04 (−0.43)	**−0.06** (−1.87)	−0.10 (−1.01)
$FDI \times NX/NO$	0.16 (0.10)	0.68 (0.89)	**−2.07** (−1.82)	**1.06** (1.70)	−1.01 (−0.78)
$Industry$	**−0.17** (−2.75)	**0.07** (1.88)	0.03 (0.54)	**0.05** (1.86)	**0.09** (1.75)
$Industry \times NX/NO$	0.44 (0.42)	0.05 (0.10)	0.08 (0.09)	−0.48 (−1.49)	−0.40 (−0.41)

The table reports the results of panel regressions of the form $x_t^k = \alpha + \tau_t + \mu^k + \beta_x \times \left(\frac{NX}{NO}\right)_t^k + \gamma_x' \times \left(\frac{NX}{NO}\right)_t^k \times z_t^k + \psi' \times z_t^k + \varepsilon_t^k$, where x_t^k stands in turn for the VAR-implied expectations of the respective channel. The vector z_t^k stands for the different potential explanatory variables. T-statistics clustered by regions based on Liang and Zeger (1986) in parentheses. Bold type indicates significancy at the 10% confidence level. The last column (Total) results from merging the domestic with the international interest rate channel. Factors are cross-sectionally demeaned.

$\overline{SexRatio}$: male to female ratio (2000 Census). $SOGIOV$: share of state-owned gross industrial output value. $EmplPrivate$: share of private and self-employed relative to total employment. $(Deposits - Loans)/GDP$: deposits minus loans in banks and financial institutions normalized by GDP. FDI: used FDI over GDP. $Industry$: industry sector share of GDP.

Figure 4.3: Channels of adjustment for four regions, 1986-2010

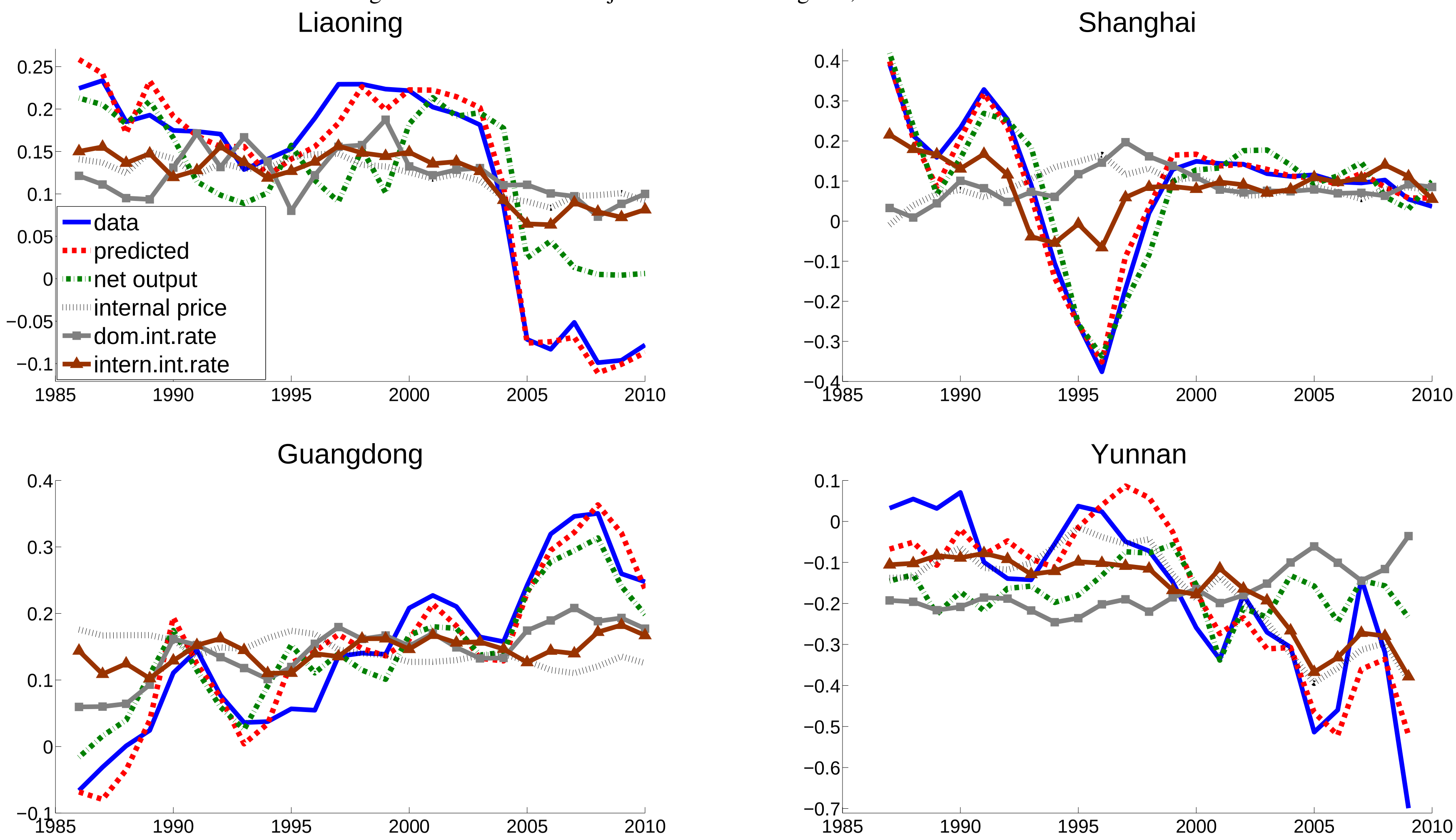


Figure 4.4: Net output channel β, 1986-2010

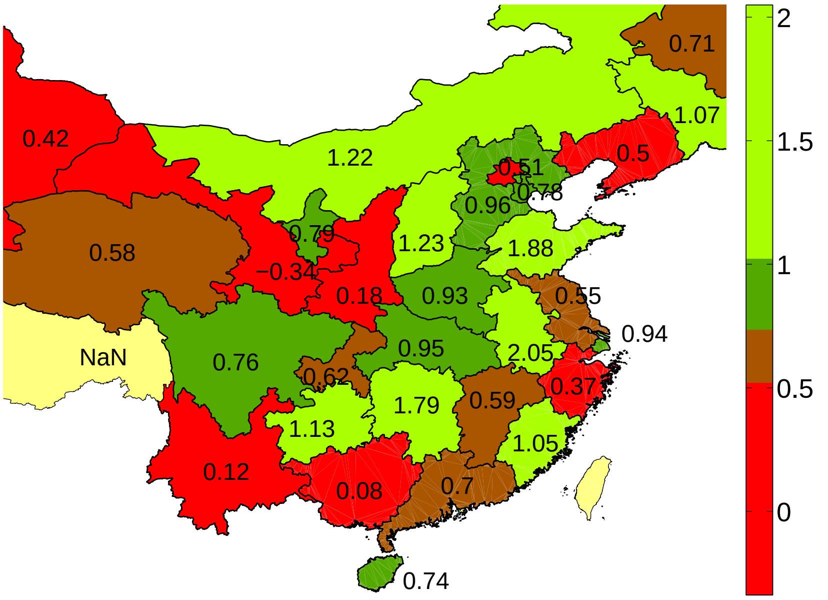


Figure 4.5: Internal price channel β, 1986-2010

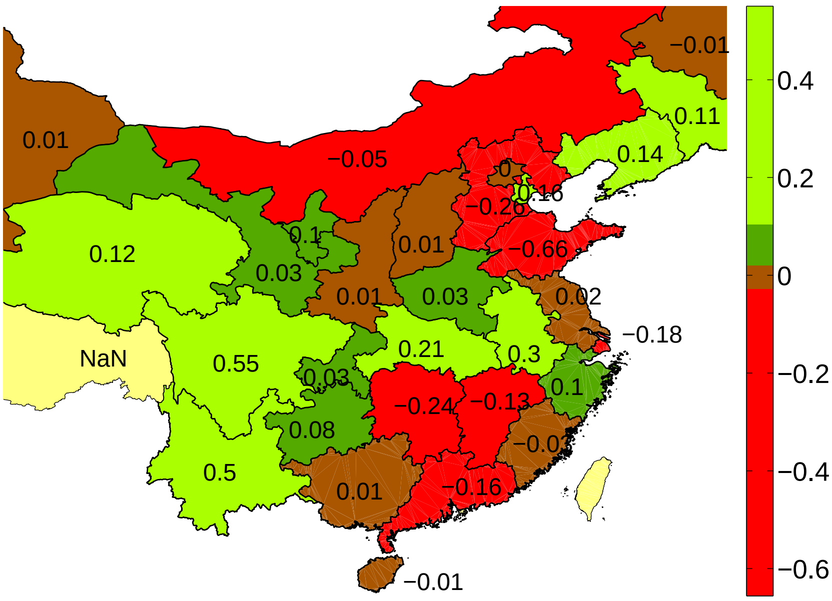


Figure 4.6: Interest rate channel β (world and domestic), 1986-2010

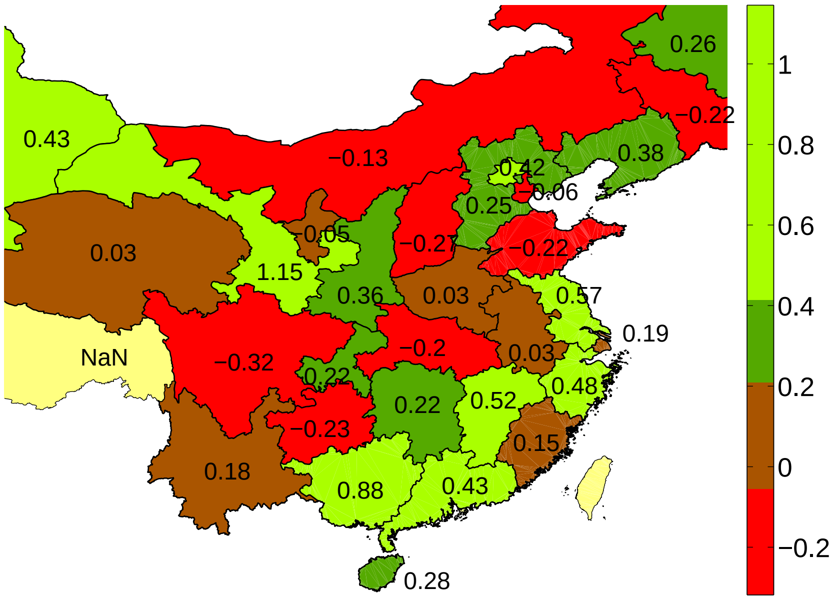


Figure 4.7: Cumulated nominal net exports (100 million RMB), 1986-2010

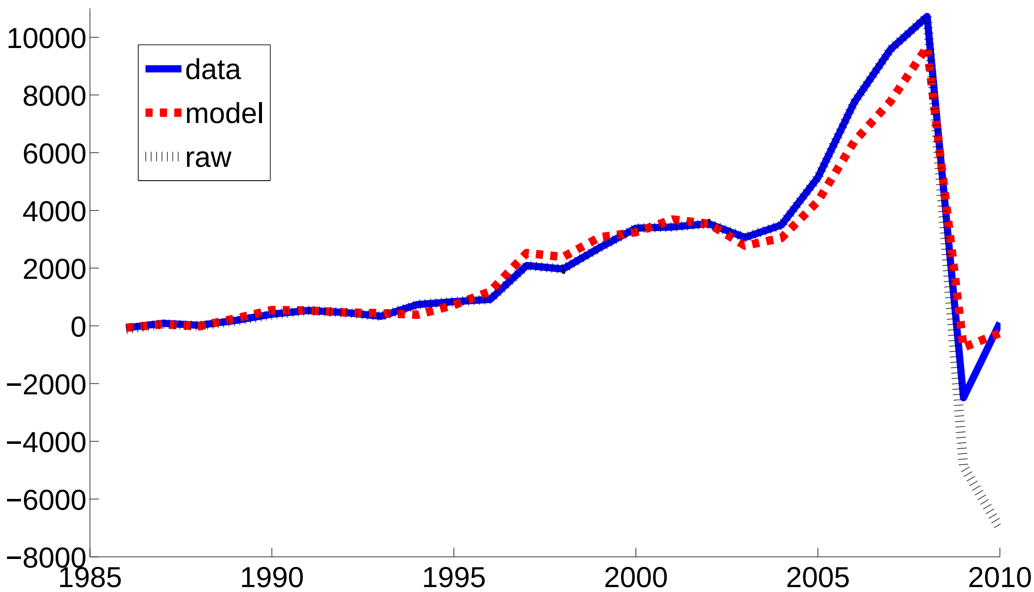
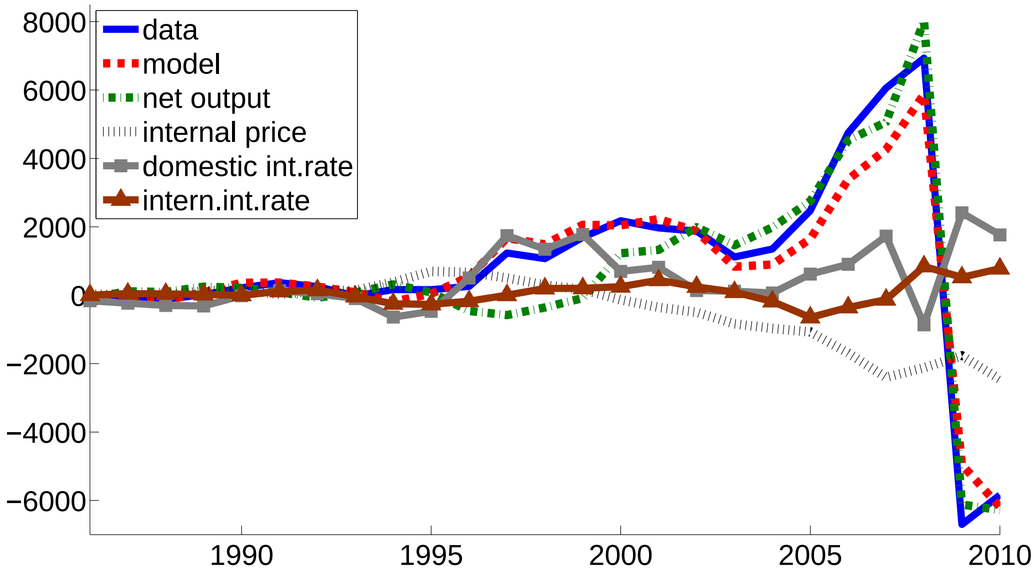


Figure 4.8: Nominal aggregate channels of net exports (100 million RMB, demeaned), 1986-2010



Bibliography

Adams, F. and Y. Chen: 1996, 'Skepticism about Chinese GDP growth–the Chinese GDP elasticity of energy consumption'. *Journal of Economic and Social Measurement* **22**(4), 231–240.

Aguiar, M. and M. Amador: 2011, 'Growth in the Shadow of Expropriation'. *The Quarterly Journal of Economics* **126**(2), 651–697.

Aizenman, J. and J. Lee: 2007, 'International reserves: precautionary versus mercantilist views, theory and evidence'. *Open Economies Review* **18**(2), 191–214.

Aizenman, J. and J. Lee: 2008, 'Financial versus Monetary Mercantilism: Long-run View of Large International Reserves Hoarding'. *The World Economy* **31**(5), 593–611.

Aizenman, J. and J. Lee: 2010, 'Real exchange rate, mercantilism and the learning by doing externality'. *Pacific Economic Review* **15**(3), 324–335.

Alder, S., L. Shao, and F. Zilibotti: 2013, 'The Effect of Economic Reform and Industrial Policy in a Panel of Chinese Cities'. *CEPR Discussion Paper* (3).

Alfaro, L., S. Kalemli-Ozcan, and V. Volosovych: 2011, 'Sovereigns, Upstream Capital Flows and Global Imbalances'. *NBER Working Paper*.

Anderson, J.: 2007, 'Is China export-led'. *UBS Investment Research Asian Focus* **27**.

Anderson, J.: 2009, 'The myth of Chinese savings'. *Far Eastern Economic Review* **172**(9), 24–30.

Aziz, J. and L. Cui: 2007, 'Explaining China's low consumption: the neglected role of household income'. *IMF Working Papers* pp. 1–36.

Bacchetta, P. and K. Benhima: 2010, 'The demand for liquid assets, corporate saving, and global imbalances'. *Département d'économétrie et d'économie politique*.

Banerjee, A., X. Meng, and N. Qian: 2010, 'The life cycle model and household savings: Micro evidence from urban china'. *mimeo*.

Bergin, P. R. and S. M. Sheffrin: 2000, 'Interest Rates, Exchange Rates and Present Value Models of the Current Account'. *Economic Journal* **110**(463), 535–58.

Bernanke, B.: 2007, 'Global Imbalances: Recent Developments and Prospects'. *Bundesbank Lecture speech*.

Borst, N.: 2012a, 'Land reform and local government finances'. *Peterson Institute*.

Borst, N.: 2012b, 'The politics of a Chinese slowdown'. *Peterson Institute*.

Borst, N.: 2012c, 'SOE dividends and economic rebalancing'. *Peterson Institute*.

Borst, N.: 2012d, 'Urbanization and economic growth'. *Peterson Institute*.

Bouakez, H. and T. Kano: 2009, 'Tests of the present-value model of the current account: a note'. *Applied Economics Letters* **16**(12), 1215–1219.

Boyreau-Debray, G. and S. Wei: 2004, 'Can China grow faster? A diagnosis of the fragmentation of its domestic capital market'. *IMF Working Papers*.

Bradsher, K.: 2012, 'Chinese data mask depth of slowdown, executives say'. *The New-York Times*.

Brandt, L. and C. Holz: 2006, 'Spatial price differences in China: estimates and implications'. *Economic Development and Cultural Change* **55**(1), 43–86.

Brandt, L. and T. G. Rawski (eds.): 2008, *China's Great Economic Transformation*. Cambridge University Press.

Brandt, L., T. Tombe, and X. Zhu: 2012, 'Factor market distortions across time, space and sectors in China'. *Review of Economic Dynamics*.

Brandt, L. and X. Zhu: 2010, 'Accounting for China's growth'. *Institute for the Study of Labor (IZA) Discussion Paper*.

Brun, J., J. Combes, and M. Renard: 2002, 'Are there spillover effects between coastal and noncoastal regions in China?'. *China Economic Review* **13**(2), 161–169.

Caballero, R. J., E. Farhi, and P.-O. Gourinchas: 2008, 'An equilibrium model of "global imbalances" and low interest rates'. *The American Economic Review* **98**(1), 358–393.

Cai, Y.: 2000, 'Between state and peasant: local cadres and statistical reporting in rural China'. *The China Quarterly* (163), 783–805.

Campbell, R.: 1985, *The conversion of national income data of the USSR to concepts of the system of national accounts in dollars and estimation of growth rate*, Vol. 777. World bank Washington, DC.

Chamon, M., K. Liu, and E. Prasad: 2013, 'Income uncertainty and household savings in China'. *Journal of Development Economics* **105**, 164–177.

Chamon, M. D. and E. S. Prasad: 2010, 'Why are saving rates of urban households in China rising?'. *American Economic Journal: Macroeconomics* pp. 93–130.

Chan, K.: 2010, 'The household registration system and migrant labor in China: notes on a debate'. *Population and Development Review* **36**(2), 357–364.

Chan, K. and M. Wang: 2008, 'Remapping China's Regional Inequalities, 1990-2006: A New Assessment of de Facto and de Jure Population Data'. *Eurasian Geography and Economics* **49**(1), 21–55.

Chan, K. W.: 2013, *The encyclopedia of global migration (China, internal migration)*. Blackwell Publishing.

Chang, G. H.: 2002, 'The cause and cure of China's widening income disparity'. *China Economic Review* **13**(4), 335–340.

Chao, C., J. Laffargue, and E. Yu: 2011, 'The Chinese saving puzzle and the life-cycle hypothesis: A revaluation'. *China Economic Review* **22**(1), 108–120.

Chen, B. and Y. Yao: 2011, 'The Cursed Virtue: Government Infrastructural Investment and Household Consumption in Chinese Provinces*'. *Oxford Bulletin of Economics and Statistics* **73**(6), 856–877.

Chen, X. and W. Nordhaus: 2010, 'The value of luminosity data as a proxy for economic statistics'. *NBER working paper*.

Chen, Y., M. Funke, and A. Mehrotra: 2011, 'What drives urban consumption in Mainland China? The role of property price dynamics'. *BOFIT Discussion Paper No. 13/2011*.

Chou, S.: 1971, 'Railway development and economic growth in Manchuria'. *The China Quarterly* **45**, 57–84.

Chow, G.: 2006, 'Are Chinese official statistics reliable?'. *CESifo Economic Studies* **52**(2), 396–414.

Curtis, C., S. Lugauer, and N. Mark: 2011, 'Demographic Patterns and Household Saving in China'. *NBER Working Paper*.

Curtis, C. and N. Mark: 2010, 'Business Cycles, Consumption and Risk-Sharing: How Different Is China?'. *NBER Working Paper*.

Dadush and Stancil: 2011, 'The Capital Flow Conundrum'. *Vox*.

Demurger, S.: 2001, 'Infrastructure development and economic growth: an explanation for regional disparities in China?'. *Journal of Comparative economics* **29**(1), 95–117.

Demurger, S., J. Sachs, W. Woo, S. Bao, G. Chang, and A. Mellinger: 2002, 'Geography, Economic Policy, and Regional Development in China*'. *Asian Economic Papers* **1**(1), 146–197.

Dollar, D. and S. Wei: 2007, 'Das (wasted) Kapital: firm ownership and investment efficiency in China'. *NBER Working Paper*.

Du, Q. and S. Wei: 2010, 'A sexually unbalanced model of current account imbalances'. *NBER Working Paper*.

Eckstein, A., K. Chao, and J. Chang: 1974, 'The economic development of Manchuria: the rise of a frontier economy'. *The Journal of Economic History* **34**(1), 239–264.

Economist, T.: 2011, 'Exports to Mars'. *The Economist* **November 12th**.

Economist, T.: 2013, 'The old regime and the revolution'. *The Economist* **March 16th**.

Fan, G., X. Wang, and L. Zhang: 2001, 'Annual report 2000: Marketization index for China's provinces'. *China & World Economy (5)*.

Fleisher, B. and J. Chen: 1997, 'The coast-noncoast income gap, productivity, and regional economic policy in China'. *Journal of Comparative Economics* **25**(2), 220–236.

Gaulier, G., F. Lemoine, and D. Unal: 2011, 'China's Foreign Trade in the Perspective of a More Balanced Economic Growth'. *CEPI Working Papers*.

Girardin, E. and R. Owen: 2011, 'Global Trade Imbalances, Structural Change and China: What Scope for Fundamental Adjustment?'. *Working Paper*.

Gourinchas, P.-O. and O. Jeanne: 2013, 'Capital flows to developing countries: The allocation puzzle'. *The Review of Economic Studies* **80**(4), 1484–1515.

Gourinchas, P.-O. and H. Rey: 2007, 'International Financial Adjustment'. *Journal of Political Economy* **115**(4), 665–703.

Gruber, J. and S. Kamin: 2009, 'Do differences in financial development explain the global pattern of current account imbalances?'. *Review of International Economics* **17**(4), 667–688.

Hannum, E. and M. Wang: 2006, 'Geography and educational inequality in China'. *China Economic Review* **17**(3), 253–265.

Hansakul, S., S. Dyck, S. Kern, S. Kaiser, and N. Walter: 2009, 'China's financial markets–a future global force?'. *Deutsche Bank Research. March* **16**, 2009.

Henderson, V., A. Storeygard, and D. Weil: 2011, 'A Bright Idea for Measuring Economic Growth'. *The American Economic Review* **101**(3), 194–199.

Herzog, F.: 2013, 'Multivariate Test of the Export-Led Growth Hypothesis in Chinese Regions'. *Bachelor Thesis* **University of Zurich**.

Ho, C., W. Ho, and D. Li: 2010, 'Consumption Fluctuations and Welfare: Evidence from China'. *World Development* **38**(9), 1315–1327.

Hoffmann, M.: 2013, 'What drives China's current account?'. *Journal of International Money and Finance* **32**, 856–883.

Hoffmann, M. and U. Woitek: 2011, 'Emerging from the war: Gold Standard mentality, current accounts and the international business cycle 1885-1939'. *ECON-Working Papers*.

Holz, C.: 2004a, 'China's statistical system in transition: Challenges, data problems, and institutional innovations'. *Review of Income and Wealth* **50**(3), 381–409.

Holz, C.: 2004b, 'Deconstructing China's GDP statistics'. *China Economic Review* **15**(2), 164–202.

Holz, C.: 2008, 'China's 2004 economic census and 2006 benchmark revision of GDP statistics: more questions than answers?'. *China Quarterly, London* **193**, 150.

HSBC: 2010, 'Inside the Growth Engine'. *Global Research*.

HSBC: 2012, 'China Inside Out'. *Global Research*.

Hsieh, C. and P. Klenow: 2009, 'Misallocation and manufacturing TFP in China and India'. *The Quarterly Journal of Economics* **124**(4), 1403–1448.

Huang, Y.: 2010, 'Rebalancing China's economic structure'. *East Asia Forum*.

Huang, Y. and X. Wang: 2011, 'Does Financial Repression Inhibit or Facilitate Economic Growth? A Case Study of Chinese Reform Experience*'. *Oxford Bulletin of Economics and Statistics* **73**(6), 833–855.

Jia, R.: 2012, 'The legacies of forced freedom: China's treaty ports'. *Mimeographed, IIES, Stockholm University*.

Jian, T., J. Sachs, and A. Warner: 1996, 'Trends in regional inequality in China'. *China economic review* **7**(1), 1–21.

Jiang, Y.: 2011, 'Understanding openness and productivity growth in China: An empirical study of the Chinese provinces'. *China Economic Review*.

Jin, K.: 2012, 'Industrial structure and capital flows'. *The American Economic Review* **102**(5), 2111–2146.

Jin, K., S. Guibaud, and N. Coeurdacier: 2012, 'Credit constraints and growth in a global economy'. *CEPR*.

Johansson, A. and X. Wang: 2011, 'Financial repression and structural imbalances'. *China Economic Research Center Working Paper* **19**.

Johansson, A. and X. Wang: 2012, 'Financial repression and external imbalances'. *China Economic Research Center Working Paper* **20**.

Johansson, A. C.: 2012, 'Financial Repression and China's Economic Imbalances'. *Rebalancing and Sustaining Growth in China* p. 45.

Jun, Z. and Z. Tian: 2013, 'Chinese consumption grossly underestimated'. *Global Times*.

Kanbur, R. and X. Zhang: 1999, 'Which regional inequality? The evolution of rural-urban and inland-coastal inequality in China from 1983 to 1995'. *Journal of Comparative Economics* **27**(4), 686–701.

Kano, T.: 2008, 'A structural VAR approach to the intertemporal model of the current account'. *Journal of International Money and Finance* **27**(5), 757–779.

Kuijs, L.: 2006, 'How will China's saving-investment balance evolve?'. *World Bank Policy Research Working Paper No. 3958*.

Lardy, N. and N. Borst: 2013, 'A blueprint for rebalancing the Chinese economy'. *Peterson Institute* (3).

Lettau, M. and S. Ludvigson: 2001, 'Consumption, aggregate wealth, and expected stock returns'. *the Journal of Finance* **56**(3), 815–849.

Li, C.: 2010, 'Savings, investment, and capital mobility within China'. *China Economic Review* **21**(1), 14–23.

Lijuan, D.: 2010, 'An overview of China's real estate price index'. *National Bureau of Statistics of China*.

Liu, T. and K. Li: 2006, 'Disparity in factor contributions between coastal and inner provinces in post-reform China'. *China Economic Review* **17**(4), 449–470.

Ma, G. and W. Yi: 2011, 'China's high saving rate: myth and reality'. *Economie internationale* (2), 5–39.

Mendoza, E. G., V. Quadrini, and J.-V. Rios-Rull: 2009, 'Financial integration, financial development, and global imbalances'. *Journal of Political Economy* **117**(3), 371–416.

Modigliani, F. and S. Cao: 2004, 'The Chinese saving puzzle and the life-cycle hypothesis'. *Journal of Economic Literature* **42**(1), 145–170.

Naughton, B.: 2003, 'How much can regional integration do to unify China's markets?'. *How far across the river* pp. 204–232.

Obstfeld, M. and K. Rogoff: 1996, *Foundations of International Macroeconomics*. The MIT Press.

Poncet, S.: 2003, 'Is China disintegrating? The magnitude of Chinese provinces' domestic and international integration'. *China Economic Review*.

Poncet, S.: 2005, 'A fragmented China: measure and determinants of Chinese domestic market disintegration'. *Review of International Economics* **13**(3), 409–430.

Rawski, T.: 2000, 'China by the numbers: How reform affected Chinese economic statistics'. *Unpublished manuscript*.

Rawski, T.: 2001, 'What is happening to China's GDP statistics?'. *China Economic Review* **12**(4), 347–354.

Rawski, T. and R. Mead: 1998, 'On the trail of China's phantom farmers'. *World Development* **26**(5), 767–781.

Sachs, J., R. Cooper, and S. Fischer: 1981, 'The current account and macroeconomic adjustment in the 1970s'. *Brookings Papers on Economic Activity* **1981**(1), 201–282.

Scharping, T.: 2001, 'Hide-and-seek: China's elusive population data'. *China Economic Review* **12**(4), 323–332.

Shen, J.: 2006, 'Estimating Urbanization Levels in Chinese Provinces in 1982-2000'. *International statistical review* **74**(1), 89–107.

Smith, C.: 2001, 'China reports 7.8*The New-York Times*.

Soh, K. and A. Wang: 2011, 'Special report: China's debt pileup raises risk of hard landing'. *Reuters*.

Song, Z., K. Storesletten, and F. Zilibotti: 2011, 'Growing like China'. *The American Economic Review* **101**(1), 196–233.

Storesletten, K. and F. Zilibotti: 2014, 'China's Great Convergence and Beyond'. *Annual Economic Review*.

University, S. C. and C. People's Bank: 2012, 'Chinese Household Finance Survey'. *Southwest China University and People's Bank of China*.

Voigtlaender, N. and H.-J. Voth: 2013a, 'How the West" invented" fertility restriction'. *The American Economic Review* **103**(6), 2227–2264.

Voigtlaender, N. and H.-J. Voth: 2013b, 'The three horsemen of riches: Plague, war, and urbanization in early modern Europe'. *The Review of Economic Studies* **80**(2), 774–811.

WB: 2002, 'Building Statistical Capacity to Monitor Development Progress'. *The World Bank*.

Wei, S.-J. and X. Zhang: 2009, 'The competitive saving motive: Evidence from rising sex ratios and savings rates in China'. *NBER Working Paper*.

Xiaolu, W. and M. Lian: 2001, 'A reevaluation of China's economic growth'. *China Economic Review* **12**(4), 338–346.

Xu, X.: 2008, 'Consumption Risk-Sharing in China'. *Economica* **75**(298), 326–341.

Yang, D.: 2002, 'What has caused regional inequality in China?'. *China Economic Review* **13**(4), 331–334.

Yang, D., J. Zhang, and S. Zhou: 2011, 'Why are saving rates so high in China?'. *NBER Working Paper*.

Yao, S. and Z. Zhang: 2001, 'On regional inequality and diverging clubs: a case study of contemporary China'. *Journal of Comparative Economics* **29**(3), 466–484.

Yongding, Y.: 2012, 'China's struggle to slow'. *Project Syndicate*.

Zhang, Z.: 2008, 'Dark Matter in China's Current Account'. Unpublished manuscript, International Monetary Fund.

Appendix A

Appendix to “Capital’s Long March West”

A.1 Data and robustness checks appendix

A.1.1 Factors

Investment structure:

SOInvFA state-owned share of investment in fixed assets (by status of registration/ownership).
FOInvFA: foreign-owned share of investment in fixed assets (HK, TW and MO included).
REInvFA: residual share of investment in fixed assets (contains collectively-owned, private and self-employed).

State vs Private:

SOGIOV: state-owned share in gross industrial output value.
EmplPrivate: private and self-employed employment share.
Market: mean of marketization index (1997-2005 average). Scale 0-10. Factors: government and market, ownership structure, goods market dvpt, factor market dvpt and legal framework. See Fan et al. (2001).
SOCGOV: state-owned share in construction gross output value.

Economic structure:

SectorPrim: share of primary sector in GDP (production approach).
SectorInd: share of industrial sector in GDP (production approach)
SectorConst: share of construction sector in GDP (production approach).
SectorTert: share of tertiary sector in GDP (production approach).
StructConc: Index of structural economic concentration using primary, industrial, construction and tertiary sector.
HousingPrice: growth of housing prices 1999-2010 (selling price of commercialized buildings, includes business and residential use, in RMB/sq.m).
CoalOil: coal and crude oil production value over GDP (using world prices for 1984-1988 and national price indices for non-ferrous metal and raw material/fuel for 1989-2010).

International:

Openness: international exports + imports over GDP (by place of destination and origin).
MNE: share of multinational enterprises in provincial international exports, mean of 1996 and 2008, from Girardin and Owen (2011).
FDI: share of used FDI over GDP.

Financial Dvpt:

Deposits: deposits in banks and financial institutions over GDP.
Loans: loans in banks and financial institutions over GDP.

Human Capital:

TertiaryEduc: enrollment in tertiary school over population.
HighEduc: enrollment in higher education over population.
Patents: patents granted per 10 000 habitants.

Social Security:

SOCSEC: index of social security coverage. Mean of pension insurance (average of urban contributors over urban employment 2005-2010 and rural contributors over rural employment 2005-2009, mixed using urbanization rate), unemployment insurance 2000-2010 (average of contributors over employment) and medical insurance 2000-2010 (average of contributors over population).

Demographics:

OldDepRatio: people 65 and more over population, mean of 2000 and 2010 Censuses.
SexRatio: nb of men for 1 woman, mean of 2000 and 2010 Censuses.
UrbRate: urbanization rate (Shen, 2006 and statistical yearbooks).
EthnicShare: share of non-Han relative to population. 2000 Census.

A.1.2 Data

A.1.2.1 Capital stock

National and regional capital stocks are estimated using the perpetual inventory method. Business cycles are removed from the initial value by taking the mean of f (real gross fixed capital formation, GFCF) adjusted for its growth rate.[1] In accordance with the model's notation, 1984 is $t = 0$ and 2010 is $t = T = 26$.

The initial gross fixed capital formation (F_0) is

$$F_0 = \frac{1}{T+1} \sum_{t=0}^{T} \frac{f_t}{((\frac{f_T}{f_0})^{\frac{1}{T}})^t}$$

To transform data into real terms, we use Brandt and Holz (2006)'s consumption basket expenditure for the initial price level, the regional consumer price index (CPI) for 1984-1991 and the regional price index of investment in fixed assets (PIFA) for 1992-2010 (i.e. as soon as available).[2]

The initial capital stock (K_0) is

$$K_0 = \frac{F_0}{\delta + k}$$

where δ is the yearly depreciation rate of capital and k is the growth rate of capital. As in Gourinchas and Jeanne, a value of $\delta = 0.06$ is assumed. The choice of a region-specific k is more controversial. Usually, a mean of past growth rates is used. Unfortunately, this method is impossible to apply to all regions due to the lack of data on GFCF and CPI for the pre-1984 period.[3] As an alternative, we use the mean value for the 1984-2010 period.[4] By using the initial capital stock and real gross fixed capital formation, we obtain provincial and national time series for capital stock. We discuss the effects of capital adjustment costs in Section A.1.4.1.

In 1984, the initial capital stock to real output ratio is on average 1.2 with values between 0.6 and 2.1. At the end of the sample (2010), values for relative capital stock range between 1.6 and 3.8 with a mean of 2.4. Average provincial relative capital stock more than doubled between 1984 and 2010. The 2010 value for China (2.3) is in line with estimates found in the literature and on the private research market (HSBC, 2012, Dragonomics). Nevertheless, our estimation strategy has some shortcomings.[5]

[1]We are thankful to Prof. Dr. Woitek for suggesting that approach.

[2]We thus take into account differences in inflation as well as initial differences in price level. The reference is the price level for Chinese national value in 1984 Renminbi.

[3]Jiangxi, Guangdong, Hainan, Chongqing, Sichuan, Tibet and Ningxia have missing GFCF values before economic reforms. As for CPI, more than half of provinces have missing values.

[4]In the literature, the growth of output of past periods is sometimes used as a proxy for capital growth. Here again, the poor output data for some provinces before 1984 refrain us from giving it a try. Using post 1984 output growth would lead to a roughly similar k compared to using GFCF growth. For China, real output growth over the sample period has been of 10% while real gross fixed capital formation growth has been of 12%.

[5]Intuitively, relatively rich provinces should have a higher 1984 K/Y ratio than less developped ones. In our case,

A.1.2.2 Technology catch-up

The time path of total factor productivity (A) is estimated using a labor share $(1-\alpha)$ of 0.70.[6] Real output (expenditure approach) is in 100 million Renminbi for Y.[7] We use Brandt and Holz (2006) consumption expenditure basket price data and official CPI to convert nominal into real output.[8] The exactness of GDP figures in China is highly controversial. For a general discussion of aggregation properties and quality, we refer to the second chapter of the thesis. The model assumes that labor supply is exogenous and equal to population. For L, Gourinchas and Jeanne used working-age population. We refrain from using regional active population because of data issues and take employed persons in million as a proxy for L.

As population, Chinese employment measurement is an issue of its own: bad aggregation properties as well as biases arising from migration and erroneous sectoral reporting in the agricultural and state sector are common. Keeping that in mind, we assemble our own employment dataset using provincial and national statistical yearbooks. In Section 3.5.4, we address that issue by using data from Brandt et al. (2012). The fact that we are primarily interested in capturing long run trends and the smoothing of the TFP series should minimize potential biases. In order to get rid of transitory fluctuations, we use the Hodrick-Prescott filter.[9] The annual growth rate of TFP is then obtained using filtered data (i.e. $g^* = \left(A_T^{trend}/A_0^{trend}\right)^{1/T}$). In contrast to Gourinchas and Jeanne, we adopt a new definition of g^* as being the growth of the Chinese productivity frontier instead of the world.[10]

The last value of the catch-up parameter is defined as the steady-state one ($\pi_T = \pi$), which is

this is generally true (e.g. Shanghai 1.4 vs Guangxi 0.7). There are some important exceptions: although being below average in terms of wealth, western regions like Xinjiang (2.1), Qinghai (1.5) and Ningxia (1.6) start with relatively high initial value. The reason is that many less developed western provinces started with low output and experienced huge capital formation flows (i.e. high F_0) relative to their economic size.

[6]Our labor share is similar to Gourinchas and Jeanne but may be slightly too high compared to the aggregate value of 0.60 suggested by Brandt and Zhu (2010). Our results would not be seriously affected by such a change. Assuming the same factor elasticities for all regions is not controversial: Brandt et al. (2012) found them to be very similar.

[7]The use of GDP data computed following the production approach would not influence results much. On the national level, differences between expenditure and production GDP approach figures have been of a little less than 2% of expenditure output over the reform period on average. No systematic discrepancies on the provincial level have been observed. The regions of Shanxi, Zhejiang, Anhui, Hubei, Hainan and Chongqing have average errors of more than 1% of output.

[8]GDP deflators are not directly available at the provincial level and would have to be inferred from the real growth of the agriculture, manufacturing and service sector (see Brandt and Zhu, 2010). We use their productivity figures in Section 3.5.4 as a robustness check.

[9]Gourinchas and Jeanne used a value of 1600 that filters out more than 70% of cycles lower than 32 years. Although it certainly fits within the long run focus of the model, we stick to the usual macroeconomic value of 100. The reason is specific to the huge transformation of the Chinese economy: the steep rise in TFP experienced in most provinces causes the filter to start with a value far below initial data points. As a consequence, TFP growth would be unrealistically high. Furthermore, we are not convinced of using such a heavy filter on our short time series. In the end, it primarily affects the absolute value of the catch-up parameters rather than their distribution across provinces.

[10]To be conceptually correct, one should compute the catch-up values not relative to national figures but to a mix of world and local TFP growth as provincial net exports contain international as well as interprovincial flows. In fact, compared to the rest of the world, all regions have extremely large catch-up values. We find it more convenient to refer to national values to look at cross-sectional patterns. We motivate our approach in Section A.1.3.1 and show that taking alternative values of g^* does not invalidate our main conclusions.

used for the estimation of the wedges. Our empirical value for China is $g^* = 7.07\%$. Compared to values obtained in recent studies, our estimate appears to be an upper bound. It seems to make sense to the extent that we do not control for other potential factors inside TFP (e.g. human capital).[11]

A.1.2.3 Capital flows

In the baseline version of the estimation procedure, we refrain from using estimates of the initial debt position.[12] Still, we discuss the effects of considering a synthetic initial debt in Section A.1.3.3. For that reason, we keep the methodology as general as possible and already include initial debt in the discussion.

In Gourinchas and Jeanne, the change in external position over the sample (ΔD) was approximated by using a measure of initial net external debt as well as the sum of negative current accounts and net overseas development assistance.[13] We could in principle approximate the initial external positions by using (negative) net exports from 1952 to 1983 and deflate them using the cumulated CPI inflation of China over the period:[14]

$$D_0^n = -\sum_{t=1952}^{1983} \frac{NX_t}{CPI_{1983}^{nat}}$$

Define Q as the regional deflator. Thereafter, we follow Gourinchas and Jeanne and start with the external accumulation equation and some definitions:

$$\begin{aligned} D_T^n &= D_0^n - \sum_{t=0}^{T-1} CA_t^n \\ \frac{D_T^n}{Q_T} &= D_T^r \\ \frac{D_0^n}{Q_0} &= D_0^r \end{aligned}$$

which gives

[11] Note that, unlike in the literature on growth accounting where TFP is defined as Solow residual, in this framework TFP is labor-increasing and can thus can *de facto* be interpreted as a mix of human capital and TFP. Without accounting for human capital, Brandt and Zhu (2010) found an average Solow residual growth of 3.05% over 1988-1998 and 4.58% for 1998-2007.

[12] First, transfers between the central government and regions are conceptually not comparable to movements in international assets as they could be unlimited and free of interests. Second, data availability issues do not allow us to include income flows and transfers in the net exports measure.

[13] For initial debt, they used the difference between the opposite of net international investment position (NIIP) and cumulated errors and omissions.

[14] Many regions lack CPI data before 1984. As for net exports, we had no choice but to infer the entire 1952-1983 time series from 1978-1983 for Jiangxi, Guangdong, Sichuan and Ningxia. Note that as initial periods were quite flat in terms of net exports, we chose to consider only half the obtained amount. The value for Hainan is set to zero corresponding to the first available values between 1984 and 1986. A gap for Chongqing in 1957 is ignored.

$$
\begin{aligned}
D_T^r Q_T &= D_0^n - \sum_{t=0}^{T-1} CA_t^n \\
D_T^r &= D_0^n / Q_T - \sum_{t=0}^{T-1} CA_t^n / Q_T \\
D_T^r - D_0^r &= D_0^n \left(\frac{1}{Q_T} - \frac{1}{Q_0}\right) - \sum_{t=0}^{T-1} \frac{CA_t^n}{Q_T}
\end{aligned}
$$

We use net exports instead of current account as, to our knowledge, no income flows statistic at the regional level is available over the sample period. Ideally, the deflator Q should capture the evolution of the price of traded goods. Gourinchas and Jeanne used the price of investment goods as a proxy due to data restriction. Our deflator for cumulated net exports and debt (PTG) is a combination of regional CPI (1984 to 1996) and of the regional producer price index of manufactured goods (PPI) (1997-2010) as a proxy for the price of tradable goods. Thus, our final expression for relative capital flows from 0 to $T-1$ is

$$
\frac{\Delta D^r}{Y_0} = \frac{D_0^n}{Y_0} \left(\frac{1}{PTG_T} - \frac{1}{PTG_0} \right) - \frac{\sum_{t=0}^{T-1} NX_t^n / PTG_T}{Y_0}
$$

It is normalized by initial real GDP based on regional CPI ($Y_0 = Y_0^n / CPI_0$). The initial debt or asset position enters our flow indicator with the following reasoning: in steady-state, provinces with initial debt position D_0^n – negative cumulated net exports in the past – will experience capital outflows depending on the pace of price level growth as they hold their debt ratio constant. Regions with initial assets will experience capital inflows.

A.1.3 Sensitivity analysis

A.1.3.1 Reference productivity growth rate

The reference rate of productivity growth (g^*) is an important parameter of the model as it determines the benchmark against which provinces catch-up or fall behind in TFP terms. It directly influences the reference interest rate.[15] While the authors of the initial paper chose US TFP growth (1.7%), we used our estimated value for China (7.07%). Obviously, this high national rate only plainly makes sense when the focus lies on intranational capital flows. The purely international and mixed flows should be discussed using a productivity reference rate rather similar in value to world TFP growth rate. In the past sections, we refrained from it.[16]

[15] In the baseline model, the assumption that $R^* = g^{*\gamma}/\beta$ is used.

[16] Four arguments can be made to justify that choice. 1.The interpretation of results is more intuitive using zero-centered saving wedges. 2.All regions without exception massively caught up relative to the world technology frontier. As a consequence, we would end up with abnormally high π. For a given set of "extreme" parameters, the model has difficulties to match precisely the data and to identify the frictions. 3.As discussed in the second chapter of the thesis, much of the dynamics and discrepancies in external balances inside China seems to arise from interregional capital flows rather than purely international ones. 4.We are primarily interested in determining and looking into

A lower benchmark productivity value raises the level of catch-up parameters. For example, by using an hypothetical reference rate of 3%, the value range would expand from $[-0.45\ \ 0.86]$ to $[-0.51\ \ 4.11]$. In Figure 3.17, we provide a graphical representation of the provincial frictions for different reference rates. We observe only minor variations in the rank of regional frictions. The general patterns are identical and the wedges using a 3% TFP would be highly correlated with their counterpart using national TFP estimates.[17] The saving wedge - TFP relationship becomes increasingly negative and remains significant as the reference rate grows. Qualitatively, econometric results are comparable.

A.1.3.2 Coefficient of relative risk aversion

Up to this point, the entire procedure has been run with log utility (i.e. unit intertemporal elasticity of substitution). Alternative risk aversion coefficients (γ) influence investment wedges in a near linear way: the entire distribution shifts down following a parameter increase (Figure 3.18 on the left). Changes in relative risk aversion affect saving wedges (Figure 3.18 on the right). The range and the cross-sectional variability in saving wedges increases with higher coefficient. The relationship with TFP becomes more and more negative. Minor changes in ranking are observed but the correlation with the baseline case remains high.[18]

A.1.3.3 Initial external position

Although the general capital flows concept has already been presented in Section A.1.2.3, we have refrained from assuming an initial debt or asset position in the wedges' computation yet. In this section, we include them in the analysis. The initial debt level enters the model via the capital flows estimation and the debt in efficiency units in the trend channel ($\Delta D^t/Y_0$) of the relative capital flows decomposition used to estimate saving wedge.

The initial position is estimated using half of cumulated negative net exports from 1952 to 1983 divided by the national CPI index. In Figure 3.19, we observe that initial external positions are large. The Metropolises – Shanghai, Beijing and Tianjing – and to a lesser extent Manchurian regions had accumulated large assets over the pre-reform period and, as a consequence, experienced capital inflows from their net external position in the following periods. West and South China recorded large initial debt (i.e. subsequent capital outflows) while most of the Center was roughly neutral.

Even though the initial debt part reveals itself to be negatively correlated with the net exports part of our flow indicator, the latter primarily drives the pattern of the capital flows estimation. Our final flows measure is still nearly perfectly correlated with the one of the baseline case.

cross-regional patterns of wedges rather than their absolute value.

[17] For investment wedges, the correlation is of 0.94 and the rank correlation 0.92, while corresponding values for saving wedges amount to 0.97 and 0.96.

[18] For $\gamma = 2$, the correlation of saving wedges is of 0.97 and the rank correlation of 0.96.

The minor changes in the relative ranking of the flows do not translate into substantial changes in saving wedges.[19] Still, there are some major differences for Metropolises and West China. Beijing, Tianjin and Shanghai accumulated large external assets of more than 200% of GDP before economic reforms. Implicit capital inflows over the next decades are automatically making their saving wedge more positive. The situation is reversed for some less developed western regions (e.g. Xinjiang) where high initial debt – and subsequent capital outflows – lowers the net inflows and the saving wedge.

These changes do not invalidate the *capital allocation puzzle*. In Figure 3.20, where we let the weight on initial external position vary between 0 and 1, the negative correlation of saving wedge with productivity is maintained but the substantially affected regions that we mentioned end up on the flip side of the regression line without affecting much its slope and significancy. All in all, taking net external position into account only influences the saving friction of a few provinces and does not affect our conclusions.

A.1.4 Model extensions

A.1.4.1 Capital adjustment costs

In preceding sections, following Gourinchas and Jeanne, we abstracted from capital adjustment costs. They argued that their results were robust to that extension. In our case, they could potentially affect less developed regions that accumulated huge relative GFCF relative to their economic size. Thus, the introduction of capital adjustment costs is particularly interesting as its effect on the distribution of investment wedges could lead to a change in the investment puzzle's pattern. We introduce two adjustment costs. The first one influences the initial jump in capital to reach the steady-state level. The second one takes effect in the dynamics of capital over time:

$$I_t = K_{t+1} - (1-\delta)K_t + \kappa_2 (K_{t+1} - K_t)^{\theta}$$

The parameters κ_1 and κ_2 enable us to test the effects of various costs of adjusting capital stock (see Section A.2.4):

$$\begin{aligned} i \quad = \quad & \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{\alpha}} + \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^{\alpha}}\kappa_1 + g^*\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)} + (g^*n + \delta - 1)\tilde{k}^{*(1-\alpha)} \\ & + \frac{1}{T}\sum_{t=0}^{T-1}\kappa_2 \frac{(A_{t+1}N_{t+1}\tilde{k}^* - A_t N_t \tilde{k}^*)^{\theta}}{A_t N_t \tilde{k}^{*\alpha}} \end{aligned}$$

The implementation of the initial jump cost driven by κ_1 presents no major difficulty. For a value of $\kappa_1 = 0.5$, capital adjustment needed to reach steady-state capital value makes up 3 percentage points of investment rate on average. While it is quite large for a typical high investment

[19] The correlation and rank correlation between saving wedges of the baseline model and this version are 0.96.

region (e.g. Ningxia, 5.0pp), it is negligible for initially more developed provinces (Shandong, 2.4pp).

The second adjustment implies that economies have to use a supplementary part of their investment flow to maintain capital in efficiency unit of labor at steady-state level. In terms of the grid-search, it means that provinces with high average investment rate are attributed a lower $\tilde{k}^*$and a higher relative adjustment cost channel. The κ_2 channel influences three steps of the model estimation and its implementation raises several issues.[20]

The adjustment costs are substantial. Economically large and/or fast investment growing regions are particularly affected. Capital stock is lower and the technology reference productivity growth rate rises. In spite of the adjustment in the value of κ_2, national values end up being more affected by adjustment costs. Consequently, most provinces end up with lower catch-up parameter compared to the baseline case. In the estimation of the investment wedge components, we have two new channels. Over all provinces and given our specification, the κ_1 channel accounts for between 0.8 and 5.5pp of average investment. The κ_2 channel is roughly similar in terms of relative size (between 0.3 and 6.4pp). As expected, western regions with high investment (e.g. Inner Mongolia, Ningxia and Shaanxi) and Metropolises (e.g. Beijing) have relatively high adjustment costs relative to their investment level. We register only minor changes in the distribution of frictions. Interestingly, as adjustment costs increase, the positive relationship between investment wedges and catch-up gets stronger (higher slope coefficient and lower p-value).

This somewhat counterintuitive result is due the following trade-off: on one side, our methodology attributes relatively high adjustment costs to regions with high investment rate and high GFCF growth while on the other side, larger provinces register higher relative adjustment costs due to the similarity of the κ_2s across provinces. Typically, high investment regions in the West are smaller. Thus, both effects run in opposite direction. One would have to let κ_2 vary with economic size (i.e. higher for smaller regions) in order to be able to put the positive investment wedge - productivity relationship at risk. Patterns of the factor regressions are similar.

A.1.4.2 Exogenous interest rate

In the baseline version, $R^* = g^{*\gamma}/\beta$ is assumed. It implicitly means the rest of the world is composed of steady-state economies that have no saving wedge and preferences similar to the domestic small economy. This assumption is by no way innocuous. It is used in the second step of our detailed derivation of their model (see Section A.2.2). It enables to obtain a closed-form

[20] 1.The choice of the θ parameter is not innocuous as quadratic or cubic terms may lead to convergence problems. We choose $\theta = 2.5$. 2.κ_2 enters the computation of the national capital stock time series and thus influences the reference productivity growth rate. We typically need low κ_2 values (e.g. 0.0000002) as we deal with large macroeconomic figures. The capital stock time series is approximated using a grid-search procedure. It implies substantial capital adjustment costs of 9% of GDP on average and lowers the final Chinese K/Y ratio to 1.9 compared with 2.3 in the baseline case. As capital stock is lower, average TFP growth rises from 7.07 to 7.32%. 3.For the computation of regional technology catch-up parameters, we adapt κ_2 to 0.000002 in order to get comparable adjustment costs. 4.The estimation of the wedges is based on normalized values. That is why we modify κ_2 once again (0.05).

solution that is more easily estimated (i.e. a stable long-run distribution). We crunch through an alternative version refraining from any restriction on the functional form of world interest rates. The convergence and investment channels are identical. The new trend and saving components are:

$$\frac{\Delta D^t}{Y_0} = \frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \Omega\psi(\tau_s)\,[n\phi(\tau_s)]^T\,\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}$$

and

$$\frac{\Delta D^s}{Y_0} = \frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

where

$$\Omega = \frac{(R^* - n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^* - ng^*}$$

One cannot use ψ to get rid of the infinite sum term anymore. Clearly, the saving channel will only converge if $R^* > ng^*$. For our dataset, it implies that aggregate interest rates have to be at least higher than 10%. Even for sufficient rates, any estimation of the infinite sum is only precise for a long time period.

We use different interest rates to shed light on whether the main findings are robust to exogenous interest rates. The flexible R^* has no impact on TFP. The distribution of investment wedges remains identical. Saving wedges are positively related to interest rates. Although the location of some provinces in the catch-up vs saving wedge representation is highly sensitive to the interest rate, the negative significant relationship is maintained. The correlations of saving wedges with explanatory factors is not particularly robust and often, significancy is lost. This is not surprising given the lower precision of the identification.

A.1.4.3 Implicit wage frictions

There are large differences in productivity (and wages) among regions that triggered large population migration flows. In that respect, discussing the potential effects of the introduction of a labor wedge in the model seems natural.[21]

As long as we exclude feedback effects of households on the aggregate labor supply, investment wedges would not be influenced by such a friction. The wage friction could directly be implemented in the representative agent's budget constraint (i.e. in a similar fashion as the saving wedge). However, as total revenues from frictions are rebated to households, transfers end up

[21] In their paper on factor market misallocation, Brandt et al. (2012) established that frictions in labor allocation, as opposed to capital frictions, are the driver of cross-regional distortions in TFP. They find them to be constant over time in spite of the large migration flows experienced from low to high productivity provinces. Interestingly, capital frictions explained misallocation inside regions (i.e. between the state and the non-state sector) and have been on the rise since the mid-1990s.

being only driven by investment frictions. As briefly discussed in Section A.2.6, one would need to introduce an explicit labor-leisure choice in the maximization problem in order to change our results.[22] Even if exogenous differences in wages were imposed by, for example, not rebating revenues from wages frictions, the effect would be similar in spirit to a variation in investment frictions (i.e. a level effect on the saving channel via transfers) and thus insufficient to turn the *capital allocation puzzle* upside down.

All the same, we still at least partly address that issue by computing the real wages implied by the model and compare them with regional wage income data. We compute the model-induced regional and national steady-state real wages as:

$$w = \underbrace{(1-\alpha)k^{*\alpha}}_{\tilde{w}} A_T$$

and capture the difference between provincial and national values with a labor wedge:

$$w^* = (1-\tau_w)w$$

Our proxy for real wage is urban income from wages and salaries deflated by CPI (2010). In Figure 3.21, we compare the implicit wedge between regional and national values ($-\tau_w = -(1-\frac{w^*}{w})$) of the model with the data. First, the relationship is clearly positive and the correlation is high (0.61): regions with lower predicted wages relative to China (positive $-\tau_w$) tend to have lower wages in the data as well. Second, while 22 out of 31 regions are in the "right" quadrant, there seems to be differences between the model's implications and the data for a few provinces. For instance, Beijng and Shanghai (northwest quadrant) seem to have higher wages than predicted by the model. Some western regions (e.g. Xinjiang, Shaanxi and Inner Mongolia in the southeast quadrant) turn out to have the opposite pattern.[23]

[22] We refrain from it for three reasons. First, this would be a major theoretical extension. Second, there is uncertaintly concerning data availability over the entire sample. Third, the labor-leisure trade-off on the intensive margin is possibly less relevant for developing countries like China. In our opinion, the extensive margin (i.e. labor force participation) is definitely more of an issue.

[23] Brandt and Zhu (2010) found a large state sector wage premium in the non-agricultural sector. In our case, it does not seem to be the case that regions where the state is typically more present enjoy higher real wages. This effect may manifest itself rather within regional economies.

A.2 Mathematical appendix[24]

A.2.1 Decomposition of average investment over GDP (investment wedge identification)

Dynamics of capital stock:

$$I_t = K_{t+1} - (1-\delta)K_t$$

Divide by GDP in period t:

$$i_t = \frac{K_{t+1} - (1-\delta)K_t}{Y_t}$$

Find an alternative expression for Y_t assuming that labor supply is equal to the entire population ($L_t = N_t$):

$$\begin{aligned} Y_t &= K_t^{\alpha}(A_t L_t)^{1-\alpha} \\ &= K_t^{\alpha}(A_t N_t)^{1-\alpha} \\ &= \left(\frac{K_t}{A_t N_t}\right)^{\alpha} A_t N_t \\ &= \tilde{k}_t^{\alpha} A_t N_t \end{aligned}$$

For $t \geq 1$we have a constant steady-state capital in efficiency units. Thus:

$$\begin{aligned} Y_t &= A_t N_t \tilde{k}^{*\alpha} \\ K_{t+1} &= A_{t+1} N_{t+1} \tilde{k}^{*} \\ K_t &= A_t N_t \tilde{k}^{*} \end{aligned}$$

Using $g_{t+1} = \frac{A_{t+1}}{A_t}$ and the assumption that population grows at a constant rate ($n_{t+1} = n = \frac{N_{t+1}}{N_t}$), we can rewrite i_t as

$$\begin{aligned} i_t &= \frac{A_{t+1}N_{t+1}\tilde{k}^{*} - (1-\delta)A_t N_t \tilde{k}^{*}}{A_t N_t \tilde{k}^{*\alpha}} \\ &= \frac{A_{t+1}N_{t+1}\tilde{k}^{*}}{A_t N_t \tilde{k}^{*\alpha}} - (1-\delta)\frac{\tilde{k}^{*}}{\tilde{k}^{*\alpha}} \\ &= g_{t+1} n \tilde{k}^{*(1-\alpha)} - (1-\delta)\tilde{k}^{*(1-\alpha)} \\ &= \tilde{k}^{*(1-\alpha)}\left[g_{t+1}n - 1 + \delta\right] \end{aligned}$$

In $t = 0$, there is a term reflecting the initial jump from $\tilde{k}_0$ to $\tilde{k}^{*}$. First note that

$$\begin{aligned} K_0^{*} &= A_0 N_0 \tilde{k}^{*} \\ K_0 &= A_0 N_0 \tilde{k}_0 \\ Y_0 &= A_0 N_0 \tilde{k}_0^{\alpha} \end{aligned}$$

One gets the following jump term:

[24] The baseline derivations are own detailed explanations of the standard setting in Gourinchas and Jeanne (2013). Capital adjustment costs, exogenous interest rate and wage friction are extensions.

$$\begin{aligned}\frac{K_0^* - K_0}{Y_0} &= \frac{A_0 N_0 \tilde{k}^* - A_0 N_0 \tilde{k}_0}{A_0 N_0 \tilde{k}_0^\alpha} \\ &= \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha}\end{aligned}$$

Add the jump term to the initial expression for i_t to get i_0:

$$i_0 = \tilde{k}^{*(1-\alpha)}(g_1 n + \delta - 1) + \frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha}$$

Time frame: start at $t = 0$ until $t = T$, which is the last observation (steady-state). Thus, the last data is ignored. In our sample ($1984 - 2010$), we have $\frac{1}{26}\sum_{t=0}^{t=25} i_t$ although we have a total of 27 periods.

The average investment rate between t and $T-1$ is

$$\begin{aligned} i &= \frac{1}{T}\sum_{t=0}^{T-1} i_t \\ &= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \frac{1}{T}\sum_{t=0}^{T-1}(g_{t+1} n + \delta - 1)\tilde{k}^{*(1-\alpha)}\end{aligned}$$

The first part is the initial jump term in i_0 whereas the second one gathers the standard elements from 0 to $T-1$. Define average productivity growth rate as $\frac{1}{T}\sum_{t=0}^{T-1} g_{t+1} = \bar{g}$. Rewrite the second term of the last expression and expand with $g^* n \tilde{k}^{*(1-\alpha)}$ (note that the sum from 0 to $T-1$ divided by T has no effect on constants):

$$\begin{aligned}(\bar{g}n + \delta - 1)\tilde{k}^{*(1-\alpha)} &- g^* n \tilde{k}^{*(1-\alpha)} + g^* n \tilde{k}^{*(1-\alpha)} \\ (\bar{g}n - g^* n)(\tilde{k}^{*(1-\alpha)}) &+ (\delta - 1 + g^* n)\tilde{k}^{*(1-\alpha)} \\ (\bar{g} - g^*) n \tilde{k}^{*(1-\alpha)} &+ (g^* n + \delta - 1)\tilde{k}^{*(1-\alpha)}\end{aligned}$$

We want to express g_{t+1} relative to the catchup parameters π_t and π_{t+1}. First note that:

$$\begin{aligned} A_t \leq A_t^* &= A_0^* g^{*t} \\ \pi_t &= \frac{A_t}{A_0 g^{*t}} - 1 \\ \pi_{t+1} &= \frac{A_{t+1}}{A_0 g^{*t+1}} - 1\end{aligned}$$

Start with the definition of technology growth rate and use A_t and $A_{t+1}$from former equations:

$$\begin{aligned} g_{t+1} &= \frac{A_{t+1}}{A_t} \\ &= \frac{1+\pi_{t+1}}{1+\pi_t}\frac{A_0 g^{*t+1}}{A_0 g^{*t}} \\ &= \frac{1+\pi_{t+1}}{1+\pi_t} g^*\end{aligned}$$

where one can approximate $\frac{1+\pi_{t+1}}{1+\pi_t}$ with $1+\pi_{t+1}-\pi_t$.
Rewrite $\bar{g}$:

$$
\begin{aligned}
\bar{g} &= \frac{1}{T}\sum_{t=0}^{T-1} g_{t+1} \\
&= \frac{1}{T}\sum_{t=0}^{T-1} \frac{1+\pi_{t+1}}{1+\pi_t} g^* \\
&\approx \frac{1}{T}\sum_{t=0}^{T-1} (1+\pi_{t+1}-\pi_t) g^*
\end{aligned}
$$

Focus on the concrete form of the summation, taking into account that $\pi_0 = 0$ and $\pi_T = \pi$ (i.e. steady-state at $T = 26$, in the 27th period):

$$
\begin{aligned}
\frac{1}{T}[(1+\pi_1-\pi_0 \quad)+ &\quad (1+\pi_2-\pi_1)+(1+\pi_3-\pi_2)+\ldots+(1+\pi_T-\pi_{T-1})] \\
&\frac{1}{T}[1+1+1+...+\pi] \\
&\frac{26}{26}+\frac{\pi}{26} \\
&1+\frac{\pi}{T}
\end{aligned}
$$

Thus we have that $\bar{g} = (1+\frac{\pi}{T})g^*$, which we use in i:

$$
\begin{aligned}
i &= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \frac{1}{T}\sum_{t=0}^{T-1}(g_{t+1}n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + (\bar{g}-g^*)n\tilde{k}^{*(1-\alpha)} + (g^*n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&= \frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha} + \left[g^*(1+\frac{\pi}{T})-g^*\right]n\tilde{k}^{*(1-\alpha)} + (g^*n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&= \underbrace{\frac{1}{T}\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{k}_0^\alpha}}_{convergence} + \underbrace{g^*\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)}}_{catch-up} + \underbrace{(g^*n+\delta-1)\tilde{k}^{*(1-\alpha)}}_{depreciation}
\end{aligned}
$$

Convergence component: initial investment at $t = 0$ required to put capital at its steady-state level.
Catch-up component: additional investment required by the productivity catch-up.
Depreciation component: investment required to offset capital depreciation taking into account productivity and population/labor force growth.

A.2.2 Closed-form expression for relative cumulated capital flows (saving wedge identification)

Step 1: debt ratio

Note that $\triangle D = D_T - D_0$, $\tilde{d}_T = \frac{D_T}{A_T N_T}$ and $\tilde{y}_0 = \frac{Y_0}{A_0 N_0}$. We can rewrite relative change in debt as

$$\begin{aligned}\frac{\triangle D}{Y_0}=\frac{D_T-D_0}{Y_0} &= \frac{\tilde{d}_T A_T N_T-\tilde{d}_0 A_0 N_0}{A_0 N_0 \tilde{y}_0}\\ &= \frac{\tilde{d}_T A_T N_T/A_0 N_0-\tilde{d}_0}{\tilde{y}_0}\end{aligned}$$

Notice that

$$\begin{aligned}\frac{A_t}{A_0 g^{*t}} &= 1+\pi_t\\ \pi_T &= \pi\\ \frac{A_T}{A_0} &= g^{*T}(1+\pi)\\ \frac{N_{t+1}}{N_t} &= n_{t+1}=n\\ \frac{N_T}{N_0} &= n^T\end{aligned}$$

Use expression for A_T/A_0 and N_T/N_0:

$$\begin{aligned}\frac{\triangle D}{Y_0} &= \frac{\tilde{d}_T g^{*T}(1+\pi)n^T-\tilde{d}_0}{\tilde{y}_0}\\ & \frac{\tilde{d}_T (g^* n)^T(1+\pi)-\tilde{d}_0}{\tilde{y}_0}\end{aligned} \tag{A.1}$$

At $t=0$, there is a jump in external debt to $\tilde{d}_0^+=\tilde{d}_0+\tilde{k}^*-\tilde{k}_0$ to finance the initial increase in capital from $\tilde{k}_0$ to $\tilde{k}^*$. Normalization by initial output level occurs before capital jump.

Now we look for $\tilde{d}_T$ using the *BC* of the representative HH:

$$N_t w_t+N_t z_t=C_t+K_{t+1}-(1-\tau_s)R^*K_t+(1-\tau_s)R^*D_t-D_{t+1}$$

HH resources: work income, transfers, net returns from capital, higher debt and interests on assets.
HH expenditures: consumption, investment in capital for next period, debt repayment and interests on debt.
The marginal influence of the saving wedge is $\frac{\partial .}{\partial \tau_s}=R^*(K_t-D_t)$. Thus, if $K_t>D_t$ we have a positive effect of an increase in τ_s on the expenditures (less ressources). If $K_t<D_t$, HH have more resources at disposal. There are two opposite effects given higher τ_s: HH get less returns on capital but pay less on debt. Which effects dominates depends on the size of K_t and D_t. The former situation is considered as the standard case.

Divide *BC* by N_t and use $c_t=\frac{C_T}{N_T}$ and $N_t=\frac{N_{t+1}}{n}$:

$$\begin{aligned}w_t+z_t &= \frac{C_t}{N_t}+\frac{K_{t+1}-D_{t+1}}{N_t}+(1-\tau_s)\frac{R^*(D_t-K_t)}{N_t}\\ &= c_t+n(k_{t+1}-d_{t+1})+R^*(d_t-k_t)-\tau_s R^*(d_t-k_t)\end{aligned}$$

In a next step, the terms involving the saving wedge are consolidated.
The total revenue from the wedges in per capita unit is

$$z_t=\tau_k R_t k_t+\tau_s R^*(k_t-d_t)$$

In the BC, focus on the terms involving τ_s and use z_t:

$$\begin{aligned} z_t &+ \tau_s R^*(d_t - k_t) \\ \tau_k R_t k_t + \tau_s R^*(k_t - d_t) &+ \tau_s R^*(d_t - k_t) \\ \tau_k R_t k_t & \end{aligned}$$

Thus, the size of transfers is influenced by investment frictions.
Using $R_t = \frac{R^*}{1-\tau_k}$, we get z_{kt}, the lump-sum transfers financed by capital wedge:

$$z_{kt} = \frac{\tau_k}{1-\tau_k} R^* k_t$$

The BC is:

$$z_{kt} + w_t = c_t + n(k_{t+1} - d_{t+1}) + R^*(d_t - k_t)$$

Divide the BC by A_t and use $A_t = \frac{A_{t+1}}{g_{t+1}}$:

$$\begin{aligned} \frac{z_{kt}}{A_t} + \frac{w_t}{A_t} &= \frac{c_t}{A_t} + n\left(\frac{k_{t+1}}{A_t} - \frac{d_{t+1}}{A_t}\right) + R^*\left(\frac{d_t - k_t}{A_t}\right) \\ \tilde{z}_{kt} + \tilde{w}_t &= \tilde{c}_t + n g_{t+1}(\tilde{k}_{t+1} - \tilde{d}_{t+1}) + R^*(\tilde{d}_t - \tilde{k}_t) \end{aligned}$$

In steady-state, $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$ and $\tilde{z}_k = \frac{\tau_k}{1-\tau_k} R^* \tilde{k}^*$.
The normalized BC is

$$\tilde{z}_k + \tilde{w} = \tilde{c}_t + n g_{t+1}(\tilde{k}^* - \tilde{d}_{t+1}) + R^*(\tilde{d}_t - \tilde{k}^*)$$

where $\tilde{w} = (1-\alpha)\tilde{k}^{*\alpha}$ (competitive factor markets). At time T, the economy is in steady-state and the saving wedge disappears. Using $g_{t+1} = g^*$, $\tilde{d}_t = \tilde{d}_T$ and $\tilde{c}_t = c_T$, we get the SS debt:

$$\begin{aligned} \tilde{z}_k + \tilde{w} &= \tilde{c}_T + n g^*(\tilde{k}^* - \tilde{d}_T) + R^*(\tilde{d}_T - \tilde{k}^*) \\ &= \tilde{c}_T - \tilde{k}^*(R^* - n g^*) + \tilde{d}_T(R^* - n g^*) \\ \tilde{d}_T &= \frac{\tilde{z}_k + \tilde{w} - \tilde{c}_T}{R^* - n g^*} + \tilde{k}^* \end{aligned} \tag{A.2}$$

Step 2: consumption

Start with the Euler equation and use $R^* = \frac{g^{*\gamma}}{\beta}$ to get rid of β with $\beta = \frac{g^{*\gamma}}{R^*}$:

$$\begin{aligned} c_t^{-\gamma} &= \beta R^*(1-\tau_s) c_{t+1}^{-\gamma} \\ c_{T-1}^{-\gamma} &= \frac{g^{*\gamma}}{R^*} R^*(1-\tau_s) c_T^{-\gamma} \\ c_T^{-\gamma} &= \frac{c_{T-1}^{-\gamma}}{g^{*\gamma}(1-\tau_s)} \\ c_T &= \frac{c_{T-1}}{g^{*-1}(1-\tau_s)^{-\frac{1}{\gamma}}} \\ c_T &= c_{T-1} g^*(1-\tau_s)^{\frac{1}{\gamma}} \end{aligned}$$

This result is used to get c_T relative to c_0 with $\phi(\tau_s) = (1-\tau_s)^{\frac{1}{\gamma}}$:

$$\begin{aligned} c_T &= c_0\left[g^*(1-\tau_s)^{\frac{1}{\gamma}}\right]^T \\ &= c_0\left[g^*\phi(\tau_s)\right]^T \end{aligned}$$

Using $A_T = (1+\pi)A_0 g^{*T}$, consumption in efficiency unit of labor can be expressed as

$$\begin{aligned} \tilde{c}_T &= \frac{c_0\left[g^*\phi(\tau_s)\right]^T}{(1+\pi)A_0 g^{*T}} \\ &= \frac{\tilde{c}_0\phi(\tau_s)^T}{1+\pi} \end{aligned} \tag{A.3}$$

The next step is to get the intertemporal *BC*. Rewrite the per capita *BC* of households:

$$\begin{aligned} t=0: &\quad z_{k0}+w_0 &= c_0+n(k_1-d_1)+R^*(d_0-k_0) \\ t=1: &\quad z_{k1}+w_1 &= c_1+n(k_2-d_2)+R^*(d_1-k_1) \\ t=2: &\quad z_{k2}+w_2 &= c_2+n(k_3-d_3)+R^*(d_2-k_2) \end{aligned}$$

From $t=1$, get

$$k_1-d_1=\frac{c_1+n(k_2-d_2)-z_{k1}-w_1}{R^*}$$

and plug it in $t=0$

$$z_{k0}+w_0=c_0+n\frac{c_1}{R^*}+\frac{n^2(k_2-d_2)}{R^*}-\frac{nz_{k1}}{R^*}-\frac{nw_1}{R^*}+R^*(d_0-k_0)$$

Rearrange:

$$z_{k0}+\frac{nz_{k1}}{R^*}+w_0+\frac{nw_1}{R^*}=c_0+\frac{nc_1}{R^*}+R^*(d_0-k_0)+\frac{n^2(k_2-d_2)}{R^*}$$

From $t=2$ get

$$k_2-d_2=\frac{c_2+n(k_3-d_3)-z_{k2}-w_2}{R^*}$$

and plug it in the result from before:

$$z_{k0}+\frac{nz_{k1}}{R^*}+\frac{n^2z_{k2}}{R^{*2}}+w_0+\frac{nw_1}{R^*}+\frac{n^2w_2}{R^{*2}}=c_0+\frac{nc_1}{R^*}+\frac{n^2c_2}{R^{*2}}+R^*(d_0-k_0)+\frac{n^3}{R^{*2}}(k_3-d_3)$$

By repeating the process until ∞, the $k-d$ term on the right disappears if $R^*>n$ and we are left with the intertemporal *BC*:

$$\sum_0^\infty\left(\frac{n}{R^*}\right)^t(z_{kt}+w_t)=\sum_0^\infty\left(\frac{n}{R^*}\right)^t c_t+R^*(d_0-k_0) \tag{A.4}$$

From before, we know that consumption per capita grows at rate $g^*\phi(\tau_s)$ until T and at g^*afterwards ($\tau_s=0$ and $\phi=1$):

$$c_t=A_0\phi^{min(t,T)}g^{*t}\tilde{c}_0$$

Rewrite the consumption part of the intertemporal *BC*:

$$
\begin{aligned}
\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t c_t &= \sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t A_0\phi^{min(t,T)} g^{*t}\tilde{c}_0 \\
&= A_0\tilde{c}_0\sum_{t=0}^{T}\left(\frac{n}{R^*}\right)^t \phi^t g^{*t} + A_0\tilde{c}_0\phi^T\sum_{t=T+1}^{\infty}\left(\frac{n}{R^*}\right)^t g^{*t} \\
&= A_0\tilde{c}_0\left[\sum_{t=0}^{T}\left(\frac{\phi n g^*}{R^*}\right)^t + \phi^T\sum_{t=T+1}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right] \\
&= \frac{A_0\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)}
\end{aligned} \tag{A.5}
$$

All elements influenced by τ_s are gathered in $\psi(\tau_s)$:

$$
\psi(\tau_s) = \left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\sum_{t=0}^{T}\left(\frac{\phi n g^*}{R^*}\right)^t + \phi^T\sum_{t=T+1}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right]^{-1} \tag{A.6}
$$

The following part aims at getting an expression for $\psi(\tau_s)$:

$$
\begin{aligned}
\sum_{t=0}^{n-1} x^t &= \frac{1-x^n}{1-x} \\
\sum_{t=0}^{\infty} x^t &= \frac{1}{1-x} \\
\sum_{t=1}^{\infty} x^t &= \sum_{t=0}^{\infty} x^t - 1 = \frac{x}{1-x}
\end{aligned}
$$

Use them in $\psi(\tau_s)$:

$$
\left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\frac{1-(\frac{\phi n g^*}{R^*})^{T+1}}{1-\frac{\phi n g^*}{R^*}} + \phi^T\frac{1}{1-\frac{ng^*}{R^*}} - \phi^T\frac{1-\frac{ng^*}{R^*}^{T+1}}{1-\frac{ng^*}{R^*}}\right]
$$

$$
\left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\frac{1-(\frac{\phi n g^*}{R^*})^{T+1}}{1-\frac{\phi n g^*}{R^*}} + \phi^T\frac{\frac{ng^*}{R^*}^{T+1}}{1-\frac{ng^*}{R^*}}\right]^{-1}
$$

$$
\frac{R^*}{R^*-ng^*}\left[\frac{1-(\frac{\phi n g^*}{R^*})^{T+1}}{\frac{R^*-\phi n g^*}{R^*}} + \phi^T\frac{\frac{ng^*}{R^*}^{T+1}}{\frac{R^*-ng^*}{R^*}}\right]^{-1}
$$

$$
\frac{R^*}{R^*-ng^*}\left[R^*\frac{1-(\frac{\phi n g^*}{R^*})^{T+1}}{R^*-\phi n g^*} + R^*\phi^T\frac{\frac{ng^*}{R^*}^{T+1}}{R^*-ng^*}\right]^{-1}
$$

$$
\frac{1}{R^*-ng^*}\left[\frac{(1-(\frac{\phi n g^*}{R^*})^{T+1})(R^*-ng^*)+\phi^T\frac{ng^*}{R^*}^{T+1}(R^*-\phi n g^*)}{(R^*-\phi n g^*)(R^*-ng^*)}\right]^{-1}
$$

Focus on the numerator. First gather terms containing ng^* (without any R^*):

$$
-ng^* + \left[\frac{\phi n g^*}{R^*}\right]^{T+1} ng^* - \phi^T\left[\frac{ng^*}{R^*}\right]^{T+1}\phi n g^* = -ng^*
$$

then, concentrate on R^* terms:

$$R^* - \frac{\phi n g^{*\,T+1}}{R^*}R^* + \phi^T \frac{n g^{*\,T+1}}{R^*}R^*$$

$$R^* - R^*\left[\frac{\phi n g^*}{R^*}\right]^T \frac{\phi n g^*}{R^*} + R^*\left[\frac{\phi n g^*}{R^*}\right]^T \frac{n g^*}{R^*}$$

$$R^* - \left[\frac{\phi n g^*}{R^*}\right]^T \phi n g^* + \left[\frac{\phi n g}{R^*}\right]^T n g^*$$

$$R^* + \left[\frac{\phi n g^*}{R^*}\right]^T n g^*(1-\phi)$$

We get

$$\frac{1}{R^* - ng^*}\left[\frac{R^* - ng^* + \left[\frac{\phi n g^*}{R^*}\right]^T n g^*(1-\phi)}{(R^* - \phi n g^*)(R^* - n g^*)}\right]^{-1}$$

$$\psi(\tau_s) = \frac{R^* - \phi n g^*}{R^* - n g^* + \left[\frac{n g^* \phi(\tau_s)}{R^*}\right]^T n g^*(1-\phi(\tau_s))} \tag{A.7}$$

The consumption in period 0 in efficiency units can now be computed.
Note that:

$$\begin{aligned}
(1+\pi_T)A_0 g^{*T} &= A_T \\
(1+\pi_t)A_0 g^{*t} &= A_t \\
\tilde{w} = \frac{w_t}{A_t} &= \frac{w_t}{A_0(1+\pi_t)g^{*t}} \\
\tilde{z}_k = \frac{z_{kt}}{A_t} &= \frac{z_{kt}}{A_0(1+\pi_t)g^{*t}} \\
w_t + z_{kt} &= (\tilde{w}+\tilde{z}_k)A_0(1+\pi_t)g^{*t}
\end{aligned} \tag{A.8}$$

Plug (A.5) and (A.8) in the intertemporal version of the budget constraint (A.4):

$$\begin{aligned}
\sum_0^\infty \left(\frac{n}{R^*}\right)^t c_t &= \sum_0^\infty \left(\frac{n}{R^*}\right)^t (z_{kt}+w_t) + R^*(k_0 - d_0) \\
\frac{A_0\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} &= \sum_{t=0}^\infty \left(\frac{n}{R^*}\right)^t (\tilde{w}+\tilde{z}_k)A_0(1+\pi_t)g^{*t} + R^*(k_0-d_0) \\
\tilde{c}_0 &= \frac{R^*-ng^*}{R^*}\psi(\tau_s)\frac{1}{A_0}(\tilde{w}+\tilde{z}_k)A_0\sum_{t=0}^\infty \left(\frac{n}{R^*}\right)^t (1+\pi_t)g^{*t} \\
&\quad + \frac{R^*-ng^*}{R^*}\psi(\tau_s)\frac{1}{A_0}R^*(k_0-d_0) \\
&= \frac{R^*-ng^*}{R^*}\psi(\tau_s)(\tilde{w}+\tilde{z}_k)\sum_{t=0}^\infty \left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) \\
&\quad + (R^*-ng^*)\psi(\tau_s)\frac{k_0-d_0}{A_0} \\
&= \ldots + (R^*-ng^*)\psi(\tau_s)(\tilde{k}_0 - \tilde{d}_0) \\
\tilde{c}_0 &= (R^*-ng^*)\psi(\tau_s)\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^\infty \left(\frac{ng^*}{R^*}\right)^t (1+\pi_t) + \tilde{k}_0 - \tilde{d}_0\right]
\end{aligned} \tag{A.9}$$

The saving wedge τ_s enters consumption choice through the marginal propensity to consume $(R^* - ng^*)\psi(\tau_s)$ out of wealth [...]. If τ_s is higher, then $\phi(\tau_s) = (1-\tau_s)^{\frac{1}{\gamma}}$ is lower, $\psi(\pi_s)$ is higher and the marginal propensity to consume out of wealth rises.

From (A.5) we get an expression for $\tilde{c}_t$

$$\begin{aligned}\tilde{c}_t = \frac{c_t}{A_t} &= \frac{1}{(1+\pi_t)A_0 g^{*t}} \frac{A_0\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} \frac{1}{\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t} \\ &= \frac{1}{(1+\pi_t)g^{*t}} \frac{\tilde{c}_0}{(1-\frac{ng^*}{R^*})\psi(\tau_s)} \frac{1}{\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t}\end{aligned}$$

and by using $\tilde{c}_0$, for $\tilde{c}_T$ as well:

$$\tilde{c}_T = \frac{c_T}{A_T} = \frac{\tilde{c}_0\phi(\tau_s)^T}{1+\pi_T} \tag{A.10}$$

Step 3: closed-form expression for the relative flows

Use the expression for $\tilde{d}_T$ (A.2) in the initial $\frac{\triangle D}{Y_0}$ (A.1):

$$\begin{aligned}\frac{\triangle D}{Y_0} &= \frac{\tilde{d}_T(g^*n)^T(1+\pi)-\tilde{d}_0}{\tilde{y}_0} \\ &= \left(\frac{\tilde{z}_k+\tilde{w}-\tilde{c}_T}{R^*-ng^*}+\tilde{k}^*\right)\left(\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}\right) - \frac{\tilde{d}_0}{\tilde{y}_0} \\ &= \frac{\tilde{k}^*(g^*n)^T(1+\pi)}{\tilde{y}_0} + \frac{\tilde{w}+\tilde{z}_k}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} - \frac{\tilde{c}_T}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} - \frac{\tilde{d}_0}{\tilde{y}_0}\end{aligned}$$

Focus on the last term containing $\tilde{c}_T$ (A.10) and $\tilde{c}_0$ (A.9) :

$$-\frac{\tilde{c}_0\phi(\tau_s)^T}{1+\pi}\frac{1}{(R^*-ng^*)}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}$$

$$-(R^*-ng^*)\psi(\tau_s)\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right]\frac{\phi(\tau_s)^T}{1+\pi}\frac{1}{(R^*-ng^*)}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}$$

$$-\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^*n)^T\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right]$$

The entire expression is

$$\begin{aligned}\frac{\triangle D}{Y_0} = &\frac{\tilde{k}^*(g^*n)^T(1+\pi)}{\tilde{y}_0} + \frac{\tilde{w}+\tilde{z}_k}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} \\ &-\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^*n)^T\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+\tilde{k}_0-\tilde{d}_0\right] - \frac{\tilde{d}_0}{\tilde{y}_0}\end{aligned}$$

The initial part of the equation gathering $\tilde{k}^*$ is the first term:

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi) \ \ (\mathbf{I})$$

Focusing on parts containing $\tilde{k}_0$, one gets the second term:

$$-\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^*n)^T\tilde{k}_0$$

$$-\frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[ng^*\phi(\tau_s)]^T \quad \textbf{(II)}$$

For the third one, take the terms containing $\tilde{d}_0$:

$$\frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}(g^*n)^T\tilde{d}_0 - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right) \quad \textbf{(III)}$$

Collecting the left-over terms of the initial expression, we have

$$\frac{\tilde{w}+\tilde{z}_k}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} - \frac{\psi(\tau_s)\phi(\tau_s)^T(g^*n)^T}{\tilde{y}_0}\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\left[\frac{1}{R^*-ng^*}(1+\pi) - \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\left[\frac{1}{R^*-ng^*}(1+\pi) - \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi) - \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right] \tag{A.11}$$

where the assumption that, from T on, $\pi_t = \pi$ has been used.
From now on, one needs to proceed sequentially. Put the third term aside for the moment:

$$-\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t) \tag{A.12}$$

Focus on the first and second part:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\left[\frac{1}{R^*-ng^*}(1+\pi) - \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi)\right]$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\left[(1+\pi)\frac{1}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t - (1+\pi)\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right]$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\frac{(1+\pi)}{R^*}\left[\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t - \psi(\tau_s)\phi(\tau_s)^T\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right] \tag{A.13}$$

Before, we had defined

$$\psi(\tau_s) = \left(1-\frac{ng^*}{R^*}\right)^{-1}\left[\sum_{t=0}^{T}\left(\frac{\phi ng^*}{R^*}\right)^t + \phi(\tau_s)^T\sum_{t=T+1}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right]^{-1}$$

Use this definition to get an expression for $\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t = \left(1-\frac{ng^*}{R^*}\right)^{-1}$:

$$\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t = \psi(\tau_s)\left[\sum_{t=0}^{T}\left(\frac{\phi ng^*}{R^*}\right)^t + \phi(\tau_s)^T\sum_{t=T+1}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right]$$

Plug it in (A.13) and focus on the interior of the bracket of the following expression:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\frac{(1+\pi)}{R^*}\left[\psi(\tau_s)\sum_{t=0}^{T}\left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s)\phi(\tau_s)^T\sum_{t=T+1}^{\infty}\left(\frac{ng^*}{R^*}\right)^t - \psi(\tau_s)\phi(\tau_s)^T\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\right]$$

Note that we can rewrite the sum terms. For the first one, $\sum_{t=0}^{T} = \sum_{t=0}^{T-1} + \sum_{t=T}^{T}$. For the second one, $\sum_{t=T+1}^{\infty} = \sum_{t=T}^{\infty} - \sum_{t=T}^{T}$.

$$\begin{aligned}\psi(\tau_s)\sum_{t=0}^{T-1}\left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s)\sum_{t=T}^{T}\left(\frac{\phi ng^*}{R^*}\right)^t + \psi(\tau_s)\phi(\tau_s)^T\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t \\ -\psi(\tau_s)\phi^T\sum_{t=T}^{T}\left(\frac{ng^*}{R^*}\right)^t - \psi(\tau_s)\phi(\tau_s)^T\sum_{t=T}^{\infty}\left(\frac{ng^*}{R^*}\right)^t\end{aligned}$$

All parts but the first one disappear. By using the external part we left behind at the beginning we get

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\frac{(1+\pi)}{R^*}\left[\psi(\tau_s)\sum_{t=0}^{T-1}\left(\frac{\phi ng^*}{R^*}\right)^t\right]$$

This is an expression for the first and second parts of (A.11). Use it with the terms we put aside (A.12) to get the fourth part:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\frac{(1+\pi)}{R^*}\left[\psi(\tau_s)\sum_{t=0}^{T-1}\left(\frac{\phi ng^*}{R^*}\right)^t\right] - \frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t\left[\phi(\tau_s)^{t-T}(1+\pi)-(1+\pi_t)\right] \quad \textbf{(IV)}$$

The equation of the flows decomposition is composed of the four expressions we have just computed (I), (II), (III) and (IV):

$$\begin{aligned}\frac{\triangle D}{Y_0} &= \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi) - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[ng^*\phi(\tau_s)]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right) \\ &\quad + \frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t\left[\phi(\tau_s)^{t-T}(1+\pi)-(1+\pi_t)\right] \qquad \text{(A.14)}\end{aligned}$$

Gourinchas and Jeanne (2013) show that under certain conditions:

$$\frac{\triangle D}{Y_0} = D\left(\underset{+}{\tilde{d}_0}, \underset{-}{\tilde{k}_0}, \underset{+}{\pi}, \underset{-}{\tau_k}, \underset{+}{\tau_s}\right)$$

Cumulated relative capital inflows increase with $\tilde{d}_0$ if $\psi(\tau_s)[ng^*\phi(\tau_s)]^T > 1$, which is the case if τ_s is small enough. They clearly decrease with higher $\tilde{k}_0$. An increase in π causes the first term to be higher. For the fourth one, concentrate on the bracket. First, $\pi_t = f(t)\pi$ with $f(t) = min(\frac{t}{T}, 1) \leq 1$. Second, $\phi(\tau_s)^{t-T} = (1-\tau_s)^{\frac{1}{\gamma}(t-T)} > 1$. Thus, an increase in π leads to more inflows. Countries with a relatively higher τ_k have lower $\tilde{k}^*$and thus always less inflows given part (IV) is positive. At last, a relatively higher

τ_s implies higher inflows if $\tilde{k}_0 \geq \tilde{d}_0$.

Step 4: channel decomposition

Focus on the first three terms of (A.14) and distribute:

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T + \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\,[ng^*\phi(\tau_s)]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right)$$

Expand with $\frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T$:

$$\underbrace{\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{y}_0}(ng^*)^T}_{convergence} + \underbrace{\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi}_{investment} + \frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\,[ng^*\phi(\tau_s)]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\left(\psi(\tau_s)[ng^*\phi(\tau_s)]^T - 1\right)$$

Focus on the left-over terms:

$$\frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\,[ng^*\phi(\tau_s)]^T + \frac{\tilde{d}_0}{\tilde{y}_0}\psi(\tau_s)[ng^*\phi(\tau_s)]^T - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\underbrace{\frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \psi(\tau_s)\,[ng^*\phi(\tau_s)]^T\,\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}}_{trend}$$

The saving channel is the fourth part of (A.14):

$$\underbrace{\frac{\tilde{w} + \tilde{z}_k}{\tilde{y}_0}\,\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T \sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t\left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right]}_{saving}$$

A.2.3 Relative flows with no saving wedge

With $\tau_s = 0$, we have

$$\begin{aligned}
\phi(\tau_s) &= (1-\tau_s)^{\frac{1}{\gamma}} = 1 \\
\psi(\tau_s) &= \frac{R^* - ng^*}{R^* - ng^* + \left(\frac{ng^*}{R^*}\right)^T ng^*(1-1)} = 1
\end{aligned}$$

The convergence and investment channels are identical:

$$\frac{\tilde{k}^* - \tilde{k}_0}{\tilde{y}_0}(ng^*)^T$$

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T\pi$$

The trend channel becomes

$$\frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + (ng^*)^T\,\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0(ng^*)^T}{\tilde{y}_0} - \frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T + \frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0}{\tilde{y}_0}\left[(ng^*)^T - 1\right]$$

The saving channel is:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}\frac{1}{R^*}(ng^*)^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t[(1+\pi)-(1+\pi_t)]$$

Remember that $\pi_t = f(t)\pi$:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}\frac{1}{R^*}(ng^*)^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)\pi[1-f(t)]$$

Note that in a model without investment frictions as well, all the channels would be similar with the exception of the saving channel ($\tilde{z}_k$ would disappear).

A.2.4 Extension 1: capital adjustment costs

A capital adjustment cost is introduced in the equation for the dynamics of capital stock:

$$I_t \quad = \quad K_{t+1}-(1-\delta)K_t+\kappa_2(K_{t+1}-K_t)^\theta$$

Divide by GDP in period t

$$i_t = \frac{K_{t+1}-(1-\delta)K_t}{Y_t}+\kappa_2\frac{(K_{t+1}-K_t)^\theta}{Y_t}$$

We can rewrite it as

$$i_t \quad = \quad \tilde{k}^{*(1-\alpha)}[g_{t+1}n-1+\delta]+\kappa_2\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^\theta}{A_tN_t\tilde{k}^{*\alpha}}$$

The second capital adjustment cost enters in the initial jump in capital stock:

$$\frac{K_0^*-K_0}{Y_0} \quad = \quad \frac{(1+\kappa_1)(\tilde{k}^*-\tilde{k}_0)}{\tilde{k}_0^\alpha}$$

Add the jump term to the expression from before to get a new i_0

$$i_0 \quad = \quad \tilde{k}^{*(1-\alpha)}(g_1n+\delta-1)+\kappa_2\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^\theta}{A_tN_t\tilde{k}^{*\alpha}}+\frac{(1+\kappa_1)(\tilde{k}^*-\tilde{k}_0)}{\tilde{k}_0^\alpha}$$

The average investment rate between t and $T-1$ is

$$\begin{aligned} i \quad &= \quad \frac{1}{T}\sum_{t=0}^{T-1}i_t \\ &= \quad \frac{1}{T}\frac{(1+\kappa_1)(\tilde{k}^*-\tilde{k}_0)}{\tilde{k}_0^\alpha}+\frac{1}{T}\sum_{t=0}^{T-1}(g_{t+1}n+\delta-1)\tilde{k}^{*(1-\alpha)}+\frac{1}{T}\sum_{t=0}^{T-1}\kappa_2\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^\theta}{A_tN_t\tilde{k}^{*\alpha}} \end{aligned}$$

Two new channels appear: the second term is the initial capital adjustment cost and the fifth term represents standard capital adjustment costs.

$$
\begin{aligned}
i &= \frac{1}{T}\frac{\tilde{k}^*-\tilde{k}_0}{\tilde{k}_0^{\alpha}}+\frac{1}{T}\frac{\tilde{k}^*-\tilde{k}_0}{\tilde{k}_0^{\alpha}}\kappa_1+g^*\frac{\pi}{T}n\tilde{k}^{*(1-\alpha)}+(g^*n+\delta-1)\tilde{k}^{*(1-\alpha)} \\
&+\frac{1}{T}\sum_{t=0}^{T-1}\kappa_2\frac{(A_{t+1}N_{t+1}\tilde{k}^*-A_tN_t\tilde{k}^*)^{\theta}}{A_tN_t\tilde{k}^{*\alpha}}
\end{aligned}
$$

A.2.5 Extension 2: exogenous interest rate

The debt ratio is not influenced, à savoir:

$$
\tilde{d}_T=\frac{\tilde{z}_k+\tilde{w}-\tilde{c}_T}{R^*-ng^*}+\tilde{k}^* \tag{A.15}
$$

Start with the Euler equation but do not use $R^*=\frac{g^{*\gamma}}{\beta}$:

$$
\begin{aligned}
c_t^{-\gamma} &= \beta R^*(1-\tau_s)c_{t+1}^{-\gamma} \\
c_T^{-\gamma} &= \frac{c_{T-1}^{-\gamma}}{\beta R^*(1-\tau_s)} \\
c_T &= \frac{c_{T-1}}{[\beta R^*(1-\tau_s)]^{-\frac{1}{\gamma}}} \\
c_T &= c_{T-1}[\beta R^*(1-\tau_s)]^{\frac{1}{\gamma}}
\end{aligned}
$$

This result is used to get c_T relative to c_0 with $\phi(\tau_s)=(1-\tau_s)^{\frac{1}{\gamma}}$:

$$
\begin{aligned}
c_T &= c_0\left[(\beta R^*)^{\frac{1}{\gamma}}(1-\tau_s)^{\frac{1}{\gamma}}\right]^T \\
&= c_0\left[(\beta R^*)^{\frac{1}{\gamma}}\phi(\tau_s)\right]^T
\end{aligned}
$$

Using $A_T=(1+\pi)A_0g^{*T}$, consumption in efficiency unit can be expressed as

$$
\begin{aligned}
\tilde{c}_T &= \frac{c_0\left[(\beta R^*)^{\frac{1}{\gamma}}\phi(\tau_s)\right]^T}{(1+\pi)A_0g^{*T}} \\
&= \frac{\tilde{c}_0\phi(\tau_s)^T(\beta R^*)^{\frac{T}{\gamma}}}{(1+\pi)g^{*T}}
\end{aligned} \tag{A.16}
$$

The intertemporal *BC* is the same as in the baseline case:

$$
\sum_0^{\infty}\left(\frac{n}{R^*}\right)^t(z_{kt}+w_t)=\sum_0^{\infty}\left(\frac{n}{R^*}\right)^t c_t+R^*(d_0-k_0) \tag{A.17}
$$

From before, we know that consumption per capita grows at rate $(\beta R^*)^{\frac{1}{\gamma}}\phi(\tau_s)$ until T and at $(\beta R^*)^{\frac{1}{\gamma}}$ afterwards:

$$
c_t=A_0\phi^{min(t,T)}(\beta R^*)^{\frac{t}{\gamma}}\tilde{c}_0
$$

Rewrite the consumption part of the intertemporal *BC*:

$$
\begin{aligned}
\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t c_t &= \sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t A_0\phi^{min(t,T)}(\beta R^*)^{\frac{t}{\gamma}}\tilde{c}_0 \\
&= A_0\tilde{c}_0\sum_{t=0}^{T}\left(\frac{n}{R^*}\right)^t \phi^t(\beta R^*)^{\frac{t}{\gamma}} + A_0\tilde{c}_0\phi^T\sum_{t=T+1}^{\infty}\left(\frac{n}{R^*}\right)^t(\beta R^*)^{\frac{t}{\gamma}} \\
&= A_0\tilde{c}_0\left[\sum_{t=0}^{T}\left(\frac{\phi n}{R^*}\right)^t(\beta R^*)^{\frac{t}{\gamma}} + \phi^T\sum_{t=T+1}^{\infty}\left(\frac{n}{R^*}\right)^t(\beta R^*)^{\frac{t}{\gamma}}\right] \\
&= \frac{A_0\tilde{c}_0}{(1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})\psi(\tau_s)} \qquad \text{(A.18)}
\end{aligned}
$$

All elements influenced by τ_s are gathered in $\psi(\tau_s)$:

$$
\psi(\tau_s) = \left(1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right)^{-1}\left[\sum_{t=0}^{T}\left(\frac{\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right)^t + \phi^T\sum_{t=T+1}^{\infty}\left(\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\right)^t\right]^{-1}
$$

We omit the steps to get an expression for $\psi(\tau_s)$ as they are similar as in the baseline case with $(\beta R^*)^{\frac{1}{\gamma}}$ instead of g^*. We get:

$$
\psi(\tau_s) = \frac{R^*-\phi n(\beta R^*)^{\frac{1}{\gamma}}}{R^*-n(\beta R^*)^{\frac{1}{\gamma}} + \left[\frac{n(\beta R^*)^{\frac{1}{\gamma}}\phi(\tau_s)}{R^*}\right]^T n(\beta R^*)^{\frac{1}{\gamma}}(1-\phi(\tau_s))} \qquad \text{(A.19)}
$$

The consumption in period 0 in efficiency units can now be computed similarly as in the baseline case using again:

$$
w_t + z_{kt} \quad = \quad (\tilde{w}+\tilde{z}_k)A_0(1+\pi_t)g^{*t}
$$

Plug the rewritten consumption part (A.18) and the former equation in the intertemporal version of the budget constraint (A.17):

$$
\begin{aligned}
\sum_{0}^{\infty}\left(\frac{n}{R^*}\right)^t c_t &= \sum_{0}^{\infty}\left(\frac{n}{R^*}\right)^t (z_{kt}+w_t)+R^*(k_0-d_0) \\
\frac{A_0\tilde{c}_0}{(1-\frac{n(\beta R^*)^{\frac{1}{\gamma}}}{R^*})\psi(\tau_s)} &= \sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t (\tilde{w}+\tilde{z}_k)A_0(1+\pi_t)g^{*t}+R^*(k_0-d_0) \\
\tilde{c}_0 &= \frac{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\psi(\tau_s)\frac{1}{A_0}(\tilde{w}+\tilde{z}_k)A_0\sum_{t=0}^{\infty}\left(\frac{n}{R^*}\right)^t(1+\pi_t)g^{*t} \\
&\quad +\frac{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\psi(\tau_s)\frac{1}{A_0}R^*(k_0-d_0) \\
&= \frac{R^*-n(\beta R^*)^{\frac{1}{\gamma}}}{R^*}\psi(\tau_s)(\tilde{w}+\tilde{z}_k)\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t) \\
&\quad +(R^*-n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)\frac{k_0-d_0}{A_0} \\
&= \ldots+(R^*-n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)(\tilde{k}_0-\tilde{d}_0) \\
\tilde{c}_0 &= (R^*-n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right] \quad \text{(A.20)}
\end{aligned}
$$

Use the expression for $\tilde{d}_T$ (A.15) in the initial (unchanged) $\frac{\triangle D}{Y_0}$ (A.1). The expression is similar as in the baseline case:

$$
\begin{aligned}
\frac{\triangle D}{Y_0} &= \frac{\tilde{d}_T(g^*n)^T(1+\pi)-\tilde{d}_0}{\tilde{y}_0} \\
&= \left(\frac{\tilde{z}_k+\tilde{w}-\tilde{c}_T}{R^*-ng^*}+\tilde{k}^*\right)\left(\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}\right)-\frac{\tilde{d}_0}{\tilde{y}_0} \\
&= \frac{\tilde{k}^*(g^*n)^T(1+\pi)}{\tilde{y}_0}+\frac{\tilde{w}+\tilde{z}_k}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}-\frac{\tilde{c}_T}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}-\frac{\tilde{d}_0}{\tilde{y}_0}
\end{aligned}
$$

Focus on the last term containing $\tilde{c}_T$ (A.16) and $\tilde{c}_0$ (A.20) :

$$
-\frac{\tilde{c}_T}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}
$$

$$
-\frac{\tilde{c}_0\phi(\tau_s)^T(\beta R^*)^{\frac{T}{\gamma}}}{(1+\pi)g^{*T}}\frac{1}{(R^*-ng^*)}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}
$$

We get

$$
-(R^*-n(\beta R^*)^{\frac{1}{\gamma}})\psi(\tau_s)\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right]\frac{\phi(\tau_s)^T(\beta R^*)^{\frac{T}{\gamma}}}{(1+\pi)g^{*T}}\frac{1}{(R^*-ng^*)}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0}
$$

which can be rewritten as

$$
-\frac{(R^*-n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^*-ng^*}\phi(\tau_s)^T n^T\frac{\psi(\tau_s)}{\tilde{y}_0}\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+(\tilde{k}_0-\tilde{d}_0)\right]
$$

The entire expression is:

$$
\begin{aligned}
\frac{\triangle D}{Y_0} \quad = \quad & \frac{\tilde{k}^*(g^*n)^T(1+\pi)}{\tilde{y}_0} + \frac{\tilde{w}+\tilde{z}_k}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} \\
& -\frac{(R^*-n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^*-ng^*}\phi(\tau_s)^T n^T \frac{\psi(\tau_s)}{\tilde{y}_0}\left[\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)+\tilde{k}_0-\tilde{d}_0\right] \\
& -\frac{\tilde{d}_0}{\tilde{y}_0}
\end{aligned}
$$

Note that only the third part differs compared to the baseline case.
For convenience, define a new variable:

$$\Omega = \frac{(R^*-n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^*-ng^*}$$

The part of the equation gathering $\tilde{k}^*$ is the first response term as in the baseline case:

$$\frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi) \ \ (\mathbf{I})$$

Focusing on terms containing $\tilde{k}_0$, one gets the second element of the answer:

$$-\frac{(R^*-n(\beta R^*)^{\frac{1}{\gamma}})(\beta R^*)^{\frac{T}{\gamma}}}{R^*-ng^*}\phi(\tau_s)^T n^T \frac{\psi(\tau_s)}{\tilde{y}_0}\tilde{k}_0$$

$$-\Omega\frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[n\phi(\tau_s)]^T \ \ (\mathbf{II})$$

As for the third one, take the terms containing $\tilde{d}_0$:

$$\Omega n^T \frac{\psi(\tau_s)\phi(\tau_s)^T}{\tilde{y}_0}\tilde{d}_0 - \frac{\tilde{d}_0}{\tilde{y}_0}$$

$$\frac{\tilde{d}_0}{\tilde{y}_0}\left(\Omega\psi(\tau_s)[n\phi(\tau_s)]^T-1\right) \ \ (\mathbf{III})$$

Collecting the left-over terms of the initial expression, we have:

$$\frac{\tilde{w}+\tilde{z}_k}{R^*-ng^*}\frac{(g^*n)^T(1+\pi)}{\tilde{y}_0} - \Omega[\phi(\tau_s)n]^T\frac{\psi(\tau_s)}{\tilde{y}_0}\frac{\tilde{w}+\tilde{z}_k}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)$$

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right] \ \ (\mathbf{IV})$$

If $(\beta R^*)^{\frac{1}{\gamma}} = g^*$, $\Omega = g^{*T}$ and the former expression is similar as in the baseline case (A.11). One cannot use $\psi(\tau_s)$ to rewrite the first term in the brackets, get rid of the $\sum^{\infty}$ and obtain a closed-form solution. A numerical approximation is possible but obviously, one needs $R^* > ng^*$ for convergence.

The equation of the flows decomposition is constituted of the four expressions we have just computed ((I), (II), (III) and (IV)):

$$\begin{aligned}\frac{\triangle D}{Y_0} &= \frac{\tilde{k}^*}{\tilde{y}_0}(ng^*)^T(1+\pi) - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)[n\phi(\tau_s)]^T\Omega + \frac{\tilde{d}_0}{\tilde{y}_0}\left[\Omega\psi(\tau_s)[n\phi(\tau_s)]^T - 1\right] \\ &\quad + \frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]\end{aligned}$$

By focusing on the first three terms, one sees the convergence and investment channels are identical to the baseline case. Focus on the left-over terms:

$$\frac{\tilde{k}_0}{\tilde{y}_0}(ng^*)^T - \frac{\tilde{k}_0}{\tilde{y}_0}\psi(\tau_s)\left[n\phi(\tau_s)\right]^T\Omega + \frac{\tilde{d}_0}{\tilde{y}_0}\left[\Omega\psi(\tau_s)[n\phi(\tau_s)]^T - 1\right]$$

$$\underbrace{\frac{\tilde{k}_0(ng^*)^T - \tilde{d}_0}{\tilde{y}_0} + \Omega\psi(\tau_s)\left[n\phi(\tau_s)\right]^T\frac{\tilde{d}_0 - \tilde{k}_0}{\tilde{y}_0}}_{trend}$$

The saving channel is the fourth part of flows decomposition:

$$\underbrace{\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}n^T\left[\frac{g^{*T}(1+\pi)}{R^*-ng^*} - \Omega\frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]}_{saving}$$

Compared to the following form in the baseline case:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}(g^*n)^T\left[\frac{(1+\pi)}{R^*-ng^*} - \frac{\psi(\tau_s)\phi(\tau_s)^T}{R^*}\sum_{t=0}^{\infty}\left(\frac{ng^*}{R^*}\right)^t(1+\pi_t)\right]$$

which Gourinchas and Jeanne were able to rewrite as:

$$\frac{\tilde{w}+\tilde{z}_k}{\tilde{y}_0}\frac{\psi(\tau_s)}{R^*}[ng^*\phi(\tau_s)]^T\sum_{t=0}^{T-1}\left(\frac{ng^*}{R^*}\right)^t\left[\phi(\tau_s)^{t-T}(1+\pi) - (1+\pi_t)\right]$$

A.2.6 Extension 3: wage friction

The friction is introduced in a similar fashion as for the investment wedge:

$$(1-\tau_w)w_t = w^*$$

In the model, factor prices are competitive. The wage in efficiency units of labor is:

$$\tilde{w} = (1-\alpha)\tilde{k}^{*\alpha}$$

Obviously, the investment wedge-implied level of capital is the only driver of differences in wages among regions as identical values for labor share $(1-\alpha)$ have been assumed. The introduction of a wage friction would take place after the computation of investment wedges.
The labor wedge would directly enter the HH's *BC*.

$$(1-\tau_k)w_t + z_t = c_t + n(k_{t+1} - d_{t+1}) + R^*(d_t - k_t) - \tau_s R^*(d_t - k_t)$$

The total revenues of the wedges per capita are

$$z_t = \tau_k w_t + \tau_k R_t k_t + \tau_s R^*(k_t - d_t)$$

In the *BC*, focus on the terms involving τ_k/τ_s and use z_t:

$$z_t - \tau_k w_t + \tau_s R^*(d_t - k_t)$$
$$\tau_k w_t + \tau_k R_t k_t + \tau_s R^*(k_t - d_t) - \tau_k w_t + \tau_s R^*(d_t - k_t)$$
$$\tau_k R_t k_t$$

Thus, the transfers are not influenced by the labor wedge. The *BC* has the same form as before and we get the same steady-state debt.
Wage frictions do not enter the Euler equation. They would only matter through the labor-leisure trade-off but one would have to introduce leisure in addition to consumption in the utility function of HH.
The expression for consumption, the intertemporal *BC* and as consequent the channels are identical.

Appendix B

Appendix to “A Provincial View of Global Imbalances”

B.1 Data Appendix

B.1.1 Population

Chinese population data are a topic of their own. Two main problems are plaguing them: the underreported birth numbers as a consequence of the one child policy (Scharping, 2001) and the *“largest (voluntary) migration in human history”* (Chan, 2013). We tried to address the second issue. Basically, three sources of population estimates exist. The Hukou *Household Registration System* population data is reported by the *Public Security Authorities*.[1] It can be considered as a *de jure* statistic because it does not capture migration flows adequately. Typically, richer coastal provinces have an underestimated population and hinterland provinces a too high population (Chan and Wang, 2008). An alternative is the use of regular sample surveys of around 1% of the population and population censuses (1982, 1990, 2000 and 2010). They should better approximate resident population but unfortunately, the time of the survey as well as the definition of permanent residents and migrants is not always consistent over time. They are usually referred to as *de facto* data.

The yearbooks population data often are a combination of the three sources that we have already mentioned. We carefully compared CDC data, recent yearbooks, sample surveys, censuses and existing studies to at least avoid sudden jumps due to changes in definition and assemble our own population time series. We tried to consider de facto data as much as possible, particularly for provinces traditionally heavily influenced by migration.[2] In the end, we are not primarily interested in a precise estimation of the correct level of population *per se* but at least want to avoid

[1]The Hukou aims at limiting rural migration by restricting access to welfare goods and services for non-urban residents such as health care, insurances or education (Chan, 2010)

[2]Central China as well as Chongqing and Sichuan have been the main outflow regions. Shanghai, Guangdong and to a lesser extent other eastern provinces have been net recipients (Chan, 2013).

potential jumps in net output per capita due to (frequent) changes in data reporting methodology.

B.1.2 Net output

Net output is computed using data on GDP, government consumption and investment from the regional statistical yearbooks. The choice of the appropriate deflator(s) of the components of net output is of great importance. No official explicit regional GDP deflator data are published. Brandt et al. (2012) constructed regional GDP deflators but their sample stops in 2007 and does not include all provinces. A nation-wide GDP deflator is available from the IMF (IFS). While CPI (consumer price index) data are largely available and capture a relatively broad price development, our model is expressed in terms of tradable goods. PPIs (producer price indices) are limited to agricultural and industrial products and are not available across the board. In our opinion, a natural proxy is RPI (retail price index) which, like CPI, has the advantage of being broadly available.

The noisiness of the data and the large differences in economic structure among provinces force us to be sophisticated in our deflating methodology. First, we choose among two main types of deflators: RPI from official statistics and GDP from IFS (both using national values). While RPI certainly is a good proxy for the price of tradable goods, it may be inappropriate for more developed provinces (e.g. the ones that have a sectoral structure close to China). We base our deflator choice on key macroeconomic indicators. Using regional data on provincial sectoral GDP, we compile statistics on the average size of the primary, construction, industry and tertiary sector relative to China and observe the correlation of these variables over time between regional and national data. Furthermore, we construct our own index of economic specialization relative to the national economy using the share of GDP arising from the four preceding sectors.

Regions very similar to China in terms of the size, correlation and economic concentration of sectors are deflated using the official GDP deflator from IFS (half of regions) while highly specialized provinces will be deflated with national RPI (other half of sample). The use of national deflators instead of regional ones is motivated by the fact that the use of the tradable good as the numéraire implies that inflation in that good should be similar inside China. Furthermore, no regional off-the-shelf GDP deflator is available.[3] At last, we use the (noisy) RPI regional data in our indicator for internal price.

If investment has been a major driver of variations in Chinese output over the last decades, this is even more the case on the regional level, particularly for less developed regions in the West. We gather descriptive statistics on investment to output ratio. When necessary (for instance when large shocks in investment and/or a very high level are observed), we deflate investment

[3] By using agriculture, industrial and service (or consumer) price indices, one could deflate the production approach GDP components separately. We refrain from it for two reasons. First, we use expenditure approach GDP data as we are interested in net exports dynamics. Second, numerous data issues strongly distort regional structural indicators (see Brandt and Zhu (2010) for more).

with regional PIFA (price of investment in fixed asset), starting as soon as data become available (1992).

We thus end up with 4 different deflators for the three components of net output: national GDP (3 provinces), national RPI (4), national GDP with regional PIFA for investment (12) and national RPI with regional PIFA for investment (11). The provincial deflators are available on Table 4.4.

B.1.3 Net exports

Net exports (i.e. external surplus or deficit) correspond to the regional difference between saving and investment. Note that this indicator includes international and interprovincial flows in goods and services. In the second chapter, we showed that large discrepancies in regional external balances exist in China. As most provinces have near neutral or positive international trade balance, a substantial part of theses cross-sectional differences stems from interregional capital flows. Unfortunately, we were unable to include income and current transfers to extend the analysis to the current account level.[4]

For regions having a considerable share of migrant workers in their labor force, we would expect a high share of household remittances to lower their current account and increase it in hinterland provinces. Another important pattern is certainly linked to the capital outflows generated by the returns on FDI of foreign firms. Here again, well-integrated coastal provinces certainly have a lower true current account than we may think by using net exports. The potential large transfers between government and/or state-owned enterprises among provinces are another issue. One would expect them to raise the current account of less developed provinces. At last, it could well be that Zhang's argument that overreporting (underreporting) of exports (imports) has magnified national net exports statistics affects more surplus provinces with a large share of foreign and private firms (i.e. the East Coast and the Metropolises).

B.1.4 Domestic interest rate

The relevant domestic nominal interest rate is computed using the mean of the official deposit and lending rate from the People's Bank of China (IFS, May 2012 CD).[5] The expected common inflation (in terms of tradable goods) is proxied with national RPI (retail price index) inflation of the preceding period. Note that regional inflation in RPI is used in the internal price indicator.

[4] By comparison, for China as a whole, trade and services capture most of the current account dynamics. Over the last decades, income flows have been slightly negative with the exception of 2007 and 2008. Current transfers have been more sizeable and stabilized at a positive level since the mid-2000s. Still, they only amount to 15% of the trade balance between 2005 and 2010 on average.

[5] The PBC fixes an upper bound for deposit rate and a lower bound for lending rate. Both time series are highly correlated.

B.1.5 International interest rate

The nominal world interest rate is proxied with the yearly average of the *Federal Reserve Board* 3-Months Treasury Bill. Ex-post changes in exchange rate are proxied by the next period growth rate of the nominal effective exchange rate index (IFS, May 2012 CD). The extent to which regions are sensitive to the world interest rate is varying depending on their level of integration with the world economy. This parameter (δ) is integrated in the grid-search procedure.

B.1.6 Internal price

Numerous possibilities arise for computing a regional relative price index of non-tradable relative to tradable goods ($\Delta q_{t+1} = (1-\alpha)\Delta p_{t+1}$). For the regional share of non-tradables in consumption expenditure $(1-\alpha)$, we use data from the urban and rural *Household Survey* available from 1993 to 2010. We define tradables as expenditures on food and clothes while non-tradables is composed of healthcare, transport/communication, education/culture as well as residence/housing. As household surveys expenditure data are separated between urban and rural population, we take the average of both shares in non-tradables weighted by regional urbanization rate.[6] We end up with regional shares of non-tradables between 0.32 and 0.45.[7] For the price of tradable goods (i.e. the denominator of p), we take regional RPI (retail price index) data.

Approximating the price of non-tradable goods (the numerator of p) is more challenging. To get a complete time series over the period, we combine different sources according to data availability and scope. For 1984, we use regional CPI. From 1985 to 1999, we use SPI (services price index) as we expect it to capture non-tradable expenditures better than CPI. The initial years are exclusively urban observations (1985-1988) while the rest (1989-1999) is available at the provincial level. Unfortunately, SPI stopped to be computed in the 2000s. We use data on regional CPI categories to construct a non-tradable CPI index from 2000 to 2010 using the relative mean expenditure weight of each category over urban and rural data.[8]

Against a backdrop of financial repression, the progressive liberalization of the housing market in the 2000s led to a fast growth in real estate prices. Household quickly redirected their savings towards housing and the ownership rate increased substantially. Unfortunately, housing price is not included directly in Chinese CPI but in fixed capital formation (Lijuan, 2010). Rents, interest rates of housing loans and maintenance costs are considered but they certainly miss the bulk of the dynamics. To correct for that, we integrate the regional average selling price

[6] For urbanization, we use Shen (2006), data from the *Statistical Yearbooks* and interpolated assuming constant growth rates.

[7] The rapid increase in the expenditure share on non-tradable goods is a stylized fact of the reform period. While our model does not allow for a time-varying $1-\alpha$, the fact that we only consider later reform years (1993-2010) because of data availability issues means that our value is already relatively high. Furthermore, the upward trend is very similar among provinces.

[8] For China, CPI on health expenditures would enter with a weight of 16%, transport/communication with 28%, education/culture 23% and residence/housing 33%. In the tradables, food (82%) has a higher weight than clothing (18%).

of housing per square meter in the CPI of non-tradables from 2000 to 2010. We replace the corresponding category of CPI (residence/housing) with the housing price index but keep its relative weight unchanged.[9]

On average, our indicator of relative prices more than tripled between 1984 and 2010. While variations were low in the 1980s, the increase was most pronounced in the 1990s and continued on a somewhat lower trend in the 2000s.

[9]Besides being available for a relative long period (1999-2010), the average selling price contains residential and business transactions. It should thus be representative of the price patterns prevailing on the housing market.

B.2 Mathematical Appendix

B.2.1 Bergin and Sheffrin model

B.2.1.1 The maximization problem and its solution (Bergin and Sheffrin, 2000)

The representative consumer maximizes $\sum_{t=0}^{\infty}\beta^t E_0 U(C_{Nt}, C_{Tt})$. The relevant variable is a consumption bundle $C_t^* = C_{Tt}^{\alpha} C_{Nt}^{1-\alpha}$ with CRRA utility function $C_t^{*(1-\gamma)}/(1-\gamma)$. Remember that this kind of utility function implies an elasticity of $(1-\gamma)$. Thus it is isoelastic: a relative change in consumption is linked to the same relative change in utility. By using the standard measure of risk aversion $-CU''(C)/U'(C)$, one can see that it is constant and equal to γ. The higher γ, the more risk averse you are. The intertemporal elasticity of substitution is given by $-U'(C)/(CU''(C))$ for the typical Euler equation. It is equal to the inverse of the coefficient of risk aversion (i.e. $1/\gamma$). The higher it is, the more one is willing to let his consumption react to a change in interest rate.

The traded good is the numeraire. Thus, $P_t = P_N/P_T$ and P_T can be normalized to 1 wlog. The budget constraint is $Y_t + r_t^w B_{t-1} = B_t - B_{t-1} + C_t + G_t + I_t$ where B is the stock of foreign assets at the beginning of the period. The current account can be written in the following way:

$$\begin{aligned} Y_t - G_t - I_t - C_t &= B_t - (1+r_t^w)B_{t-1} \\ NO_t - C_t &= B_t - B_{t-1} - r_t^w B_{t-1} \\ \Delta B_t = CA_t &= NO_t - C_t + r_t^w B_{t-1} \end{aligned}$$

In a first step, we solve the standard problem by using the BC expressed in tradable goods. The value of consumption in terms of tradables is $C_t = C_{Tt} + P_t C_{Nt}$.

$$L = \beta \frac{\left(C_{Tt}^{\alpha} C_{Nt}^{1-\alpha}\right)^{1-\gamma}}{1-\gamma} + \beta\lambda_t \left(Y_t + (1+r_t^w)B_{t-1} - B_t - C_{Tt} - P_t C_{Nt} - I_t - G_t\right)$$

$$\begin{aligned} \frac{\partial L}{\partial C_{Tt}} &\Rightarrow (C_{Tt}^{\alpha} C_{Nt}^{1-\alpha})^{-\gamma} \alpha C_{Tt}^{\alpha-1} C_{Nt}^{1-\alpha} = \lambda_t \\ \frac{\partial L}{\partial C_{Nt}} &\Rightarrow (C_{Tt}^{\alpha} C_{Nt}^{1-\alpha})^{-\gamma} C_{Tt}^{\alpha}(1-\alpha) C_{Nt}^{-\alpha} = \lambda_t P_t \end{aligned}$$

With optimality condition

$$P_t = \frac{P_{Nt}}{P_{Tt}} = \frac{1-\alpha}{\alpha}\frac{C_{Tt}}{C_{Nt}}$$

The intratemporal elasticity of subsitution between tradable and non-tradable goods is constant and equal to 1. Remember that the elasticity can be written using

$$\begin{aligned} \frac{d\log(x)}{dx} &= \frac{1}{x} \\ \varepsilon &= \frac{d\log(x)}{d\log(y)} = \frac{dx/x}{dy/y} \end{aligned}$$

Rearrange the result and take $dlog$:

$$
\begin{aligned}
\frac{C_{Tt}}{C_{Nt}} &= \frac{\alpha}{1-\alpha}\frac{P_{Nt}}{P_{Tt}} \\
d\log(C_{Tt}/C_{Nt}) &= d\log(\alpha/1-\alpha) + d\log(P_{Nt}/P_{Tt}) \\
\frac{d\log(C_{Tt}/C_{Nt})}{d\log(P_{Nt}/P_{Tt})} &= 1
\end{aligned}
$$

Note that relative demand will only depend on relative prices, not income.

The next step is to compute the demand functions. Use $C_{Tt} = C_t - P_t C_{Nt}$ and $C_{Nt} = (C_t - C_{Tt})/P_t$ in the preceding optimality condition to get them. First, find the optimal C_N:

$$
\begin{aligned}
P_t &= \frac{1-\alpha}{\alpha}\frac{C_t - P_t C_{Nt}}{C_{Nt}} \\
P_t C_{Nt} &= \frac{1-\alpha}{\alpha}(C_t - P_t C_{Nt}) \\
\frac{1-\alpha}{\alpha}C_t &= P_t C_{Nt} + \frac{1-\alpha}{\alpha}P_t C_{Nt} \\
\frac{1-\alpha}{\alpha}C_t &= P_t C_{Nt}\left(1 + \frac{1-\alpha}{\alpha}\right) \\
C_{Nt} &= (1-\alpha)\frac{C_t}{P_t}
\end{aligned}
$$

and the optimal C_T:

$$
\begin{aligned}
P_t &= \frac{1-\alpha}{\alpha}\frac{C_{Tt}}{(C_t - C_{Tt})/P_t} \\
C_t P_t - C_{Tt} P_t &= \frac{1-\alpha}{\alpha}C_{Tt} P_t \\
P_t C_t &= \frac{1-\alpha}{\alpha}C_{Tt} P_t + P_t C_{Tt} \\
P_t C_t &= P_t C_{Tt}(\frac{1-\alpha}{\alpha} + 1) \\
C_{Tt} &= \alpha C_t
\end{aligned}
$$

These expressions can be used in the consumption bundle C_t^*:

$$
\begin{aligned}
C_t^* &= C_{Tt}^{\alpha} C_{Nt}^{(1-\alpha)} \\
&= (\alpha C_t)^{\alpha}\left((1-\alpha)\frac{C_t}{P_t}\right)^{1-\alpha}
\end{aligned}
$$

Now the Obstfeld and Rogoff (1996) trick is used: define P_t^* as the price index of aggregate consumption C_t^* that minimizes consumption expenditure C_t given P_t:

$$\begin{aligned}
C_t^* &= (\alpha P_t^* C_t^*)^{\alpha}((1-\alpha)\frac{P_t^* C_t^*}{P_t})^{1-\alpha} \\
C_t^* &= \alpha^{\alpha} P_t^{*\alpha} C_t^{*\alpha}(1-\alpha)^{1-\alpha}\frac{P_t^{*1-\alpha} C_t^{*1-\alpha}}{P_t^{1-\alpha}} \\
P_t^* &= P_t^{1-\alpha}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}
\end{aligned}$$

It is now possible to rewrite the maximization problem using the consumption bundle C_t^*:

$$L = \beta^t \frac{C_t^*}{1-\gamma}^{1-\gamma} + \beta^t \lambda_t (Y_t + B_{t-1}(1+r_t^w) - B_t - P_t^* C_t^* - I_t - G_t)$$

$$\begin{aligned}
\frac{\partial L}{\partial C_t^*} \quad &\Rightarrow \quad C_t^{*-\gamma} = \lambda_t P_t^* \\
& \quad\quad\ \lambda_t = \frac{C_t^{*-\gamma}}{P_t^*} \\
& \quad\quad\ \lambda_{t+1} = \frac{C_{t+1}^{*-\gamma}}{P_{t+1}^*} \\
\frac{\partial L}{\partial B_t} \quad &\Rightarrow \quad \beta^{t+1}\lambda_{t+1}(1+r_{t+1}^w) = \beta^t \lambda_t
\end{aligned}$$

$$\beta \frac{C_{t+1}^{*-\gamma}}{P_{t+1}^*}(1+r_{t+1}^w) = \frac{C_t^{*-\gamma}}{P_t^*}$$

$$E_t\left[\beta(1+r_{t+1}^w)\frac{P_t^*}{P_{t+1}^*}(\frac{C_t^*}{C_{t+1}^*})^{\gamma}\right] = 1 \tag{B.1}$$

The expressions for C_t^* and P_t^* from before are used to transform the Euler back.

$$\begin{aligned}
(\frac{C_t^*}{C_{t+1}^*})^{\gamma} &= \frac{(\alpha C_t)^{\alpha\gamma}\left((1-\alpha)\frac{C_t}{P_t}\right)^{(1-\alpha)\gamma}}{(\alpha C_{t+1})^{\alpha\gamma}\left((1-\alpha)\frac{C_{t+1}}{P_{t+1}}\right)^{(1-\alpha)\gamma}} \\
&= \frac{\alpha^{\alpha\gamma} C_t^{\alpha\gamma}(1-\alpha)^{(\gamma-\gamma\alpha)}(C_t/P_t)^{(\gamma-\gamma\alpha)}}{\alpha^{\alpha\gamma} C_{t+1}^{\alpha\gamma}(1-\alpha)^{(\gamma-\gamma\alpha)}(C_{t+1}/P_{t+1})^{(\gamma-\gamma\alpha)}} \\
&= (\frac{C_t}{C_{t+1}})^{\alpha\gamma}(\frac{P_{t+1}}{P_t})^{(\gamma-\gamma\alpha)}\frac{C_t^{(\gamma-\gamma\alpha)}}{C_{t+1}^{(\gamma-\gamma\alpha)}} \\
&= (\frac{C_t}{C_{t+1}})^{\gamma}(\frac{P_{t+1}}{P_t})^{(\gamma-\gamma\alpha)}
\end{aligned}$$

$$\begin{aligned}
\frac{P_t^*}{P_{t+1}^*} &= \frac{P_t^{1-\alpha}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}}{P_{t+1}^{1-\alpha}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}} \\
&= (\frac{P_t}{P_{t+1}})^{1-\alpha}
\end{aligned}$$

Use them in Euler:

$$
\begin{aligned}
E_t\left[\beta(1+r^w_{t+1})(\frac{P_t}{P_{t+1}})^{1-\alpha}\left(\frac{P_{t+1}}{P_t}\right)^{\gamma(1-\alpha)}(\frac{C_t}{C_{t+1}})^{\gamma}\right] &= 1 \\
E_t\left[\beta(1+r^w_{t+1})(\frac{C_t}{C_{t+1}})^{\gamma}\left(\frac{P_t}{P_{t+1}}\right)^{(1-\alpha)(1-\gamma)}\right] &= 1 \qquad \text{(B.2)}
\end{aligned}
$$

From that point on, some assumptions concerning the distribution are made. Remember that if X is a random variable with normal distribution, then $\exp(X) = Y$ has a log-normal distribution (if Y is log-normal, then $X = log(Y)$ is normally distributed).
The first and second moments are:

$$
\begin{aligned}
E(Y) &= exp(\mu + \frac{1}{2}\sigma^2) \\
Var(Y) &= (e^{\sigma^2} - 1)e^{2\mu+\sigma^2}
\end{aligned}
$$

Now assume that the entire Euler, say Y_t, is jointly log normal distributed. Variance and covariance between them are not time-varying. Thus, $\log(Y_t)$ has a normal distribution with moments μ and σ^2. For the variance part, use $var(\alpha x + \beta y) = \alpha^2\sigma_x + \beta^2\sigma_y + \alpha\beta 2cov(x,y)$:

$$
\begin{aligned}
\mu &= E_t(\log(Y_t)) \\
&= \log(\beta) + E_t(r^w_{t+1}) - \gamma E_t(\Delta c_{t+1}) - (1-\gamma)(1-\alpha)E_t(\Delta p_{t+1})
\end{aligned}
$$

$$
\begin{aligned}
\sigma^2 &= var(\log(Y_t)) \\
&= var(r^w_{t+1}) + \gamma^2 var(\Delta c_{t+1}) + (1-\gamma)^2(1-\alpha)^2 var(\Delta p_{t+1}) \\
&\quad -2\gamma cov(r^w_{t+1}, \Delta c_{t+1}) - 2(1-\gamma)(1-\alpha)cov(r^w_{t+1}, \Delta p_{t+1}) \\
&\quad +2\gamma(1-\gamma)(1-\alpha)\mathrm{cov}(\Delta c_{t+1}, \Delta p_{t+1}) \\
&= \sigma_r^2 + \gamma^2\sigma_c^2 + (1-\gamma)^2(1-\alpha)^2\sigma_p^2 - 2\gamma\sigma_{r,c} - 2(1-\gamma)(1-\alpha)\sigma_{r,p} \\
&\quad +2\gamma(1-\gamma)(1-\alpha)\sigma_{c,p}
\end{aligned}
$$

At this point one gets the expectation of Y_t as:

$$E(Y_t) = \exp(\mu + \frac{1}{2}\sigma^2) = 1$$

Take log on both sides and rearrange:

$$
\begin{aligned}
0 &= \mu + 0.5\sigma^2 \\
&= \log(\beta) + E_t(r^w_{t+1}) - \gamma E_t(\Delta c_{t+1}) - (1-\gamma)(1-\alpha)E_t(\Delta p_{t+1}) \\
&\quad +0.5\sigma_r^2 + 0.5\gamma^2\sigma_c^2 + 0.5(1-\gamma)^2(1-\alpha)^2\sigma_p^2 - \gamma\sigma_{r,c} - (1-\gamma)(1-\alpha)\sigma_{r,p} \\
&\quad +\gamma(1-\gamma)(1-\alpha)\sigma_{c,p}
\end{aligned}
$$

Put $\gamma E_t(\Delta c_{t+1})$ on the left-hand side and divide by γ:

$$
\begin{aligned}
E_t(\Delta c_{t+1}) &= \frac{1}{\gamma}\log(\beta)+\frac{1}{\gamma}E_t(r^w_{t+1})-\frac{1}{\gamma}(1-\gamma)(1-\alpha)E_t(\Delta p_{t+1}) \\
&+\frac{0.5}{\gamma}\sigma_r^2+0.5\gamma\sigma_c^2+\frac{0.5}{\gamma}(1-\gamma)^2(1-\alpha)^2\sigma_p^2-\sigma_{r,c}-\frac{(1-\gamma)}{\gamma}(1-\alpha)\sigma_{r,p} \\
&+(1-\gamma)(1-\alpha)\sigma_{c,p}
\end{aligned}
$$

Use $1/\gamma=\eta$, noting that $(1-\gamma)/\gamma=(\eta-1)$ and $(1-\gamma)^2/\gamma=(1-1/\eta)^2/(1/\eta)=((\eta-1)/\eta)^2\eta=(\eta-1)^2/\eta$:

$$
\begin{aligned}
E_t(\Delta c_{t+1}) &= \eta\log(\beta)+\eta Et(r^w_{t+1})-(\eta-1)(1-\alpha)E_t(\Delta p_{t+1}) \\
&+0.5\eta\sigma_r^2+0.5\frac{1}{\eta}\sigma_c^2+0.5\frac{(\eta-1)^2}{\eta}(1-\alpha)^2\sigma_p^2-\sigma_{r,c}-(\eta-1)(1-\alpha)\sigma_{r,p} \\
&+\frac{(\eta-1)}{\eta}(1-\alpha)\sigma_{c,p}
\end{aligned}
$$

Rearrange somewhat by taking the r and p terms to the front:

$$
\begin{aligned}
E_t(\Delta c_{t+1}) &= \eta(E_t(r^w_{t+1})-\frac{\eta-1}{\eta}(1-\alpha)E_t(\Delta p_{t+1})) \\
&+\eta\log(\beta)+0.5\eta\sigma_r^2+0.5\frac{1}{\eta}\sigma_c^2+0.5\frac{(\eta-1)^2}{\eta}(1-\alpha)^2\sigma_p^2 \\
&-\sigma_{r,c}-(\eta-1)(1-\alpha)\sigma_{r,p}+\frac{(\eta-1)}{\eta}(1-\alpha)\sigma_{c,p} \\
&= \eta(E_t(r^w_{t+1})+\frac{1-\eta}{\eta}(1-\alpha)E_t(\Delta p_{t+1}))+const
\end{aligned}
$$

The variance and covariance parts can be gathered in a time-invariant constant. Define the consumption-based real interest rate as $r^*_{t+1}=r^w_{t+1}+\frac{1-\eta}{\eta}(1-\alpha)\Delta p_{t+1}$ to get:

$$
\begin{aligned}
E_t(\Delta c_{t+1}) &= \eta E_t(r^*_{t+1})+const \\
&= \eta E_t\left[r^w_{t+1}+\frac{1-\eta}{\eta}(1-\alpha)\Delta p_{t+1}\right]+const
\end{aligned}
$$

The variable r^*_t reflects both the normal interest rate (r^w) and the change in relative price of non-traded goods (Δp). The optimal consumption profile is thus influenced by the world interest rate and the changes in relative price of goods. There are three effects: intertemporal effect 1, intratemporal effect and intertemporal effect 2.

- Intertemporal effect 1: if the world interest rate is higher, the consumption-based real interest rate is higher as well. As a consequence, consumption today is lower and future consumption higher. Intuition: current consumption is more expensive in terms of future one, the agent substitute with elasticity $\eta=1/\gamma$ and save more (intertemporal elasticity of substitution).

Assume the price of non-tradables is temporarily low (at t) and expected to rise relative to tradables (at

$t+1$) or $E_t(\Delta p_{t+1}) > 0$ as $P_t = P_N/P_T$. We have $(1-\eta)(1-\alpha)\Delta p_{t+1}$. Now the effect depends on $1-\eta$ being positive or negative. Assume that $\eta = 1/\gamma < 1$, as it is often the case in macro (i.e. with coefficients of relative risk aversion bigger than one). In this case, we have that $(1-\eta)(1-\alpha)\Delta p_{t+1}$ is positive. The term $E_t(\Delta c_{t+1})$ has to be higher. Consumption today is lower relative to future consumption. Intuition: the consumption-based real interest rate rises above the world interest rate, supplementary incentive to save because of expected increase in price of non-tradables. Let's look under the hood:

- Intratemporal effect: intra elasticity is unity and the effect is $1 \times (1-\alpha)$. This effect tends to push the consumption-based real interest rate up compared to world interest rate (incentive to save). Intuition: P_T is high today relative to P_N. Remember the value of consumption in terms of tradables is $C_t = C_{Tt} + \frac{P_N}{P_T} C_{Nt}$. As a result, total current consumption expenditures are lower by the share of non-tradables in the consumption bundle $(1-\alpha)$.
- Intertemporal effect 2: it is $-\eta(1-\alpha) = -(1/\gamma)(1-\alpha)$. This effect tends to push the consumption-based real interest rate down compared to world interest rate (incentive to save less). If P_T is high today and low tomorrow (i.e. P_N low today and high tomorrow, $E_t(\Delta p_{t+1}) > 0$), the repayment of a loan taken today in traded goods is high. A loan in traded goods will thus cost a lot. A loan in terms of the consumption bundle costs less in value because $\frac{P_N}{P_T}$ is low and the non-tradable part is thus less expensive in terms of tradables. The future repayment has thus a lower cost in terms of the consumption bundle than in terms of the tradables. Current consumption is thus higher and saving lower.

In our case, as $\eta = 1/\gamma < 1$, the intratemporal effect dominates the intertemporal effect 2 and HH save more if P_N is expected to rise. In conclusion, one sees that borrowing and lending using the world interest rate has to be thought of in terms of the price of tradable goods while using the consumption-based real interest rate has to be thought of in terms of the price of the consumption bundle.

If $\eta = 1/\gamma > 1$, $(1-\eta)(1-\alpha)\Delta p_{t+1}$ becomes negative. $E_t(\Delta c_{t+1})$ has to be lower, consumption today is higher relative to future consumption. The consumption-based real interest rate is below the world interest rate. It makes people save less.

B.2.1.2 Saving wedge extension

Follow the same steps as in the original paper but introduce a saving wedge:

$$1 + r_t^T = (1 + r_t^w)(1 - \tau_t^s)$$

The budget constraint is:

$$\begin{aligned} Y_t + (1 + r_t^T) B_{t-1} &= B_t + C_t + G_t + I_t \\ Y_t + (1 + r_t^w)(1 - \tau_t^s) B_{t-1} &= B_t + C_t + G_t + I_t \end{aligned}$$

Using $(1 + r_t^w)(1 - \tau_t^s) = 1 - \tau_t^s + r_t^w - r_t^w \tau_t^s$, we get that the current accout is:

$$\Delta B_t = CA_t = NO_t - C_t + [r_t^w(1 - \tau_t^s) - \tau_t^s] B_{t-1}$$

The solution of the maximization problem is identical to the baseline case:

$$P_t = \frac{P_{Nt}}{P_{Tt}} = \frac{1-\alpha}{\alpha}\frac{C_{Tt}}{C_{Nt}}$$

The demand functions are identical as well:

$$\begin{aligned} C_{Nt} &= (1-\alpha)\frac{C_t}{P_t} \\ C_{Tt} &= \alpha C_t \end{aligned}$$

As before, these expressions can be used in the consumption bundle C_t^*:

$$\begin{aligned} C_t^* &= C_{Tt}^{\alpha} C_{Nt}^{(1-\alpha)} \\ &= (\alpha C_t)^{\alpha}\left((1-\alpha)\frac{C_t}{P_t}\right)^{1-\alpha} \end{aligned}$$

The price of aggregate consumption is the same as before:

$$P_t^* = P_t^{1-\alpha}\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$$

It is now possible to rewrite the maximization problem using the consumption bundle C_t^* and find the new Euler:

$$E_t\left[\beta(1+r_{t+1}^w)(1-\tau_{t+1}^s)\frac{P_t^*}{P_{t+1}^*}(\frac{C_t^*}{C_{t+1}^*})^{\gamma}\right] = 1$$

Transform Euler back:

$$E_t\left[\beta(1+r_{t+1}^w)(1-\tau_{t+1}^s)(\frac{C_t}{C_{t+1}})^{\gamma}\left(\frac{P_t}{P_{t+1}}\right)^{(1-\alpha)(1-\gamma)}\right] = 1$$

In the steps following the log normal distribution assumption, the wedge behaves like r^w but with opposite sign cause $\log(1-x) \approx -x$. We end up with:

$$E_t(\Delta c_{t+1}) = \eta(E_t(r_{t+1}^w) - E_t(\tau_t^s) + \frac{1-\eta}{\eta}(1-\alpha)E_t(\Delta p_{t+1})) + const$$

Define the consumption-based real interest rate (with wedge) as $r_{t+1}^* = r_{t+1}^w - \tau_t^s + \frac{1-\eta}{\eta}(1-\alpha)\Delta p_{t+1}$ to get:

$$\begin{aligned} E_t(\Delta c_{t+1}) &= \eta E_t(r_{t+1}^*) + const \\ &= \eta E_t\left[r_{t+1}^w - \tau_t^s + \frac{1-\eta}{\eta}(1-\alpha)\Delta p_{t+1}\right] + const \end{aligned}$$

If the saving wedge is positive, the consumption-based real interest rate is lower than the world interest rate, $E_t(\Delta c_{t+1})$ has to be lower, consumption today is higher relative to future consumption, people save less. It acts like a tax on savings.

B.2.1.3 Alternative derivation of Bergin and Sheffrin (2000)[10]

The maximization problem for the representative household is

$$\max \sum_{t=0}^{\infty} \beta^t E_0 U(C_{T,t}, C_{N,t}), \quad \text{s.t.}$$
$$Y_t - C_{T,t} - Q_t C_{N,t} - I_t - G_t + r_t B_{t-1} = B_t - B_{t-1}, \forall t$$

With $U(C_{T,t}, C_{N,t}) = \frac{1}{1-\gamma}\left(C_{T,t}^{a} C_{N,t}^{1-a}\right)^{1-\gamma}$.
Note that prices are in terms of tradables ($Q_t = P_N/P_T$).
Define an aggregate consumption index:

$$C_t = C_{T,t}^{a} C_{N,t}^{1-a}$$

First, the cost of consuming a certain amount of C_t is minimized:

$$\min C_{T,t} + Q_t C_{N,t} \quad \text{s.t.}$$
$$C_{T,t}^{a} C_{N,t}^{1-a} = C_t$$

The Lagrangian is

$$\mathfrak{L} = C_{T,t} + Q_t C_{N,t} + \lambda_t (C_t - C_{T,t}^{a} C_{N,t}^{1-a})$$

The first-order conditions with respect to C_T, C_N and λ are

$$1 - \lambda_t a C_{T,t}^{a-1} C_{N,t}^{1-a} = 0$$
$$Q_t - \lambda_t (1-a) C_{T,t}^{a} C_{N,t}^{-a} = 0$$
$$C_t - C_{T,t}^{a} C_{N,t}^{1-a} = 0$$

Find λ with the first equation:

$$\lambda_t = \frac{1}{a} C_{T,t}^{1-a} C_{N,t}^{a-1}$$

Plug it in the second one:

$$\begin{aligned}
Q_t &= \lambda_t (1-a) C_{T,t}^{a} C_{N,t}^{-a} \\
&= \frac{1-a}{a} C_{T,t} C_{N,t}^{-1} \\
\frac{1}{Q_t} &= \frac{a}{1-a} \frac{C_{N,t}}{C_{T,t}}
\end{aligned}$$

[10] We are thankful to Alexander Rathke for suggesting us that approach.

Take first FOC and expand with C_T to simplify the expression using the definition of the consumption index and get the demand equation for tradable goods:

$$\frac{1}{\lambda_t} = aC_{T,t}^{a-1}C_{N,t}^{1-a}\frac{C_{T,t}}{C_{T,t}}$$
$$\frac{1}{\lambda_t} = a\frac{C_t}{C_{T,t}}$$
$$C_{T,t} = a\left(\frac{1}{\lambda_t}\right)^{-1}C_t$$

In the same spirit, take second FOC and expand with C_N to get the demand for non-tradable goods:

$$\begin{aligned} Q_t &= \lambda_t(1-a)C_{T,t}^{a}C_{N,t}^{-a}\frac{C_{N,t}}{C_{N,t}} \\ &= \lambda_t(1-a)C_tC_{N,t}^{-1} \\ C_{N,t} &= (1-a)\left(\frac{Q_t}{\lambda_t}\right)^{-1}C_t \end{aligned}$$

Plug both demands into the definition of C_t and get the shadow price:

$$C_t = C_{T,t}^{a}C_{N,t}^{1-a}$$
$$C_t = \left(a\left(\frac{1}{\lambda_t}\right)^{-1}C_t\right)^{a}\left((1-a)\left(\frac{Q_t}{\lambda_t}\right)^{-1}C_t\right)^{1-a}$$
$$C_t = a^a\left(\frac{1}{\lambda_t}\right)^{-a}C_t^a(1-a)^{1-a}\left(\frac{Q_t}{\lambda_t}\right)^{-(1-a)}C_t^{1-a}$$
$$1 = a^a\lambda_t^a(1-a)^{1-a}Q_t^{-(1-a)}\lambda_t^{(1-a)}$$
$$\lambda_t = a^{-a}(1-a)^{-(1-a)}Q_t^{(1-a)}$$
$$= P_t$$

Get consumption expenditures in terms of traded goods using both demands and P_t for λ_t:

$$C_{T,t} + Q_tC_{N,t} =$$
$$a\left(\frac{1}{P_t}\right)^{-1}C_t + Q_t(1-a)\left(\frac{Q_t}{P_t}\right)^{-1}C_t = P_tC_t$$

Now we can solve the maximization problem directly using the consumption bundle. Maximize with respect to consumption and bonds:

$$\max E_0\sum_{t=0}^{\infty}\beta^t\frac{1}{1-\gamma}C_t^{1-\gamma},$$
$$s.t.$$
$$Y_t - P_tC_t - I_t - G_t + r_tB_{t-1} = B_t - B_{t-1}, \forall t$$

$$L=\beta^t\left(\frac{1}{1-\gamma}C_t^{1-\gamma}+\Lambda_t(Y_t-P_tC_t-I_t-G_t+(1+r_t)B_{t-1}-B_t)\right)$$

FOC with respect to C_t and B_t:

$$\begin{aligned}C_t^{-\gamma}-\Lambda_tP_t=0\\ -\beta^t\Lambda_t+\beta^{t+1}E_t(1+r_{t+1})\Lambda_{t+1}=0\end{aligned}$$

Rearrange:

$$\begin{aligned}\Lambda_t &= \frac{C_t^{-\gamma}}{P_t}\\ \beta E_t(1+r_{t+1})\frac{\Lambda_{t+1}}{\Lambda_t} &= 1\end{aligned}$$

Use expression of first equation in second one:

$$\begin{aligned}\beta E_t(1+r_{t+1})\frac{C_{t+1}^{-\gamma}/P_{t+1}}{C_t^{-\gamma}/P_t} &= 1\\ \beta E_t(1+r_{t+1})\frac{C_t^{\gamma}}{C_{t+1}^{\gamma}}\frac{P_t}{P_{t+1}} &= 1 \qquad \text{(B.3)}\end{aligned}$$

Note that this equation is similar to equation (B.1) with a different notation (i.e. $C=C^*$ and $P=P^*$). Rewriting it in expanding with P_t and P_{t+1} (C ratio part) and use preceding result for P (P ratio part). In terms of consumption expenditure C_t and the real exchange rate defined as the price of non-tradables in terms of tradables, one finds:

$$\beta E_t(1+r_{t+1})\frac{(P_tC_t)^{\gamma}P_{t+1}^{\gamma}}{(P_{t+1}C_{t+1})^{\gamma}P_t^{\gamma}}\frac{a^{-a}(1-a)^{-(1-a)}Q_t^{(1-a)}}{a^{-a}(1-a)^{-(1-a)}Q_{t+1}^{(1-a)}}=1$$

Use the expression for P again:

$$\begin{aligned}\beta E_t(1+r_{t+1})\frac{(P_tC_t)^{\gamma}(a^{-a}(1-a)^{-(1-a)}Q_{t+1}^{(1-a)})^{\gamma}}{(P_{t+1}C_{t+1})^{\gamma}(a^{-a}(1-a)^{-(1-a)}Q_t^{(1-a)})^{\gamma}}\frac{Q_t^{(1-a)}}{Q_{t+1}^{(1-a)}} &= 1\\ \beta E_t(1+r_{t+1})\frac{(P_tC_t)^{\gamma}Q_{t+1}^{\gamma(1-a)}}{(P_{t+1}C_{t+1})^{\gamma}Q_t^{\gamma(1-a)}}\frac{Q_t^{(1-a)}}{Q_{t+1}^{(1-a)}} &= 1\\ \beta E_t(1+r_{t+1})\frac{(P_tC_t)^{\gamma}}{(P_{t+1}C_{t+1})^{\gamma}}\frac{Q_t^{(1-a)-\gamma(1-a)}}{Q_{t+1}^{(1-a)-\gamma(1-a)}} &= 1\\ \beta E_t(1+r_{t+1})\frac{(P_tC_t)^{\gamma}}{(P_{t+1}C_{t+1})^{\gamma}}\frac{Q_t^{(1-a)(1-\gamma)}}{Q_{t+1}^{(1-a)(1-\gamma)}} &= 1\end{aligned}$$

Assuming $\tilde{C}_t = P_t C_t$, the intertemporal Euler equation is

$$E_t\left[\beta \frac{(1+r_{t+1})\tilde{C}_{t+1}^{-\gamma}}{\tilde{C}_t^{-\gamma}} \frac{Q_t}{Q_{t+1}}^{(1-\gamma)(1-a)}\right] = 1 \tag{B.4}$$

It corresponds to equation (B.2), albeit with a different notation (i.e. $\tilde{C} = C$ and $Q = P$). Assume that $X_t = \beta \frac{(1+r_{t+1})\tilde{C}_{t+1}^{-\gamma}}{\tilde{C}_t^{-\gamma}} \frac{Q_t}{Q_{t+1}}^{(1-\gamma)(1-a)}$ is jointly log normal distributed.
We know that

$$E_t(X_t) = E_t\left[\beta \frac{(1+r_{t+1})\tilde{C}_{t+1}^{-\gamma}}{\tilde{C}_t^{-\gamma}} \frac{Q_t}{Q_{t+1}}^{(1-\gamma)(1-a)}\right] = 1$$

and $\log(X_t) = r_{t+1} + \log\beta - \gamma\Delta\tilde{c}_{t+1} - (1-\gamma)(1-a)\Delta q_{t+1}$ has a normal distribution with expected value

$$E_t\left(\log(X_t)\right) = E_t(r_{t+1}) + \log(\beta) - \gamma E_t\left(\Delta\tilde{c}_{t+1}\right) - (1-\gamma)(1-a)E_t\left(\Delta q_{t+1}\right)$$

where $\Delta\tilde{c}_{t+1} = \log\tilde{C}_{t+1} - \log\tilde{C}_t$ and $\Delta q_{t+1} = \log Q_{t+1} - \log Q_t$.
The Variance is

$$\begin{aligned}
Var(\log(X_t)) =& Var(r_{t+1}) + \gamma^2 Var(\Delta\tilde{c}_{t+1}) + (1-\gamma)^2(1-a)^2 Var(\Delta q_{t+1}) \\
& - 2\gamma Cov(r_{t+1}, \Delta\tilde{c}_{t+1}) - 2(1-\gamma)(1-a)Cov(r_{t+1}, \Delta q_{t+1}) \\
& + 2\gamma(1-\gamma)(1-a)Cov(\Delta\tilde{c}_{t+1}, \Delta q_{t+1}) \\
=& \sigma_r^2 + \gamma^2\sigma_c^2 + (1-\gamma)^2(1-a)^2\sigma_q^2 \\
& - 2\gamma\sigma_{r,c} - 2(1-\gamma)(1-a)\sigma_{r,q} + 2\gamma(1-\gamma)(1-a)\sigma_{c,q}
\end{aligned}$$

Remember the first moment of a log normal distribution is $\exp\left(\mu + 0.5\sigma^2\right)$.
Start with

$$\exp(E_t(\log X_t) + 0.5 Var(\log X_t)) = 1$$

or

$$\begin{aligned}
0 =& E_t(\log X_t) + 0.5 Var(\log X_t) \\
0 =& E_t\left(r_{t+1}\right) + \log\left(\beta\right) - \gamma E_t\left(\Delta\tilde{c}_{t+1}\right) - (1-\gamma)(1-a)E_t\left(\Delta q_{t+1}\right) + \\
& 0.5(\sigma_r^2 + \gamma^2\sigma_c^2 + (1-\gamma)^2(1-a)^2\sigma_q^2) + \\
& 0.5(-2\gamma\sigma_{r,c} - 2(1-\gamma)(1-a)\sigma_{r,q} + 2\gamma(1-\gamma)(1-a)\sigma_{c,q}) \\
\gamma E_t\left(\Delta\tilde{c}_{t+1}\right) =& E_t\left(r_{t+1}\right) + \log\left(\beta\right) - (1-\gamma)(1-a)E_t\left(\Delta q_{t+1}\right) + \\
& 0.5(\sigma_r^2 + \gamma^2\sigma_c^2 + (1-\gamma)^2(1-a)^2\sigma_q^2) + \\
& 0.5(-2\gamma\sigma_{r,c} - 2(1-\gamma)(1-a)\sigma_{r,q} + 2\gamma(1-\gamma)(1-a)\sigma_{c,q}) \\
E_t\left(\Delta\tilde{c}_{t+1}\right) =& (1/\gamma)E_t\left(r_{t+1}\right) + \log\left(\beta\right)/\gamma - ((1-\gamma)(1-a)/\gamma)E_t\left(\Delta q_{t+1}\right) + \\
& 0.5(\sigma_r^2/\gamma + \gamma\sigma_c^2 + \left((1-\gamma)^2(1-a)^2/\gamma\right)\sigma_q^2) \\
& 0.5(-2\sigma_{r,c} - (2(1-\gamma)(1-a)/\gamma)\sigma_{r,q} + 2(1-\gamma)(1-a)\sigma_{c,q})
\end{aligned}$$

Now $\eta = 1/\gamma$, note that $\frac{1-\gamma}{\gamma} = \frac{1-1/\eta}{1/\eta} = \eta - 1$ and $\frac{(1-\gamma)^2}{\gamma} = \left(\frac{\eta-1}{\eta}\right)^2 \eta = \frac{(\eta-1)^2}{\eta}$:

$$\begin{aligned} E_t\left(\Delta\tilde{c}_{t+1}\right) = \eta E_t\left(r_{t+1}\right) + \eta\log\left(\beta\right) - (\eta-1)(1-a)E_t\left(\Delta q_{t+1}\right) + \\ 0.5(\eta\sigma_r^2 + \frac{\sigma_c^2}{\eta} + \frac{(\eta-1)^2}{\eta}(1-a)^2\sigma_q^2) + \\ 0.5(-2\sigma_{r,c} - 2(\eta-1)(1-a)\sigma_{r,q} + 2\frac{(\eta-1)}{\eta}(1-a)\sigma_{c,q}) \end{aligned}$$

Rearrange

$$\begin{aligned} E_t\left(\Delta\tilde{c}_{t+1}\right) = \eta[E_t\left(r_{t+1}\right) + \frac{1-\eta}{\eta}(1-a)E_t\Delta q_{t+1}] + \\ + \eta[\log\beta + 0.5(\sigma_r^2 + \frac{\sigma_c^2}{\eta^2} + \frac{(\eta-1)^2}{\eta^2}(1-a)^2\sigma_q^2)] + \\ 0.5\eta[-\frac{2}{\eta}\sigma_{r,c} - 2\frac{(\eta-1)}{\eta}(1-a)\sigma_{r,q} + 2\frac{(\eta-1)}{\eta^2}(1-a)\sigma_{c,q}] \\ E_t(\Delta\tilde{c}_{t+1}) = \eta E_t\left(r_{t+1} + \frac{1-\eta}{\eta}(1-a)\Delta q_{t+1}\right) + const \end{aligned}$$

which is exactly the result obtained in the original paper (remember Δq stands for relative prices Δp and $\Delta\widetilde{c}$ corresponds to Δc in original paper).

One can get this result by log-linearizing around the steady-state. That's the methodology that we use later for the CES case (I) in Section B.2.1.4. Start with the Euler equation (B.3) and use $1 + r_{t+1} = R_{t+1}$:

$$\begin{aligned} E_t\left[R_{t+1}\frac{C_t^{\gamma}}{C_{t+1}^{\gamma}}\frac{P_t}{P_{t+1}}\right] &= 1/\beta \\ E_t\left[R_{t+1}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{P_{t+1}}{P_t}\right)^{-1}\right] &= 1/\beta \end{aligned}$$

In SS it is:

$$\begin{aligned} \bar{R}(\frac{\bar{C}}{\bar{C}})^{\gamma}\frac{\bar{P}}{\bar{P}} &= \frac{1}{\beta} \\ \bar{R} &= \frac{1}{\beta} \end{aligned}$$

Remember:

$$\hat{x}\frac{\partial f(\bar{x},\bar{y})}{\partial\bar{x}}\bar{x} + \hat{y}\frac{\partial f(\bar{x},\bar{y})}{\partial\bar{y}}\bar{y} \approx 0$$

Log-linearizing around the steady-state:

$$
\begin{aligned}
E_t\left[R_{t+1}C_{t+1}^{-\gamma}C_t^{\gamma}P_{t+1}^{-1}P_t\right] &= 1/\beta \\
E_t\left[\hat{R}_{t+1}\bar{R}\bar{C}^{-\gamma}\bar{C}^{\gamma}\bar{P}^{-1}\bar{P}+\hat{C}_{t+1}\bar{C}(-\gamma)\bar{C}^{-\gamma-1}\bar{C}^{\gamma}\bar{P}^{-1}\bar{P}\bar{R}+\hat{C}_t\bar{C}\gamma\bar{C}^{\gamma-1}\bar{C}^{-\gamma}\bar{P}^{-1}\bar{P}\bar{R}\right]+ & \\
\hat{P}_{t+1}\bar{P}(-1)\bar{P}^{-2}\bar{P}\bar{R}\bar{C}^{-\gamma}\bar{C}^{\gamma}+\hat{P}_t\bar{P}\bar{P}^{-1}\bar{R}\bar{C}^{-\gamma}\bar{C}^{\gamma} &\approx 0 \\
E_t\left[\hat{R}_{t+1}\bar{R}-\gamma\hat{C}_{t+1}\bar{R}+\gamma\hat{C}_t\bar{R}-\hat{P}_{t+1}\bar{R}+\hat{P}_t\bar{R}\right] &\approx 0 \\
E_t\left[\hat{R}_{t+1}-\gamma(\hat{C}_{t+1}-\hat{C}_t)-(\hat{P}_{t+1}-\hat{P}_t)\right]\approx 0 &
\end{aligned}
$$

Remember that $\hat{X}_t=\log(X_t)-\log(\bar{X})=\log(X_t/\bar{X})\approx\frac{X_t-\bar{X}}{\bar{X}}=\frac{dX_t}{X}$.
Note that $\hat{R}_{t+1}=(R_{t+1}-\bar{R})=(1+r_{t+1})-(1+\bar{r})=r_{t+1}-\bar{r}$. Use it:

$$
\begin{aligned}
E_t\left[r_{t+1}-\bar{r}-\gamma\Delta c_{t+1}-\Delta p_{t+1}\right] &\approx 0 \\
E_t\left(\Delta c_{t+1}\right) &\approx \frac{1}{\gamma}E_t\left(r_{t+1}\right)-\frac{1}{\gamma}E_t\left(\Delta p_{t+1}\right)-\frac{1}{\gamma}\bar{r}
\end{aligned}
$$

So the only difference between a standard log-linearization and Bergin and Sheffrin is the constant. Ignore it and use $\frac{1}{\gamma}=\eta$:

$$
E_t\left(\Delta c_{t+1}\right)=\eta E_t\left(r_{t+1}\right)-\eta E_t\left(\Delta p_{t+1}\right)
$$

Linearize the consumption price inflation $\frac{P_t}{P_{t-1}}=\frac{a^{-a}(1-a)^{-(1-a)}Q_t^{(1-a)}}{a^{-a}(1-a)^{-(1-a)}Q_{t-1}^{(1-a)}}$ by taking logs:

$$
\begin{aligned}
\frac{P_t}{P_{t-1}} &= \left(\frac{Q_t}{Q_{t-1}}\right)^{1-\alpha} \\
\Delta p_t &= (1-a)\Delta q_t
\end{aligned}
$$

Alternatively, could log-linearize the former expression:

$$
\begin{aligned}
P_tP_{t-1}^{-1} &= Q_t^{1-\alpha}Q_{t-1}^{\alpha-1} \\
\hat{P}_t\bar{P}\bar{P}^{-1}+\hat{P}_{t-1}\bar{P}(-1)\bar{P}^{-2}\bar{P} &\approx \hat{Q}_t\bar{Q}(1-\alpha)\bar{Q}^{-\alpha}\bar{Q}^{\alpha-1}+\hat{Q}_{t-1}\bar{Q}(\alpha-1)\bar{Q}^{\alpha-2}\bar{Q}^{1-\alpha} \\
\hat{P}_t-\hat{P}_{t-1} &\approx (1-\alpha)(\hat{Q}_t-\hat{Q}_{t-1}) \\
\Delta\hat{P}_t &\approx (1-\alpha)\Delta\hat{Q}_t
\end{aligned}
$$

Remember that $\hat{P}_t=\log(P_t)-\log(\bar{P})=\log(P_t/\bar{P})\approx\frac{P_t-\bar{P}}{\bar{P}}=\frac{dP_t}{P}$.
Use this definition results in:

$$
\begin{aligned}
\log(P_t)-\log(\bar{P})-(\log(P_{t-1})-\log(\bar{P})) &\approx (1-\alpha)\left[\log(Q_t)-\log(\bar{Q})-(\log(Q_{t-1})-\log(\bar{Q}))\right] \\
\Delta p_t &\approx (1-\alpha)\Delta q_t
\end{aligned}
$$

Remember the definition of consumption expenditure and take logs:

$$
\begin{aligned}
\tilde{C}_t &= P_t C_t \\
\tilde{c}_t &= p_t + c_t \\
\Delta\tilde{c}_t &= \Delta p_t + \Delta c_t \\
\Delta\tilde{c}_{t+1} &= \Delta p_{t+1} + \Delta c_{t+1}
\end{aligned}
$$

Again, as we take the first difference log-linearizing would be equivalent. The steady-state is:

$$\bar{\tilde{C}} = \bar{P}\bar{C}$$

The log-linearization is:

$$
\begin{aligned}
\hat{\tilde{C}}_t \bar{\tilde{C}} &\approx \hat{P}_t \bar{P}\bar{C} + \hat{C}_t \bar{C}\bar{P} \\
\hat{\tilde{C}}_t &\approx \hat{P}_t + \hat{C}_t \\
\log \tilde{C}_t - \log \bar{\tilde{C}} &\approx \log P_t - \log \bar{P} + \log C_t - \log \bar{C}
\end{aligned}
$$

Now by taking the first difference for period $t+1$ one gets:

$$
\begin{aligned}
\Delta\hat{\tilde{C}}_{t+1} &\approx \Delta\hat{P}_{t+1} + \Delta\hat{C}_{t+1} \\
\Delta\tilde{c}_{t+1} &\approx \Delta p_{t+1} + \Delta c_{t+1}
\end{aligned}
$$

Rewriting the linear Euler equation in terms of consumption expenditure $\Delta c_{t+1} = \Delta\tilde{c}_{t+1} - \Delta p_{t+1}$:

$$
\begin{aligned}
E_t(\Delta c_{t+1}) &= \eta E_t(r_{t+1}) - \eta E_t(\Delta p_{t+1}) - \frac{1}{\gamma}\bar{r} \\
E_t(\Delta\tilde{c}_{t+1}) - E_t(\Delta p_{t+1}) &= \eta E_t(r_{t+1}) - \eta E_t(\Delta p_{t+1}) + const \\
E_t(\Delta\tilde{c}_{t+1}) &= \eta E_t(r_{t+1}) - (\eta - 1) E_t(\Delta p_{t+1}) + const
\end{aligned}
$$

Use the log-linearized price inflation equation $\Delta p_t = (1-\alpha)\Delta q_t$:

$$
\begin{aligned}
E_t(\Delta\tilde{c}_{t+1}) &= \eta E_t(r_{t+1}) - (\eta - 1)(1-a) E_t(\Delta q_{t+1}) + const \\
&= \eta \left[E_t(r_{t+1}) + \frac{(1-\eta)}{\eta}(1-a) E_t(\Delta q_{t+1}) \right] + const
\end{aligned}
$$

B.2.1.4 The CES case (I)[11]

Now assume a CES aggregator. Again, the household will minimize the cost of buying a certain level of the composite consumption index C_t:

$$\min_{C_t^T, C_t^N} C_t^T + Q_t C_t^N$$

s.t.

$$C_t = [(1-\alpha)^{\frac{1}{\theta}} (C_t^T)^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} (C_t^N)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}.$$

As before prices are in terms of tradables ($Q_t = P_N/P_T$). Note that now α is the expenditure share of non-tradable goods whereas, before, it was the share of tradable goods. The intratemporal elasticity of substitution is θ. The coefficient of relative risk aversion is γ and the intertemporal elasticity of substitution is $\eta = 1/\gamma$.

The first-order conditions with respect to tradables and non-tradables are:

$$\begin{aligned} 1 - \psi_t[(1-\alpha)^{\frac{1}{\theta}} (C_t^T)^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} (C_t^N)^{\frac{\theta-1}{\theta}}]^{\frac{1}{\theta-1}} (1-\alpha)^{\frac{1}{\theta}} C_t^{T^{-\frac{1}{\theta}}} &= 0, \\ Q_t - \psi_t[(1-\alpha)^{\frac{1}{\theta}} (C_t^T)^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} (C_t^N)^{\frac{\theta-1}{\theta}}]^{\frac{1}{\theta-1}} \alpha^{\frac{1}{\theta}} C_t^{N^{-\frac{1}{\theta}}} &= 0, \end{aligned}$$

where ψ_t is the Lagrange multiplier of the minimization problem.

Define $X_t = [(1-\alpha)^{\frac{1}{\theta}} (C_t^T)^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} (C_t^N)^{\frac{\theta-1}{\theta}}]^{\frac{1}{\theta-1}}$ and rewrite both FOCs:

$$\begin{aligned} 1 &= \psi_t X_t (1-\alpha)^{\frac{1}{\theta}} C_t^{T^{-\frac{1}{\theta}}} \\ Q_t &= \psi_t X_t \alpha^{\frac{1}{\theta}} C_t^{N^{-\frac{1}{\theta}}} \end{aligned}$$

From first one, get that:

$$\psi_t = X_t^{-1} (1-\alpha)^{-\frac{1}{\theta}} C_t^{T^{\frac{1}{\theta}}}$$

Plug in the second FOC:

$$\begin{aligned} Q_t &= \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\theta}} \left(\frac{C_t^T}{C_t^N}\right)^{\frac{1}{\theta}} \\ \frac{C_t^T}{C_t^N} &= \frac{1-\alpha}{\alpha} \left(\frac{1}{Q_t}\right)^{-\theta} \end{aligned}$$

The demand functions for C_t^T and C_t^N can be derived from the first-order conditions. Note that $X_t^\theta = C_t$. Take first FOC at the power of θ and use definition of C_t:

[11] We are thankful to Alexander Rathke for suggesting us that approach.

$$
\begin{aligned}
1 &= \psi_t^{\theta} X_t^{\theta} (1-\alpha) C_t^{T-1} \\
1 &= \psi_t^{\theta} C_t (1-\alpha) C_t^{T-1} \\
C_t^T &= (1-\alpha) \left(\frac{1}{\psi_t}\right)^{-\theta} C_t
\end{aligned}
$$

Do same with second FOC:

$$
\begin{aligned}
Q_t^{\theta} &= \psi_t^{\theta} X_t^{\theta} \alpha C_t^{N-1} \\
Q_t^{\theta} &= \psi_t^{\theta} C_t \alpha C_t^{N-1} \\
C_t^N &= \alpha \left(\frac{Q_t}{\psi_t}\right)^{-\theta} C_t
\end{aligned}
$$

Plugging into the definition of the consumption index (C_t) and get the shadow price:

$$
\begin{aligned}
C_t &= \left[(1-\alpha)^{\frac{1}{\theta}} \left((1-\alpha) \left(\frac{1}{\psi_t}\right)^{-\theta} C_t \right)^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} \left(\alpha \left(\frac{Q_t}{\psi_t}\right)^{-\theta} C_t \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\
C_t^{\frac{\theta-1}{\theta}} &= (1-\alpha)^{\frac{1}{\theta}} (1-\alpha)^{\frac{\theta-1}{\theta}} \psi_t^{\theta-1} C_t^{\frac{\theta-1}{\theta}} + \alpha^{\frac{1}{\theta}} \alpha^{\frac{\theta-1}{\theta}} Q_t^{1-\theta} \psi_t^{\theta-1} C_t^{\frac{\theta-1}{\theta}} \\
1 &= (1-\alpha) \psi_t^{\theta-1} + \alpha Q_t^{1-\theta} \psi_t^{\theta-1} \\
\psi_t^{1-\theta} &= (1-\alpha) + \alpha Q_t^{1-\theta} \\
\psi_t &= \left((1-\alpha) + \alpha Q_t^{1-\theta} \right)^{\frac{1}{1-\theta}} \\
&\equiv P_t
\end{aligned}
$$

Using $\psi_t = P_t$, optimal behavior also implies that minimum expenditures for C_t are given as

$$
\begin{aligned}
C_t^T + Q_t C_t^N = (1-\alpha) \left(\frac{1}{P_t}\right)^{-\theta} C_t + Q_t \alpha \left(\frac{Q_t}{P_t}\right)^{-\theta} C_t \\
C_t((1-\alpha) + \alpha Q_t^{1-\theta}) P_t^{\theta} \\
C_t P_t^{1-\theta} P_t^{\theta} = C_t P_t
\end{aligned}
$$

The optimization problem of the general consumption bundle and the Euler equation are the same as in the baseline case (B.3):

$$
\beta E_t \left[(1+r_{t+1}) \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{P_{t+1}}{P_t}\right)^{-1} \right] = 1
$$

Log-linearizing around the steady-state (or taking logs) as before:

$$
E_t(\Delta c_{t+1}) = \eta E_t(r_{t+1}) - \eta E_t(\Delta p_{t+1}) + const
$$

From this section, we know that the new consumption price index is

$$P_t = \left((1-\alpha) + \alpha Q_t^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

We log-linearize. The steady-state is:

$$\bar{P} = \left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{\frac{1}{1-\theta}}$$

Log-linearize the consumption-price ratio:

$$\begin{aligned}
P_t P_{t-1}^{-1} &= \left((1-\alpha) + \alpha Q_t^{1-\theta}\right)^{\frac{1}{1-\theta}} \left((1-\alpha) + \alpha Q_{t-1}^{1-\theta}\right)^{-\frac{1}{1-\theta}} \\
\bar{P}\hat{P}_t\bar{P}^{-1} + \bar{P}\hat{P}_{t-1}(-1)\bar{P}^{-2}\bar{P} &\approx \bar{Q}\hat{Q}_t \frac{1}{1-\theta} \left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{\frac{1}{1-\theta}-1-\frac{1}{1-\theta}} (1-\theta)\alpha\bar{Q}^{-\theta} \\
&+ \bar{Q}\hat{Q}_{t-1}\left(-\frac{1}{1-\theta}\right)\left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{-\frac{1}{1-\theta}-1+\frac{1}{1-\theta}} (1-\theta)\alpha\bar{Q}^{-\theta} \\
\hat{P}_t - \hat{P}_{t-1} &\approx \alpha\bar{Q}^{1-\theta}\left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{-1}(\hat{Q}_t - \hat{Q}_{t-1}) \\
\Delta\hat{P}_t &\approx \alpha\bar{Q}^{1-\theta}\left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{-1}\Delta\hat{Q}_t
\end{aligned}$$

As in the baseline case, because $\hat{X}_t = \log(X_t) - \log(\bar{X}) = \log(X_t/\bar{X}) \approx \frac{X_t - \bar{X}}{\bar{X}} = \frac{dX_t}{X}$, the steady-state variables disappear in first differences and one gets that:

$$\Delta p_t \approx \alpha\bar{Q}^{1-\theta}\left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{-1}\Delta q_t$$

This means that if $Q = 1$ then consumption price inflation becomes:

$$\Delta p_t = \alpha \Delta q_t$$

Thus, the coefficient of the intratemporal elasticity of substitution disappears and we are left with the same expression as in the baseline case (beware the change in notation of the share of non-tradables). Note that if $\theta = 1$, one finds the initial equation as well.

Rewrite the linear Euler equation in terms of consumption expenditure $\Delta c_{t+1} = \Delta\tilde{c}_{t+1} - \Delta p_{t+1}$ and use the preceding log-linearized inflation equation as in the baseline case:

$$\begin{aligned}
E_t\Delta\tilde{c}_{t+1} &= \eta E_t r_{t+1} - (\eta - 1)E_t\Delta p_{t+1} + const \\
E_t\Delta\tilde{c}_{t+1} &= \eta E_t r_{t+1} - (\eta - 1)\alpha\bar{Q}^{1-\theta}\left((1-\alpha) + \alpha \bar{Q}^{1-\theta}\right)^{-1} E_t\Delta q_{t+1} + const \\
E_t\Delta\tilde{c}_{t+1} &= \eta \left(E_t r_{t+1} + \frac{\alpha\frac{(1-\eta)}{\eta}\bar{Q}^{1-\theta}}{(1-\alpha) + \alpha\bar{Q}^{1-\theta}} E_t\Delta q_{t+1}\right) + const
\end{aligned}$$

B.2.1.5 The CES case (II)

Problem of the preceding approach: one has to make a statement about $\bar{Q}$. We could include it in the grid-search as $\bar{Q}^{(1-\theta)}$ or estimate it from the data but in the case of a fast growing economy like China,

it definitely would be more sensible to linearize not around a constant steady-state level but around a constant steady-state growth path of Q.
We start with the price ratio of the preceding section:

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \log\left(\left[\frac{(1-\alpha)+\alpha Q_t^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right]^{\frac{1}{1-\theta}}\right)$$

Define the growth rate of $Q = P_N/P_T$:

$$\begin{aligned}\frac{Q_t - Q_{t-1}}{Q_{t-1}} &\approx \log(Q_t) - \log(Q_{t-1}) = \Delta q_t \\ Q_t &\approx Q_{t-1}(1+\Delta q_t)\end{aligned}$$

Plug it in the first expression:

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \log\left(\left[\frac{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+\Delta q_t)^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right]^{\frac{1}{1-\theta}}\right)$$

By using a Taylor approximation of first order, we want to linearize this expression around $\Delta\bar{q}$. In the following, we will assume $\Delta\bar{q} = q \times t$. We approximate $f(\Delta q_t)$ at $\Delta\bar{q}$ (keep value flexible for the moment).

$$f(\Delta\bar{q}) + \frac{\partial f(\Delta\bar{q})}{\partial\Delta\bar{q}}(\Delta q_t - \Delta\bar{q})$$

One gets:

$$\begin{aligned}&\frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+\Delta\bar{q})^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right) + \frac{1}{1-\theta}\left(\frac{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+\Delta\bar{q})^{1-\theta}}\right) \\ &\qquad\left(\frac{\alpha Q_{t-1}^{1-\theta}(1-\theta)(1+\Delta\bar{q})^{-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right)(\Delta q_t - \Delta\bar{q})\end{aligned}$$

After simplifying:

$$\frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+\Delta\bar{q})^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right) + \left(\frac{\alpha Q_{t-1}^{1-\theta}(1+\Delta\bar{q})^{-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+\Delta\bar{q})^{1-\theta}}\right)(\Delta q_t - \Delta\bar{q})$$

Using the assumption of constant linear growth ($\Delta\bar{q} = q \times t$):

$$\log\left(\frac{P_t}{P_{t-1}}\right) \approx \frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+q\times t)^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right) + \left(\frac{\alpha Q_{t-1}^{1-\theta}(1+q\times t)^{-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}(1+q\times t)^{1-\theta}}\right)(\Delta q_t - q\times t)$$

Forward it to Δp_{t+1} and use in the final Bergin and Sheffrin equation:

$$
\begin{aligned}
E_t\Delta\tilde{c}_{t+1} &= \eta E_t r_{t+1} - (\eta-1)E_t\Delta p_{t+1} + const \\
&= \eta E_t r_{t+1} + (1-\eta)\times \\
&\quad E_t[\frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_t^{1-\theta}(1+q\times t)^{1-\theta}}{(1-\alpha)+\alpha Q_t^{1-\theta}}\right)+ \\
&\quad \left(\frac{\alpha Q_t^{1-\theta}(1+q\times t)^{-\theta}}{(1-\alpha)+\alpha Q_t^{1-\theta}(1+q\times t)^{1-\theta}}\right)(\Delta q_{t+1}-q\times t)]
\end{aligned}
$$

The preceding expressions certainly looks messy. If we approximate it around $\Delta\bar{q}=0$ (from the simplified expression) we get:

$$
\begin{aligned}
\log\left(\frac{P_t}{P_{t-1}}\right) &\approx \frac{1}{1-\theta}\log(1)+\frac{\alpha Q_{t-1}^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\Delta q_t \\
\Delta p_t &\approx \frac{\alpha Q_{t-1}^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\Delta q_t
\end{aligned}
$$

With:

$$
\begin{aligned}
\alpha \to 1 \quad & then \quad \Delta p_t \to \Delta q_t \\
Q_{t-1} \to 1 \quad & then \quad \Delta p_t \to \alpha\Delta q_t \\
Q_{t-1} \to 0 \quad & then \quad \Delta p_t \to 0 \\
Q_{t-1} \to \infty \quad & then \quad \Delta p_t \to \Delta q_t
\end{aligned}
$$

Note that if one assumes $Q_{t-1}=\bar{Q}$, one finds the same result as in the preceding section for Δp_t.

B.2.1.6 The CES case (III)

It is the same approach as before but we assume another functional growth form for Q. Note that $\exp(\log(1+\Delta q_l))\approx\exp(\Delta q_l)$ and that $\exp(x)\exp(y)=\exp(x+y)$

$$
\begin{aligned}
Q_t &= Q_0\prod_{l=1}^{t}(1+\Delta q_l) \\
&= Q_0\exp(\log(\prod_{l=1}^{t}(1+\Delta q_l))) \\
&= Q_0\exp(\sum_{l=1}^{t}\Delta q_l) \\
&= Q_0\prod_{l=1}^{t}\exp(\Delta q_l)
\end{aligned}
$$

and

$$Q_{t-1} = Q_0 \exp(\sum_{l=1}^{t-1} \Delta q_l)$$

For example:

$$\begin{aligned} Q_3 &= Q_0 \exp(\Delta q_1) \exp(\Delta q_2) \exp(\Delta q_3) \\ &= Q_0 \exp(\Delta q_1 + \Delta q_2 + \Delta q_3) \end{aligned}$$

For instance, if the steady-state is growing at rate $\Delta\bar{q}$, here in period 3:

$$\bar{Q}_3 = Q_0 \exp(3 \times \Delta\bar{q})$$

Thus, the general form is:

$$\bar{Q}_t = Q_0 \exp(t \times \Delta\bar{q})$$

We start from the development of relative prices over time found in the preceding sections:

$$\log\left(\frac{P_t}{P_{t-1}}\right) = \log\left(\left[\frac{(1-\alpha)+\alpha Q_t^{1-\theta}}{(1-\alpha)+\alpha Q_{t-1}^{1-\theta}}\right]^{\frac{1}{1-\theta}}\right)$$

Plug Q_t and Q_{t-1} in it:

$$\begin{aligned} \log\left(\frac{P_t}{P_{t-1}}\right) &= \log\left(\left[\frac{(1-\alpha)+\alpha Q_0^{1-\theta}\exp((1-\theta)\sum_{l=1}^{t}\Delta q_l)}{(1-\alpha)+\alpha Q_0^{1-\theta}\exp((1-\theta)\sum_{l=1}^{t-1}\Delta q_l)}\right]^{\frac{1}{1-\theta}}\right) \\ &= \frac{1}{1-\theta}\log\left((1-\alpha)+\alpha Q_0^{1-\theta}\exp((1-\theta)\sum_{l=1}^{t}\Delta q_l)\right) \\ &\quad -\frac{1}{1-\theta}\log\left((1-\alpha)+\alpha Q_0^{1-\theta}\exp((1-\theta)\sum_{l=1}^{t-1}\Delta q_l)\right) \end{aligned}$$

We want to elaborate on $\log(P_{t+k}/P_{t+k-1}) \approx \Delta p_{t+k}$:

$$\begin{aligned} \log\left(\frac{P_{t+k}}{P_{t+k-1}}\right) &= \log\left(\left[\frac{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k}\Delta q_{t+l})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k-1}\Delta q_{t+l})}\right]^{\frac{1}{1-\theta}}\right) \\ &= \frac{1}{1-\theta}\log\left((1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k}\Delta q_{t+l})\right) \\ &\quad -\frac{1}{1-\theta}\log\left((1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k-1}\Delta q_{t+l})\right) \end{aligned} \tag{B.5}$$

Let's stop and think this functional form through. It is a function of $\Delta q_{t+l}|_{l=1}$, $\Delta q_{t+l}|_{l=2}$, ..., $\Delta q_{t+l}|_{l=k-1}$, $\Delta q_{t+l}|_{l=k}$. It can be written as $f(\Delta q_{t+1}, \Delta q_{t+2}, ..., \Delta q_{t+l}, ..., \Delta q_{t+k})$.
If $k = 1$, then the function is

$$f(\Delta q_{t+1}) = \frac{1}{1-\theta} \log\left(\frac{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\Delta q_{t+1})}{(1-\alpha)}\right)$$

If $k = 2$, it is:

$$f(\Delta q_{t+1}, \Delta q_{t+2}) = \frac{1}{1-\theta} \log\left(\frac{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)(\Delta q_{t+1} + \Delta q_{t+2}))}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\Delta q_{t+1})}\right)$$

If $k = k$, it is:

$$f(\Delta q_{t+1}, \Delta q_{t+2}, ..., \Delta q_{t+l}, ..., \Delta q_{t+k}) =$$

$$\frac{1}{1-\theta} \log\left(\frac{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)(\Delta q_{t+1} + ... + \Delta q_{t+l} + ... + \Delta q_{t+k-1} + \Delta q_{t+k}))}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)(\Delta q_{t+1} + ... + \Delta q_{t+l} + ... + \Delta q_{t+k-1}))}\right)$$

As before, use a Taylor approximation of first order. We want to linearize this expression not around $\Delta\bar{q}$ but for each individual Δq_{t+l} at $\Delta\bar{q}$. We need $\sum_{l=1}^{t} \frac{\partial f()}{\Delta q_{t+l}}|_{\Delta q_{t+l}=\Delta\bar{q}} \times (\Delta q_{t+l} - \Delta\bar{q})$. The linearization is of the following form and we will get an approximation for each realized l:

$$f(\Delta\bar{q}) + \begin{pmatrix} \frac{\partial f()}{\partial \Delta q_{t+1}} \\ \frac{\partial f()}{\partial \Delta q_{t+2}} \\ ... \\ \frac{\partial f()}{\partial \Delta q_{t+l}} \\ ... \\ \frac{\partial f()}{\partial \Delta q_{t+k}} \end{pmatrix} \begin{pmatrix} |_{\Delta q_{t+1}=\Delta\bar{q}} \\ |_{\Delta q_{t+2}=\Delta\bar{q}} \\ ... \\ |_{\Delta q_{t+l}=\Delta\bar{q}} \\ ... \\ |_{\Delta q_{t+k}=\Delta\bar{q}} \end{pmatrix} \times \begin{pmatrix} \Delta q_{t+1} - \Delta\bar{q} \\ \Delta q_{t+2} - \Delta\bar{q} \\ ... \\ \Delta q_{t+l} - \Delta\bar{q} \\ ... \\ \Delta q_{t+k} - \Delta\bar{q} \end{pmatrix}$$

The $f(\Delta\bar{q})$ term is:

$$\frac{1}{1-\theta} \log\left(\frac{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k} \Delta\bar{q})}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k-1} \Delta\bar{q})}\right) =$$
$$\frac{1}{1-\theta} \log\left(\frac{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)k \times \Delta\bar{q})}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)(k-1) \times \Delta\bar{q})}\right)$$

Get the expression for $\frac{\partial f()}{\partial \Delta q_{t+1}}$ using the log expression (B.5):

$$\frac{1}{1-\theta} \frac{\alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k} \Delta q_{t+l})(1-\theta)}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k} \Delta q_{t+l})} - \frac{1}{1-\theta} \frac{\alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k-1} \Delta q_{t+l})(1-\theta)}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k-1} \Delta q_{t+l})}$$
$$= \frac{\alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k} \Delta q_{t+l})}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k} \Delta q_{t+l})} - \frac{\alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k-1} \Delta q_{t+l})}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)\sum_{l=1}^{k-1} \Delta q_{t+l})}$$

Realize that $\frac{\partial f()}{\partial \Delta q_{t+2}}$ and the following ones until $k-1$ are similar.
Note that $\frac{\partial f()}{\partial \Delta q_{t+k}}$ is different because $t+k$ only shows up in the numerator (i.e. in the first part of the log expression B.5):

$$\begin{aligned}
&\frac{1}{1-\theta}\frac{\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k}\Delta q_{t+l})(1-\theta)}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k}\Delta q_{t+l})} \\
&= \frac{\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k}\Delta q_{t+l})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\sum_{l=1}^{k}\Delta q_{t+l})}
\end{aligned}$$

We evaluate all $t+l$s at $\Delta\bar{q}$.
For $l=1,...,k-1$ one gets:

$$\frac{\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})} - \frac{\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\Delta\bar{q})}$$

For $l=k$:

$$\frac{\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}$$

The Taylor approximation is then:

$$\begin{aligned}
\Delta p_{t+k} \approx \frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)k\times\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\times\Delta\bar{q})}\right) \\
+\sum_{l=1}^{k-1}\left(\frac{\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})} - \frac{\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\Delta\bar{q})}\right) \quad \times(\Delta q_{t+l}-\Delta\bar{q}) \\
+\frac{\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})} \quad \times(\Delta q_{t+k}-\Delta\bar{q})
\end{aligned}$$

For convenience, rewrite the approximation using:

$$b(t,k) = \alpha Q_t^{1-\theta}\exp((1-\theta)k\times\Delta\bar{q})$$

We get:

$$\begin{aligned}
\Delta p_{t+k} \approx \frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+b(t,k)}{(1-\alpha)+b(t,k-1)}\right) \\
+\sum_{l=1}^{k-1}\left(\frac{b(t,k)}{(1-\alpha)+b(t,k)} - \frac{b(t,k-1)}{(1-\alpha)+b(t,k-1)}\right) \quad \times(\Delta q_{t+l}-\Delta\bar{q}) \\
+\frac{b(t,k)}{(1-\alpha)+b(t,k)} \quad \times(\Delta q_{t+k}-\Delta\bar{q})
\end{aligned}$$

Define

$$\begin{aligned}\Psi(t,k) &= \frac{b(t,k)}{(1-\alpha)+b(t,k)} \\ &= \frac{\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)k\Delta\bar{q})}\end{aligned}$$

So that one can rewrite the approximation as:

$$\begin{aligned}\Delta p_{t+k} \approx \frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+b(t,k)}{(1-\alpha)+b(t,k-1)}\right) \\ +\sum_{l=1}^{k-1}\Psi(t,k)\left(1-\frac{b(t,k-1)}{(1-\alpha)+b(t,k-1)}\frac{(1-\alpha)+b(t,k)}{b(t,k)}\right) \quad \times(\Delta q_{t+l}-\Delta\bar{q}) \\ +\Psi(t,k) \quad \times(\Delta q_{t+k}-\Delta\bar{q})\end{aligned}$$

Focus on the middle term:

$$\begin{aligned}\frac{b(t,k-1)}{(1-\alpha)+b(t,k-1)}\frac{(1-\alpha)+b(t,k)}{b(t,k)} &= \\ \frac{(1-\alpha)b(t,k-1)+b(t,k-1)b(t,k)}{(1-\alpha)b(t,k)+b(t,k-1)b(t,k)} &= \\ \frac{(1-\alpha)b(t,k-1)/b(t,k)+b(t,k-1)}{(1-\alpha)+b(t,k-1)}\end{aligned}$$

Focus on $b(t,k-1)/b(t,k)$:

$$\begin{aligned}\frac{b(t,k-1)}{b(t,k)} &= \frac{\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\times\Delta\bar{q})}{\alpha Q_t^{1-\theta}\exp((1-\theta)k\times\Delta\bar{q})} \\ &= \exp((1-\theta)(k-1)\Delta\bar{q}-(1-\theta)k\Delta\bar{q}) \\ &= \exp((1-\theta)(-\Delta\bar{q}))\end{aligned}$$

One gets:

$$\begin{aligned}\frac{(1-\alpha)b(t,k-1)/b(t,k)+b(t,k-1)}{(1-\alpha)+b(t,k-1)} &= \frac{(1-\alpha)\exp((1-\theta)(-\Delta\bar{q}))+b(t,k-1)}{(1-\alpha)+b(t,k-1)} \\ &= \frac{(1-\alpha)\exp((1-\theta)(-\Delta\bar{q}))+\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\times\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)(k-1)\times\Delta\bar{q})}\end{aligned}$$

Our new expression for Δp_{t+k} is:

$$\Delta p_{t+k} \approx \frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+b(t,k)}{(1-\alpha)+b(t,k-1)}\right)$$
$$+\sum_{l=1}^{k-1}\Psi(t,k)\left(1-\frac{(1-\alpha)\exp((1-\theta)(-\Delta\bar{q}))+b(t,k-1)}{(1-\alpha)+b(t,k-1)}\right) \quad \times(\Delta q_{t+l}-\Delta\bar{q})$$
$$+\Psi(t,k) \quad \times(\Delta q_{t+k}-\Delta\bar{q})$$

The expression for Δp_{t+1} simplifies to:

$$\begin{aligned}\Delta p_{t+1} &\approx \frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\times\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}}\right)\\ &+ \frac{\alpha Q_t^{1-\theta}\exp((1-\theta)\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\Delta\bar{q})}\times(\Delta q_{t+1}-\Delta\bar{q})\end{aligned}$$

Rewrite the linear Euler equation in terms of consumption expenditure $\Delta c_{t+1}=\Delta\tilde{c}_{t+1}-\Delta p_{t+1}$ and use the preceding log-linearized inflation equation as in the baseline case:

$$\begin{aligned}E_t\Delta\tilde{c}_{t+1} &= \eta E_t r_{t+1}-(\eta-1)E_t\Delta p_{t+1}+const\\ &= \eta\left(E_t r_{t+1}+\frac{1-\eta}{\eta}E_t\Delta p_{t+1}\right)+const\end{aligned}$$

Note that the first part of Δp_{t+1} does not become part of the constant, it is varying over time:

$$\begin{aligned}E_t\Delta\tilde{c}_{t+1} &= \eta[E_t r_{t+1}+\frac{1-\eta}{\eta}\frac{1}{1-\theta}\log\left(\frac{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\times\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}}\right)\\ &+\frac{1-\eta}{\eta}\frac{\alpha Q_t^{1-\theta}\exp((1-\theta)\Delta\bar{q})}{(1-\alpha)+\alpha Q_t^{1-\theta}\exp((1-\theta)\Delta\bar{q})}(E_t\Delta q_{t+1}-\Delta\bar{q})]+const\end{aligned}$$

Now one can be more general:

$$\begin{aligned}E_t\Delta\tilde{c}_{t+k} &= \eta\left(Er_{t+k}+\frac{1-\eta}{\eta}E_t\Delta p_{t+k}\right)+const\\ &= \frac{1}{\gamma}E_t r_{t+k}+\frac{\gamma-1}{\gamma}E_t\Delta p_{t+k}+const\end{aligned}$$

It becomes:

$$
\begin{aligned}
E_t \Delta \tilde{c}_{t+k} &= \frac{1}{\gamma} E_t r_{t+k} \\
&+ \frac{\gamma-1}{\gamma} \frac{1}{1-\theta} \log \left(\frac{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)k \times \Delta \bar{q})}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)(k-1) \times \Delta \bar{q})} \right) \\
&+ \frac{\gamma-1}{\gamma} \sum_{l=1}^{k-1} \Psi(t,k) \left(1 - \frac{(1-\alpha)\exp((1-\theta)(-\Delta \bar{q})) + \alpha Q_t^{1-\theta} \exp((1-\theta)(k-1) \times \Delta \bar{q})}{(1-\alpha) + \alpha Q_t^{1-\theta} \exp((1-\theta)(k-1) \times \Delta \bar{q})} \right) \\
&\quad \times (E_t \Delta q_{t+l} - \Delta \bar{q}) \\
&+ \frac{\gamma-1}{\gamma} \Psi(t,k) \times (E_t \Delta q_{t+k} - \Delta \bar{q}) \\
&+ \mathit{const}
\end{aligned}
$$

B.2.2 Kano's log-linearization with saving wedge

B.2.2.1 Rearranging the budget constraint

The budget constraint is:

$$
Y_t - G_t - I_t - C_t = B_{t+1} - (1 + r_t^T) B_t
$$

We introduce a saving wedge:

$$
1 + r_t^T = (1 + r_t^w)(1 - \tau_t^s)
$$

The new budget constraint with saving wedge is:

$$
Y_t - G_t - I_t - C_t = B_{t+1} - (1 + r_t^w)(1 - \tau_t^s) B_t
$$

Solve forward to find the ex ante intertemporal budget constraint:

$$
\begin{aligned}
B_t &= E_t \left[((1 + r_t^w)(1 - \tau_t^s))^{-1} (B_{t+1} + C_t - NO_t) \right] \\
B_{t+1} &= E_{t+1} \left[\left((1 + r_{t+1}^w)(1 - \tau_{t+1}^s) \right)^{-1} (B_{t+2} + C_{t+1} - NO_{t+1}) \right] \\
B_{t+2} &= E_{t+2} \left[\left((1 + r_{t+2}^w)(1 - \tau_{t+2}^s) \right)^{-1} (B_{t+3} + C_{t+2} - NO_{t+2}) \right] \\
B_{t+3} &= \dots
\end{aligned}
$$

$$
\begin{aligned}
B_t &= E_t \left[((1 + r_t^w)(1 - \tau_t^s))^{-1} \left(E_{t+1} \left[\left((1 + r_{t+1}^w)(1 - \tau_{t+1}^s) \right)^{-1} (B_{t+2} - NO_{t+1} + C_{t+1}) \right] + C_t - NO_t \right) \right] \\
&= E_t[((1 + r_t^w)(1 - \tau_t^s))^{-1} \left((1 + r_{t+1}^w)(1 - \tau_{t+1}^s) \right)^{-1} B_{t+2}] + E_t[((1 + r_t^w)(1 - \tau_t^s))^{-1} \\
&\quad \left((1 + r_{t+1}^w)(1 - \tau_{t+1}^s) \right)^{-1} (C_{t+1} - NO_{t+1})] + E_t \left[((1 + r_t^w)(1 - \tau_t^s))^{-1} (C_t - NO_t) \right]
\end{aligned}
$$

$$
\lim_{k \to \infty} \left(E_t \prod_{l=0}^{k} \left((1 + r_{t+l}^w)(1 - \tau_{t+l}^s) \right)^{-1} B_{t+k+1} \right) = 0
$$

$$
\begin{aligned}
B_t &= \sum_{k=0}^{\infty} E_t \left\{ (C_{t+k} - NO_{t+k}) \prod_{l=0}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)^{-1} \right\} \\
R_{t+k} &= \prod_{l=0}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)
\end{aligned}
$$

Take B_t, divide it by NO_t and expand the first part with C_t:

$$
\frac{B_t}{NO_t} = \sum_{k=0}^{\infty} E_t \left\{ R_{t+k}^{-1} \left(\frac{C_t}{NO_t} \frac{C_{t+k}}{C_t} - \frac{NO_{t+k}}{NO_t} \right) \right\}
$$

Take this expression for $k = 0$ and $k \geq 1$:

$$
\begin{aligned}
\frac{B_t}{NO_t} &= E_t \left\{ ((1+r_t^w)(1-\tau_t^s))^{-1} \left[\frac{C_t}{NO_t} \frac{C_t}{C_t} - \frac{NO_t}{NO_t} \right] \right\} + \sum_{k=1}^{\infty} E_t \left\{ R_{t+k}^{-1} \left[\frac{C_t}{NO_t} \frac{C_{t+k}}{C_t} - \frac{NO_{t+k}}{NO_t} \right] \right\} \\
(1+r_t^w)(1-\tau_t^s) \frac{B_t}{NO_t} &= \frac{C_t}{NO_t} - 1 + (1+r_t^w)(1-\tau_t^s) \times \\
&\quad \sum_{k=1}^{\infty} E_t \left\{ \prod_{l=0}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)^{-1} \left[\frac{C_t}{NO_t} \frac{C_{t+k}}{C_t} - \frac{NO_{t+k}}{NO_t} \right] \right\} \\
(1+r_t^w)(1-\tau_t^s) \frac{B_t}{NO_t} &= \frac{C_t}{NO_t} - 1 + \sum_{k=1}^{\infty} E_t \left\{ \prod_{l=1}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)^{-1} \left[\frac{C_t}{NO_t} \frac{C_{t+k}}{C_t} - \frac{NO_{t+k}}{NO_t} \right] \right\} \\
&= \frac{C_t}{NO_t} E_t \left[1 + \sum_{k=1}^{\infty} \prod_{l=1}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)^{-1} \frac{C_{t+k}}{C_t} \right] \\
&\quad - E_t \left[1 + \sum_{k=1}^{\infty} \prod_{l=1}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)^{-1} \frac{NO_{t+k}}{NO_t} \right]
\end{aligned}
$$

The next step is more tricky. Use $\exp(\log(\ldots))$ and rewrite k using l. Here we show it for the consumption term. The strategy for the net output term is similar. Remember that by using a Taylor approximation one can show that for small x, $\log(1+x) \approx x$ and $\log(1-x) \approx -x$:

$$
\sum_{k=1}^{\infty} \exp \left(\log \left(\prod_{l=1}^{k} \left((1+r_{t+l}^w)(1-\tau_{t+l}^s) \right)^{-1} \frac{C_{t+k}}{C_t} \right) \right)
$$

$$
\sum_{k=1}^{\infty} \exp \left(\sum_{l=1}^{k} \left[\log(C_{t+k}) - \log(C_t) - \log(1+r_{t+l}^w) - \log(1-\tau_{t+l}^s) \right] \right)
$$

$$
\begin{aligned}
k = 1 &\Rightarrow \exp \left(\log(C_{t+1}) - \log(C_t) - r_{t+1}^w + \tau_{t+1}^s \right) \\
k = 2 &\Rightarrow \exp \left(\log(C_{t+2}) - \log(C_t) - r_{t+1}^w + \tau_{t+1}^s - r_{t+2}^w + \tau_{t+2}^s \right) \\
k = 3 &\Rightarrow \ldots
\end{aligned}
$$

Note that it can be rewritten using Δ for first difference because some terms disappear:

$$
\sum_{k=1}^{\infty} \exp \left(\sum_{l=1}^{k} \left[\Delta c_{t+l} - r_{t+l}^w + \tau_{t+l}^s \right] \right)
$$

$$
\begin{aligned}
k=1 \quad &\Rightarrow \quad \exp(\Delta c_{t+1} - r^w_{t+1} + \tau^s_{t+1}) \\
& \quad \exp(\log(C_{t+1}) - \log(C_t) - r^w_{t+1} + \tau^s_{t+1}) \\
k=2 \quad &\Rightarrow \quad \exp(\Delta c_{t+1} - r^w_{t+1} + \tau^s_{t+1} + \Delta c_{t+2} - r^w_{t+2} + \tau^s_{t+2}) \\
& \quad \exp(\log(C_{t+1}) - \log(C_t) - r^w_{t+1} + \tau^s_{t+1} + \log(C_{t+2}) - \log(C_{t+1}) - r^w_{t+2} + \tau^s_{t+2}) \\
& \quad \exp(\log(C_{t+2}) - \log(C_t) - r^w_{t+1} + \tau^s_{t+1} - r^w_{t+2} + \tau^s_{t+2}) \\
k=3 \quad &\Rightarrow \quad \dots
\end{aligned}
$$

Thus, we can rewrite the RHS of the intertemporal BC as:

$$
\frac{C_t}{NO_t} E_t \left[1 + \sum_{k=1}^{\infty} \exp\left(\sum_{l=1}^{k} \left[\Delta c_{t+l} - r^w_{t+l} + \tau^s_{t+l} \right] \right) \right] - E_t \left[1 + \sum_{k=1}^{\infty} \exp\left(\sum_{l=1}^{k} \left[\Delta no_{t+l} - r^w_{t+l} + \tau^s_{t+l} \right] \right) \right] \quad \text{(B.6)}
$$

The LHS can be rewritten using $\exp(\log(\dots))$ as well:

$$
\begin{aligned}
&\exp\left(\log\left((1 + r^w_t)(1 - \tau^s_t) \frac{NO_{t-1}}{NO_t} \frac{B_t}{NO_{t-1}} \right) \right) \\
&\exp\left(r^w_t - \tau^s_t - \Delta no_t + \log(\frac{B_t}{NO_{t-1}}) \right) \\
&\exp\left(r^w_t - \tau^s_t - \Delta no_t \right) \frac{B_t}{NO_{t-1}} \qquad \text{(B.7)}
\end{aligned}
$$

B.2.2.2 Warm-up

At this point, a warm-up is necessary.
Remember the log-linearization using an implicit function $f(\bar{x}, \bar{y}) = 0$. By implicit differentiation get:

$$
\frac{\partial f(\bar{x}, \bar{y})}{\partial x} dx + \frac{\partial f(\bar{x}, \bar{y})}{\partial y} dy = 0
$$

Now observe that $\frac{dx}{\bar{x}} = \frac{x - \bar{x}}{\bar{x}} \approx \log(x) - \log(\bar{x}) = \tilde{x}$.

$$
\begin{aligned}
\frac{\partial f(\bar{x}, \bar{y})}{\partial x} \frac{dx}{\bar{x}} \bar{x} + \frac{\partial f(\bar{x}, \bar{y})}{\partial y} \frac{dy}{\bar{y}} \bar{y} &= 0 \\
\frac{\partial f(\bar{x}, \bar{y})}{\partial x} \tilde{x}\bar{x} + \frac{\partial f(\bar{x}, \bar{y})}{\partial y} \tilde{y}\bar{y} &= 0
\end{aligned}
$$

Alternatively, one could take the log of the expression and then directly linearize using a Taylor approximation: if have a function $f(x)$ evaluated at point a (in our case the steady-state), the general formula is:

$$
\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots
$$

With a function $f(x,y)$ evaluated at steady-states $\bar{x}$ and $\bar{y}$, the first order approximation is:

$$
\begin{aligned}
f(\bar{x},\bar{y}) + \frac{\partial f(\bar{x},\bar{y})}{\partial \bar{x}}(x-\bar{x}) + \frac{\partial f(\bar{x},\bar{y})}{\partial \bar{y}}(y-\bar{y}) \\
f(\bar{x},\bar{y}) + \frac{\partial f(\bar{x},\bar{y})}{\partial \bar{x}}\tilde{x} + \frac{\partial f(\bar{x},\bar{y})}{\partial \bar{y}}\tilde{y}
\end{aligned}
$$

Compared to the implicit function case, we have a constant and we do not need to multiply with the steady state value. Thereafter, we mostly take exp(log(...)). Thus, one does not need to log-linearize but only linearize. Ignoring the constant, one could use the expression:

$$
\frac{\partial f(\bar{x},\bar{y})}{\partial x}\tilde{x} + \frac{\partial f(\bar{x},\bar{y})}{\partial y}\tilde{y}
$$

Before beginning with the log-linearization, observe that, using the implicit theorem:

$$
\begin{aligned}
\exp(\log(1+r_t)) \\
\exp(\log(\hat{r}_t(1+\bar{r}))) \\
\exp(\log(\hat{r}_t) + \log(1+\bar{r})) \\
\exp(\log(\hat{r}_t))\exp(\log(1+\bar{r})) \\
\tilde{r}_t \exp(\bar{r})
\end{aligned}
$$

Alternatively, one could take log and use Taylor expansion::

$$
\begin{aligned}
\exp(\log(1+r_t)) \quad &\approx \quad \exp(r_t) \\
\exp(\bar{r}) + \exp(\bar{r})\tilde{r}_t
\end{aligned}
$$

See that the difference is the constant.
Use implicit function:

$$
\begin{aligned}
X_t X_{t-1}^{-1} \\
\tilde{X}_t \bar{X}\bar{X}^{-1} + \tilde{X}_{t-1}\bar{X}\bar{X}\bar{X}^{-2}(-1) \\
\tilde{X}_t - \tilde{X}_{t-1} \quad &= \quad (X_t - \bar{X}) - (X_{t-1} - \bar{X}) \\
X_t - X_{t-1}
\end{aligned}
$$

Thus, the log difference of a variable (e.g. growth rate) is identical to its the log difference of the deviation from its steady-state.
Another example. Use Taylor approximation:

$$
\begin{aligned}
\exp(\log(C_{t+1}/C_t) \quad) = \quad &\exp(\Delta c_{t+1}) \\
\exp(\bar{\Delta c}) + \exp(\bar{\Delta c})\Delta\tilde{c}_{t+1}
\end{aligned}
$$

The key to understanding the following log-linearizations is:

$$\begin{aligned}
&\exp(\log\left[\left((1+r^w_{t+1})(1-\tau^s_{t+1})\right)\frac{C_{t+1}}{C_t}\right])\\
&\exp(r^w_{t+1}-\tau^s_{t+1}+\Delta c_{t+1})\\
&\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})+\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})\tilde{r}_{t+1}-\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})\tilde{\tau}_{t+1}+\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})\Delta\tilde{c}_{t+1}\\
&\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})+\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})(\tilde{r}_{t+1}-\tilde{\tau}_{t+1}+\Delta\tilde{c}_{t+1})\\
&\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})+\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})\left((r^w_{t+1}-\bar{r})-(\tau^s_{t+1}-\bar{\tau})+(c_{t+1}-\bar{c})-(c_t-\bar{c})\right)\\
&\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})(1-\bar{r}+\bar{\tau})+\exp(\bar{r}-\bar{\tau}+\bar{\Delta c})(r^w_{t+1}-\tau^s_{t+1}+\Delta c_{t+1})
\end{aligned}$$

B.2.2.3 Log-linearization

Let us start with the log-linearization of the LHS (B.7). We use a Taylor approximation of first order and ignore the constant. Observe that $\exp(x-y)=\exp(y-x)^{-1}$:

$$\begin{aligned}
&\exp(r^w_t-\tau^s_t-\Delta no_t)\frac{B_t}{NO_{t-1}}\\
&\exp(r^w_t-\tau^s_t-\Delta no_t)B^*\\
&\tilde{r}_t\exp(\bar{r}-\bar{\tau}-\Delta\bar{no})\bar{B}^*-\tilde{\tau}_t\exp(\bar{r}-\bar{\tau}-\Delta\bar{no})\bar{B}^*-\Delta\tilde{no}_t\exp(\bar{r}-\bar{\tau}-\Delta\bar{no})\bar{B}^*+\exp(\bar{r}-\bar{\tau}-\Delta\bar{no})\tilde{B}^*\\
&(\tilde{r}_t-\tilde{\tau}_t-\Delta\tilde{no})\exp(\bar{r}-\bar{\tau}-\Delta\bar{no})\bar{B}^*+\tilde{B}^*\exp(\bar{r}-\bar{\tau}-\Delta\bar{no})\\
&(\tilde{r}_t-\tilde{\tau}_t-\Delta\tilde{no})\bar{B}^*\frac{1}{\exp\left(\Delta\bar{no}-\bar{r}+\bar{\tau}\right)}+\tilde{B}^*\frac{1}{\exp\left(\Delta\bar{no}-\bar{r}+\bar{\tau}\right)} \qquad \text{(B.8)}
\end{aligned}$$

Now we turn to the RHS (B.6). We have to rewrite both $1+\exp()$ terms (here with the first block of the RHS, the consumption part). We start without the constant with:

$$E_t\left[\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}\left[\Delta c_{t+l}-r^w_{t+l}+\tau^s_{t+l}\right]\right)\right]$$

Write it for $k=1$, $k=2$, $k=3$, ...:

$$\begin{aligned}
k=1 &\quad\Rightarrow\quad \exp(\Delta c_{t+1}-r^w_{t+1}+\tau^s_{t+1})\\
k=2 &\quad\Rightarrow\quad \exp(\Delta c_{t+1}-r^w_{t+1}+\tau^s_{t+1}+\Delta c_{t+2}-r^w_{t+2}+\tau^s_{t+2})\\
k=3 &\quad\Rightarrow\quad \exp(\Delta c_{t+1}-r^w_{t+1}+\tau^s_{t+1}+\Delta c_{t+2}-r^w_{t+2}+\tau^s_{t+2}+\Delta c_{t+3}-r^w_{t+3}+\tau^s_{t+3})\\
&\quad\quad \ldots
\end{aligned}$$

Which gives:

$$\begin{aligned}
&\exp(\Delta c_{t+1}-r^w_{t+1}+\tau^s_{t+1})+\exp(\Delta c_{t+1}-r^w_{t+1}+\tau^s_{t+1})\exp(\Delta c_{t+2}-r^w_{t+2}+\tau^s_{t+2})+\\
&\quad\exp(\Delta c_{t+1}-r^w_{t+1}+\tau^s_{t+1})\exp(\Delta c_{t+2}-r^w_{t+2}+\tau^s_{t+2})\exp(\Delta c_{t+3}-r^w_{t+3}+\tau^s_{t+3})\ldots
\end{aligned}$$

Linearizing gives:

$$(\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1})\exp(\bar{\Delta c} - \bar{r} + \bar{\tau}) + (\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1})\exp(2(\bar{\Delta c} - \bar{r} + \bar{\tau})) +$$
$$(\Delta\tilde{c}_{t+2} - \tilde{r}_{t+2} + \tilde{\tau}_{t+2})\exp(2(\bar{\Delta c} - \bar{r} + \bar{\tau})) + (\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1})\exp(3(\bar{\Delta c} - \bar{r} + \bar{\tau})) +$$
$$(\Delta\tilde{c}_{t+2} - \tilde{r}_{t+2} + \tilde{\tau}_{t+2})\exp(3(\bar{\Delta c} - \bar{r} + \bar{\tau})) + (\Delta\tilde{c}_{t+3} - \tilde{r}_{t+3} + \tilde{\tau}_{t+3})\exp(3(\bar{\Delta c} - \bar{r} + \bar{\tau})) + \dots$$

The trick is the following: for $k=1$ we have $l=1$, for $k=2$ $l=1,2$, for $k=3$ $l=1,2,3$ and so on and so forth. Thus, if one gathers all k terms for $l=1$, one gets $\sum_{k=1}^{\infty}$. Similarly, for $l=2$ one has $\sum_{k=2}^{\infty}$ and for $l=3$, $\sum_{k=3}^{\infty}$.

$$\begin{aligned} E_t\left[1+\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}\left[\Delta c_{t+l} - r^w_{t+l} + \tau^s_{t+l}\right]\right)\right] \quad \Rightarrow \quad & const \\ & + \sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}(\bar{\Delta c} - \bar{r} + \bar{\tau})\right)(\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1}) \\ & + \sum_{k=2}^{\infty}\exp\left(\sum_{l=1}^{k}(\bar{\Delta c} - \bar{r} + \bar{\tau})\right)(\Delta\tilde{c}_{t+2} - \tilde{r}_{t+2} + \tilde{\tau}_{t+2}) \\ & + \dots \end{aligned}$$

Focus on the first row. Distinguish between the first k of the series (here $k=1$) and the following ones. We can rewrite it as:

$$\begin{aligned} \exp(\Delta\bar{c} - \bar{r} + \bar{\tau})(\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1}) \quad + \\ \exp(\Delta\bar{c} - \bar{r} + \bar{\tau})(\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1})\sum_{k=1}^{\infty}\exp(\sum_{l=1}^{k}(\Delta\bar{c} - \bar{r} + \bar{\tau})) \end{aligned}$$

Do the same for the second row. Distinguish between the first k of the series (here $k=2$) and the following ones. We can rewrite it as:

$$\begin{aligned} \exp(2(\Delta\bar{c} - \bar{r} + \bar{\tau}))(\Delta\tilde{c}_{t+2} - \tilde{r}_{t+2} + \tilde{\tau}_{t+2}) \quad + \\ \exp(2(\Delta\bar{c} - \bar{r} + \bar{\tau}))(\Delta\tilde{c}_{t+2} - \tilde{r}_{t+2} + \tilde{\tau}_{t+2})\sum_{k=1}^{\infty}\exp(\sum_{l=1}^{k}(\Delta\bar{c} - \bar{r} + \bar{\tau})) \end{aligned}$$

We can rewrite the (initial) first and second rows as:

$$\begin{aligned} = \quad & \left[1+\sum_{k=1}^{\infty}\exp(\sum_{l=1}^{k}(\Delta\bar{c} - \bar{r} + \bar{\tau}))\right] \\ & [\exp(\Delta\bar{c} - \bar{r} + \bar{\tau})(\Delta\tilde{c}_{t+1} - \tilde{r}_{t+1} + \tilde{\tau}_{t+1}) + \exp(2(\Delta\bar{c} - \bar{r} + \bar{\tau}))(\Delta\tilde{c}_{t+2} - \tilde{r}_{t+2} - + \tilde{\tau}_{t+2})] \end{aligned}$$

Focus on the second term of the first parenthesis:

$$\sum_{k=1}^{\infty}\exp(\sum_{l=1}^{k}(\bar{\Delta c}-\bar{r}+\bar{\tau}))$$

$$\begin{aligned} k=1 &\Rightarrow e^{\bar{\Delta c}-\bar{r}+\bar{\tau}} \\ k=2 &\Rightarrow e^{2(\bar{\Delta c}-\bar{r}+\bar{\tau})} \\ k=3 &\Rightarrow e^{3(\bar{\Delta c}-\bar{r}+\bar{\tau})} \\ &\dots \end{aligned}$$

If one defines $x=\Delta\bar{c}-\bar{r}-\bar{\tau}$, it is obvious that the preceding expression is of the form:

$$\begin{aligned} e^x+(e^x)^2+(e^x)^3 &+ \dots \\ \sum_{k=1}^{\infty}(e^x)^k &= \frac{1}{1-e^x}-1 \end{aligned}$$

The result is that:

$$1+\sum_{k=1}^{\infty}\exp(\sum_{l=1}^{k}(\Delta\bar{c}-\bar{r}+\bar{\tau}))=1+\frac{1}{1-\exp(\Delta\bar{c}-\bar{r}+\bar{\tau})}-1$$

Thus the final result for the consumption part is:

$$\begin{aligned} E_t\left[1+\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}\left[\Delta\tilde{c}_{t+l}-\tilde{r}_{t+l}+\tilde{\tau}_{t+l}\right]\right)\right] \Rightarrow \\ const+\frac{1}{1-\exp(\Delta\bar{c}-\bar{r}+\bar{\tau})}E_t\sum_{k=1}^{\infty}\exp(k(\Delta\bar{c}-\bar{r}+\bar{\tau}))(\Delta\tilde{c}_{t+k}-\tilde{r}_{t+k}+\tilde{\tau}_{t+k})) \end{aligned} \tag{B.9}$$

Now go back to the entire expression for the right hand side:

$$\frac{C_t}{NO_t}E_t\left[1+\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}\left[\Delta c_{t+l}-r^w_{t+l}+\tau^s_{t+l}\right]\right)\right]-E_t\left[1+\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}\left[\Delta no_{t+l}-r^w_{t+l}+\tau^s_{t+l}\right]\right)\right]$$

Focus on the terms one after another:

$$\begin{aligned} E_t\left(\frac{C_t}{NO_t}\right) &\Rightarrow \tilde{C}_t\bar{C}\frac{1}{\bar{NO}}+\tilde{NO}_t\bar{NO}(-1)\frac{\bar{C}}{\bar{NO}^2} \\ &\quad \tilde{C}_t\frac{\bar{C}}{\bar{NO}}-\tilde{NO}_t\frac{\bar{C}}{\bar{NO}} \\ &\quad \left(\tilde{C}_t-\tilde{NO}_t\right)\frac{\bar{C}}{\bar{NO}} \end{aligned}$$

If we consider C_t/NO_t as a block variable C^*, the log-linearization is:

$$E_t\left(C^*\right) \Rightarrow \tilde{C}^*\bar{C}^*$$

Note we just linearize the expression (i.e. the steady-state term is not needed).
This term is multiplied with the expectation of $1+\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}\left[\Delta c_{t+l}-r_{t+l}^{w}+\tau_{t+l}^{s}\right]\right)$:

$$\begin{aligned}
1+\sum_{k=1}^{\infty}\exp\left(\sum_{l=1}^{k}[\Delta\bar{c}-\bar{r}+\bar{\tau}]\right)\\
1+\sum_{k=1}^{\infty}\exp(k[\Delta\bar{c}-\bar{r}+\bar{\tau}])\\
1+e^{(\Delta\bar{c}-\bar{r}+\bar{\tau})}+(e^{(\Delta\bar{c}-\bar{r}+\bar{\tau})})^2+(e^{(\Delta\bar{c}-\bar{r}+\bar{\tau})})^3+\dots\\
\frac{1}{1-\exp(\Delta\bar{c}-\bar{r}+\bar{\tau})}
\end{aligned}$$

In the end, one gets:

$$\tilde{C}^*\frac{1}{1-\exp(\Delta\bar{c}-\bar{r}+\bar{\tau})} \tag{B.10}$$

The second and third parts should be easier to understand as they are based on the preceding thoughts. By using (B.8), (B.9) and (B.10) we get the linearized expression for (B.6) and (B.7):

$$\begin{aligned}
(\tilde{r}_t-\tilde{\tau}_t-\Delta\tilde{no})\bar{B}^*\frac{1}{\exp\left(\Delta\bar{no}-\bar{r}+\bar{\tau}\right)}+\tilde{B}^*\frac{1}{\exp\left(\Delta\bar{no}-\bar{r}+\bar{\tau}\right)} &\approx\\
\tilde{C}^*\frac{1}{1-\exp(\Delta\bar{c}-\bar{r}+\bar{\tau})} &+\\
\bar{C}^*\frac{1}{1-\exp(\Delta\bar{c}-\bar{r}+\bar{\tau})}E_t\sum_{k=1}^{\infty}\exp(k(\Delta\bar{c}-\bar{r}+\bar{\tau}))(\Delta\tilde{c}_{t+k}-\tilde{r}_{t+k}+\tilde{\tau}_{t+k})) &-\\
\frac{1}{1-\exp(\Delta\bar{no}-\bar{r}+\bar{\tau})}E_t\sum_{k=1}^{\infty}\exp(k(\Delta\bar{no}-\bar{r}+\bar{\tau}))(\Delta\tilde{no}_{t+k}-\tilde{r}_{t+k}+\tilde{\tau}_{t+k}))
\end{aligned}$$

Define some new variables:

$$\begin{aligned}
\alpha &= \exp(\bar{\Delta c}-\bar{r}+\bar{\tau})\\
\kappa &= \exp(\Delta\bar{no}-\bar{r}+\bar{\tau})
\end{aligned}$$

Rewrite the preceding result using them:

$$\begin{aligned}
(\tilde{r}_t-\tilde{\tau}_t-\Delta\tilde{no}_t)\bar{B}^*\frac{1}{\kappa}+\tilde{B}^*\frac{1}{\kappa} &\approx\\
\tilde{C}^*\frac{1}{1-\alpha} &+\\
\bar{C}^*\frac{1}{1-\alpha}E_t\sum_{k=1}^{\infty}\exp(k(\bar{\Delta c}-\bar{r}+\bar{\tau}))(\Delta\tilde{c}_{t+k}-\tilde{r}_{t+k}+\tilde{\tau}_{t+k})) &-\\
\frac{1}{1-\kappa}E_t\sum_{k=1}^{\infty}\exp(k(\Delta\bar{no}-\bar{r}+\bar{\tau}))(\Delta\tilde{no}_{t+k}-\tilde{r}_{t+k}+\tilde{\tau}_{t+k}))
\end{aligned}$$

Solve for $\tilde{C}^*$:

$$
\begin{aligned}
\tilde{C}^* \approx \frac{1-\alpha}{\kappa}\bar{B}^*(\tilde{r}_t - \tilde{\tau}_t - \Delta\tilde{no}_t) + \frac{1-\alpha}{\kappa}\tilde{B}^* \quad - \\
\bar{C}^* E_t \sum_{k=1}^{\infty} \exp(k(\bar{\Delta c} - \bar{r} + \bar{\tau}))(\Delta\tilde{c}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k})) \quad + \\
\frac{1-\alpha}{1-\kappa} E_t \sum_{k=1}^{\infty} \exp(k(\bar{\Delta no} - \bar{r} + \bar{\tau}))(\Delta\tilde{no}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}))
\end{aligned}
$$

Use definitions of α and κ again:

$$
\begin{aligned}
\tilde{C}^* \approx \frac{1-\alpha}{\kappa}\bar{B}^*(\tilde{r}_t - \tilde{\tau}_t - \Delta\tilde{no}_t) + \frac{1-\alpha}{\kappa}\tilde{B}^* \quad - \\
\bar{C}^* E_t \sum_{k=1}^{\infty} \alpha^k (\Delta\tilde{c}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k})) \quad + \\
\frac{1-\alpha}{1-\kappa} E_t \sum_{k=1}^{\infty} \kappa^k (\Delta\tilde{no}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}))
\end{aligned} \tag{B.11}
$$

From the BC, we know that $B_{t+1} = (1+r_t^w)(1-\tau_t^s)B_t + Y_t - I_t - G_t - C_t$ and $(1+r_t^w)(1-\tau_t^s) = 1 - \tau_t^s + r_t^w - r_t^w\tau_t^s$. Ignore the very small last term ($r_t^w\tau_t^s \approx 0$). We have:

$$
\begin{aligned}
\Delta B_{t+1} = CA_t &= NO_t - C_t + [r_t^w - \tau_t^s] B_t \\
\frac{CA_t}{NO_t} &= 1 - \frac{C_t}{NO_t} + \frac{[r_t^w - \tau_t^s] B_t}{NO_t}
\end{aligned} \tag{B.12}
$$

Expand last element with NO_{t-1}:

$$
\frac{B_t}{NO_{t-1}} \frac{NO_{t-1}}{NO_t} [r_t^w - \tau_t^s] \tag{B.13}
$$

Use $\exp(\log(\ldots))$ on the second term:

$$
\begin{aligned}
\exp(\log(\frac{NO_{t-1}}{NO_t} [r_t^w - \tau_t^s])) &= \exp(-\Delta no_t + \log([r_t^w - \tau_t^s]) \\
&= \frac{r_t^w - \tau_t^s}{\exp(\Delta no_t)}
\end{aligned} \tag{B.14}
$$

Note that $\log(1+r_t) \approx \log(\exp(r_t)) = r_t$, $(1+r_t) \approx \exp(r_t)$, $r_t = \exp(r_t) - 1$. Now use it with the wedge:

$$
\begin{aligned}
\log\left((1+r_t^w)(1-\tau_t^s)\right) &\approx r_t^w - \tau_t^s = \log\left(\exp(r_t^w - \tau_t^s)\right) \\
(1+r_t^w)(1-\tau_t^s) &\approx \exp(r_t^w - \tau_t^s) \\
1 - \tau_t^s + r_t^w - \underbrace{r_t^w\tau_t^s}_{0} &\approx \exp(r_t^w - \tau_t^s) \\
\exp(r_t^w - \tau_t^s) - 1 &\approx r_t^w - \tau_t^s
\end{aligned} \tag{B.15}
$$

Use (B.15) in (B.14), plug it back in (B.13) and then (B.12):

$$
\begin{aligned}
\frac{CA_t}{NO_t} &= 1 - \frac{C_t}{NO_t} + \frac{\exp(r_t^w - \tau_t^s) - 1}{\exp(\Delta no_t)} \frac{B_t}{NO_{t-1}} \\
CA^* &= 1 - C^* + \frac{\exp(r_t^w - \tau_t^s) - 1}{\exp(\Delta no_t)} B^*
\end{aligned}
$$

Before linearizing this expression, note that:

$$
\begin{aligned}
E_t\left(\frac{\exp(r_t^w - \tau_t^s) - 1}{\exp(\Delta no_t)}\right) &= \frac{\exp(\bar{r} - \bar{\tau}) - 1}{\exp(\Delta \bar{no})} \\
&= \exp(\bar{r} - \bar{\tau})\exp(\Delta \bar{no})^{-1} - \exp(\Delta \bar{no})^{-1} \\
&= \frac{\exp(\bar{r} - \bar{\tau})}{\exp(\Delta \bar{no})} - \frac{1}{\exp(\Delta \bar{no})} \\
&= \frac{1}{\exp(\Delta \bar{no} - \bar{r} + \bar{\tau}))} - \frac{1}{\exp(\Delta \bar{no})}
\end{aligned}
$$

Use it in the expression for the relative current account:

$$
CA^* = 1 - C^* + \left(\frac{1}{\exp(\Delta no_t - r_t^w + \tau_t^s))} - \frac{1}{\exp(\Delta no_t)}\right) B^*
$$

Let's linearize using $\exp(\bar{r}) = (\exp(-\bar{r}))^{-1}$:

$$
\begin{aligned}
\tilde{CA}^* \approx -\tilde{C}^* + \tilde{B}^* &\left(\frac{1}{\exp(\Delta \bar{no} - \bar{r} + \bar{\tau})} - \frac{1}{\exp(\Delta \bar{no})}\right) + \\
\tilde{r}_t &\frac{1}{\exp(\Delta \bar{no} - \bar{r} + \bar{\tau})} \bar{B}^* - \tilde{\tau}_t \frac{1}{\exp(\Delta \bar{no} - \bar{r} + \bar{\tau})} \bar{B}^* \\
-\Delta \tilde{no}_t &\left(\frac{1}{\exp(\Delta \bar{no} - \bar{r} + \bar{\tau})} - \frac{1}{\exp(\Delta \bar{no})}\right) \bar{B}^*
\end{aligned} \tag{B.16}
$$

Now come back to $\tilde{C}^*$ (B.11). Assume that $\alpha = \kappa \rightarrow \exp(\Delta \bar{c} - \bar{r} + \bar{\tau}) = \exp(\Delta \bar{no} - \bar{r} + \bar{\tau}) \rightarrow \Delta \bar{c} = \Delta \bar{no}$. Observe that $\alpha^k = \exp(k(\Delta \bar{c} - \bar{r} + \bar{\tau})) = \exp(k(\Delta \bar{no} - \bar{r} + \bar{\tau})) = \kappa^k$. So just express it all in using κ:

$$
\begin{aligned}
\tilde{C}^* \approx \frac{1-\kappa}{\kappa} &\bar{B}^*(\tilde{r}_t - \tilde{\tau}_t - \Delta \tilde{no}_t) + \frac{1-\kappa}{\kappa} \tilde{B}^* \quad - \\
&\bar{C}^* E_t \sum_{k=1}^{\infty} \kappa^k (\Delta \tilde{c}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) \quad + \\
&\frac{1-\kappa}{1-\kappa} E_t \sum_{k=1}^{\infty} \kappa^k (\Delta \tilde{no}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k})
\end{aligned} \tag{B.17}
$$

The expression for $\tilde{CA}^*$(B.16) is not influenced by this assumption.
Plug $\tilde{C}^*$ (B.17) in $\tilde{CA}^*$ (B.16):

$$
\begin{aligned}
\tilde{CA}^* \approx \frac{\kappa-1}{\kappa}\bar{B}^*(\tilde{r}_t - \tilde{\tau}_t - \Delta\tilde{no}_t) + \frac{\kappa-1}{\kappa}\tilde{B}^* &\quad + \\
\bar{C}^* E_t \sum_{k=1}^{\infty} \kappa^k (\Delta\tilde{c}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) - E_t \sum_{k=1}^{\infty} \kappa^k (\Delta\tilde{no}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) &\quad + \\
\tilde{B}^* \left(\frac{1}{\kappa} - \frac{1}{\exp(\Delta\bar{no})}\right) + \tilde{r}_t \frac{1}{\kappa}\bar{B}^* - \tilde{\tau}_t \frac{1}{\kappa}\bar{B}^* &\quad - \\
\Delta\tilde{no}_t \left(\frac{1}{\kappa} - \frac{1}{\exp(\Delta\bar{no})}\right)\bar{B}^*
\end{aligned}
$$

Focus on the standalone $\tilde{r}$ terms:

$$
\frac{\kappa-1}{\kappa}\bar{B}^*\tilde{r}_t + \frac{1}{\kappa}\bar{B}^*\tilde{r}_t \quad = \quad \bar{B}^*\tilde{r}_t
$$

Focus on standalone wedge terms:

$$
-\frac{\kappa-1}{\kappa}\tilde{\tau}_t\bar{B}^* - \frac{1}{\kappa}\tilde{\tau}_t \quad \bar{B}^* = - \quad \bar{B}^*\tilde{\tau}_t
$$

Focus on the $\tilde{B}^*$ terms:

$$
\begin{aligned}
&\frac{\kappa-1}{\kappa}\tilde{B}^* + \tilde{B}^*\left(\frac{1}{\kappa} - \frac{1}{\exp(\Delta\bar{no})}\right) \\
&\qquad \tilde{B}^*(1 - \frac{1}{\exp(\Delta\bar{no})})
\end{aligned}
$$

Focus on the $\Delta\tilde{no}$ terms:

$$
\begin{aligned}
-\frac{\kappa-1}{\kappa}\bar{B}^*\Delta\tilde{no}_t - \left(\frac{1}{\kappa} - \frac{1}{\exp(\Delta\bar{no})}\right)\bar{B}^*\Delta\tilde{no}_t &\quad = \\
\left(\frac{1}{\exp(\Delta\bar{no})} - 1\right)\bar{B}^*\Delta\tilde{no}_t &
\end{aligned}
$$

The general expression becomes:

$$
\begin{aligned}
\tilde{CA}^* \approx & \\
& \bar{C}^* E_t \sum_{k=1}^{\infty} \kappa^k (\Delta\tilde{c}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) \\
& -E_t \sum_{k=1}^{\infty} \kappa^k (\Delta\tilde{no}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) \\
& +\bar{B}^*\tilde{r}_t - \bar{B}^*\tilde{\tau}_t \\
& +\tilde{B}^*(1 - \frac{1}{\exp(\Delta\bar{no})}) \\
& +\left(\frac{1}{\exp(\Delta\bar{no})} - 1\right)\bar{B}^*\Delta\tilde{no}_t
\end{aligned}
$$

To get the expression of Kano (4.1), we need that both last terms disappear:

$$\tilde{B}^* - \bar{B}^* \Delta \tilde{no}_t + \bar{B}^* \Delta \tilde{no}_t \frac{1}{\exp(\Delta \bar{no})} - \tilde{B}^* \frac{1}{\exp(\Delta \bar{no})} = 0$$

Let's check it. Remember that:

$$\begin{aligned}
B^* &= \frac{B_t}{NO_{t-1}} = \frac{B_t}{NO_t} \frac{NO_t}{NO_{t-1}} \\
& \exp(\log(\frac{B_t}{NO_t}) + \Delta no_t) \\
& \frac{B_t}{NO_t} \exp(\Delta no_t)
\end{aligned}$$

So that:

$$\bar{B}^* = \frac{\bar{B}}{\bar{NO}} \exp(\Delta \bar{no})$$

The linearization is:

$$\frac{\bar{B}}{\bar{NO}} \exp(\Delta \bar{no}) + \tilde{B}^* \exp(\Delta \bar{no}) + \Delta \tilde{no}_t \frac{\bar{B}}{\bar{NO}} \exp(\Delta \bar{no})$$

Find an expression for $\tilde{B}^* = \Delta \tilde{no}_t \frac{\bar{B}}{\bar{NO}} - \frac{\bar{B}}{\bar{NO}}$ and plug it in the expression to check it is true indeed (neglect constant terms):

$$\Delta \tilde{no}_t \frac{\bar{B}}{\bar{NO}} - \frac{\bar{B}}{\bar{NO}} - \bar{B}^* \Delta \tilde{no}_t + \bar{B}^* \Delta \tilde{no}_t \frac{1}{\exp(\Delta \bar{no})} - \Delta \tilde{no}_t \frac{\bar{B}}{\bar{NO}} \frac{1}{\exp(\Delta \bar{no})} + \frac{\bar{B}}{\bar{NO}} \frac{1}{\exp(\Delta \bar{no})} = 0$$

We get Kano's log-linearization (4.1) with a saving wedge:

$$\begin{aligned}
\tilde{CA}^* &\approx \bar{B}^* \tilde{r}_t - \bar{B}^* \tilde{\tau}_t + \bar{C}^* \sum_{k=1}^{\infty} \kappa^k E_t (\Delta \tilde{c}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) - \sum_{k=1}^{\infty} \kappa^k E_t (\Delta \tilde{no}_{t+k} - \tilde{r}_{t+k} + \tilde{\tau}_{t+k}) \\
&\approx \bar{B}^* \tilde{r}_t + \bar{C}^* \sum_{k=1}^{\infty} \kappa^k E_t (\Delta \tilde{c}_{t+k}) + (1 - \bar{C}^*) \sum_{k=1}^{\infty} \kappa^k E_t (\tilde{r}_{t+k}) - \sum_{k=1}^{\infty} \kappa^k E_t (\Delta \tilde{no}_{t+k}) \\
&\quad - \bar{B}^* \tilde{\tau}_t + (\bar{C}^* - 1) \sum_{k=1}^{\infty} \kappa^k E_t (\tilde{\tau}_{t+k})
\end{aligned}$$

B.2.3 Hoffmann's extension (2013)

B.2.3.1 Baseline model with saving wedge

From Kano's linearization with saving wedge in Section B.2.2, we have:

$$\tilde{CA}^* \approx \bar{B}^* \tilde{r}_t + \bar{C}^* \sum_{k=1}^{\infty} \kappa^k E_t (\Delta \tilde{c}_{t+k} - \tilde{r}_{t+k}) + \sum_{k=1}^{\infty} \kappa^k E_t (\tilde{r}_{t+k} - \Delta \tilde{no}_{t+k}) - \bar{B}^* \tilde{\tau}_t + (\bar{C}^* - 1) \sum_{k=1}^{\infty} \kappa^k E_t (\tilde{\tau}_{t+k})$$

From Bergin and Sheffrin's optimization problem with saving wedge in Section B.2.1.2, we have:

$$E_t (\Delta c_{t+1}) = \eta E_t \left(r_{t+1} - \tau_t^s + \frac{1 - \eta}{\eta} (1 - a) \Delta q_{t+1} \right) + const$$

Note that in both expressions, r stands for world interest rate. Beware: the q corresponds to Bergin and Sheffring's p, the relative price of non-tradables.
Mind that the terms could be interpreted as deviation from steady-state (ignore the constant).

$$E_t(\Delta\tilde{c}_{t+k}) = \frac{1}{\gamma}E_t\tilde{r}_{t+k} - \frac{1}{\gamma}E_t\tilde{\tau}_{t+k} + \frac{\gamma-1}{\gamma}(1-a)E_t\Delta\tilde{q}_{t+k}$$

Plug the former expression in Kano's one:

$$\begin{aligned}\tilde{CA}^* \quad &\approx \quad \bar{B}^*\tilde{r}_t + \bar{C}^*\sum_{k=1}^{\infty}\kappa^k\left(\frac{1}{\gamma}E_t\tilde{r}_{t+k} - \frac{1}{\gamma}E_t\tilde{\tau}_{t+k} + \frac{\gamma-1}{\gamma}(1-a)E_t\Delta\tilde{q}_{t+k} - E_t\tilde{r}_{t+k}\right)\\ &+ \sum_{k=1}^{\infty}\kappa^k E_t(\tilde{r}_{t+k} - \Delta\tilde{no}_{t+k}) - \bar{B}^*\tilde{\tau}_t + (\bar{C}^*-1)\sum_{k=1}^{\infty}\kappa^k E_t(\tilde{\tau}_{t+k})\end{aligned}$$

Focus on the $E_t\tilde{r}_{t+k}$ terms:

$$\left(\bar{C}^*\frac{1}{\gamma} - \bar{C}^* + 1\right)\sum_{k=1}^{\infty}\kappa^k E_t\tilde{r}_{t+k}$$
$$\left(1 - \bar{C}^*(1-\frac{1}{\gamma})\right)\sum_{k=1}^{\infty}\kappa^k E_t\tilde{r}_{t+k}$$

Focus on the $E_t\tilde{q}_{t+k}$ term, define $(1-\alpha)E_t\Delta\tilde{q}_{t+1} = E_t\Delta\tilde{d}_{t+1}$:

$$\bar{C}^*\frac{\gamma-1}{\gamma}E_t\sum_{k=1}^{\infty}\kappa^k(1-a)E_t\Delta\tilde{q}_{t+k}$$
$$\bar{C}^*(1-\frac{1}{\gamma})E_t\sum_{k=1}^{\infty}\kappa^k E_t\Delta\tilde{d}_{t+k}$$

Focus on the (new) saving wedge terms:

$$-\bar{C}^*\sum_{k=1}^{\infty}\kappa^k\frac{1}{\gamma}E_t\tilde{\tau}_{t+k} - \bar{B}^*\tilde{\tau}_t + (\bar{C}^*-1)E_t\sum_{k=1}^{\infty}\kappa^k(\tilde{\tau}_{t+k})$$
$$-\bar{B}^*\tilde{\tau}_t - \left(1 - \bar{C}^*(1-\frac{1}{\gamma})\right)E_t\sum_{k=1}^{\infty}\kappa^k(\tilde{\tau}_{t+k})$$

The equation building the socle our empirical estimation is thus:

$$
\begin{aligned}
\tilde{CA}^* \approx\ & \bar{B}^*\tilde{r}_t \\
& + \left(1-\bar{C}^*(1-\frac{1}{\gamma})\right)\sum_{k=1}^{\infty}\kappa^k E_t\tilde{r}_{t+k} \\
& + \bar{C}^*(1-\frac{1}{\gamma})\sum_{k=1}^{\infty}\kappa^k E_t\Delta\tilde{d}_{t+k} \\
& - \sum_{k=1}^{\infty}\kappa^k E_t\Delta\tilde{no}_{t+k} \\
& - \bar{B}^*\tilde{\tau}_t - \left(1-\bar{C}^*(1-\frac{1}{\gamma})\right)E_t\sum_{k=1}^{\infty}\kappa^k(\tilde{\tau}_{t+k})
\end{aligned}
$$

With some differences in notation, it corresponds to equation (4.4) in the main text. Note that the saving wedge enters with an opposite sign as we define it as excess return in the main text. Furthermore, it is multiplied by δ^k. The capital flows on the steady-state level of assets/debt disappear as we focus on net exports. At last, given our definition of financial friction in the main text, the interest rate is the national one.

We have five channels of current account adjustment. The first term measures the role of net income flows. The second one is consumption-tilting due to expected variation in the world real rate of interest (if high interest rate, want to save more). The third channel is the effect of expected real exchange rate change (incentive to save more if price of domestic consumption bundle relative to tradable goods expected to rise). The fourth term is the classical consumption smoothing channel (if output is expected to be above trend, the country should run a deficit). The last terms gather the impact of expected variation in financial frictions. An expected increase in frictions (higher tax on savings) lowers the current account (save less).

To proxy expectations, a VAR in Δno_t, Δq_t, r_t^w, CA_t/NO_t and τ_t^s is estimated and can be written in companion form as $Z_t = AZ_{t-1} + U_t$. For a variable x we have:

$$
\begin{aligned}
\sum_{k=1}^{\infty}\kappa^k E_t(x_{t+k}) &= \kappa E_t(x_{t+1}) + \kappa^2 E_t(x_{t+2}) + \dots \\
&= \kappa A Z_t + \kappa^2 A^2 Z_t + \dots \\
&= (\frac{1}{1-\kappa A} - 1)Z_t \\
&= \frac{\kappa A}{1-\kappa A}Z_t
\end{aligned}
$$

In matrix notation, use a selection vector e_x to get:

$$
\sum_{k=1}^{\infty}\kappa^k E_t(x_{t+k}) = e_x'\kappa A(I-\kappa A)^{-1}Z_t
$$

Plug it in the expectation expressions:

$$
\begin{aligned}
\tilde{CA}^* \quad \approx \quad & \bar{B}^* \tilde{r}_t \\
& + \left(1 - \bar{C}^*(1 - \frac{1}{\gamma})\right) e'_r \kappa A (I - \kappa A)^{-1} Z_t \\
& + \bar{C}^*(1 - \frac{1}{\gamma}) e'_d \kappa A (I - \kappa A)^{-1} Z_t \\
& - e'_{no} \kappa A (I - \kappa A)^{-1} Z_t \\
& - \bar{B}^* \tilde{\tau}_t - \left(1 - \bar{C}^*(1 - \frac{1}{\gamma})\right) e'_s \kappa A (I - \kappa A)^{-1} Z_t
\end{aligned}
$$

Which, at last, gives:

$$
\tilde{CA}^* = \bar{B}^* \tilde{r}_t - \bar{B}^* \tilde{\tau}_t + \left[\left(1 - \bar{C}^*(1 - \frac{1}{\gamma})\right) (e'_r - e'_s) + \bar{C}^*(1 - \frac{1}{\gamma}) e'_d - e'_{no} \right] \kappa A (I - \kappa A)^{-1} Z_t
$$

It corresponds to equation (4.6) in the main text.

B.2.3.2 Model with CES case (I) and saving wedge

By using our result from Section B.2.1.4, the equation building the socle of the empirical estimation is:

$$
\begin{aligned}
\tilde{CA}^* \quad \approx \quad & \bar{B}^* \tilde{r}_t \\
& + \left(1 - \bar{C}^*(1 - \frac{1}{\gamma})\right) \sum_{k=1}^{\infty} \kappa^k E_t \tilde{r}_{t+k} \\
& + \bar{C}^*(1 - \frac{1}{\gamma}) \sum_{k=1}^{\infty} \kappa^k \frac{\bar{Q}^{1-\theta}}{(1-\alpha) + \alpha \bar{Q}^{1-\theta}} E_t \Delta \tilde{d}_{t+k} \\
& - \sum_{k=1}^{\infty} \kappa^k E_t \Delta \tilde{no}_{t+k} \\
& - \bar{B}^* \tilde{\tau}_t - \left(1 - \bar{C}^*(1 - \frac{1}{\gamma})\right) E_t \sum_{k=1}^{\infty} \kappa^k (\tilde{\tau}_{t+k})
\end{aligned}
$$

Empirical implementation:

$$
\tilde{CA}^* = \bar{B}^* \tilde{r}_t - \bar{B}^* \tilde{\tau}_t + \left[\left(1 - \bar{C}^*(1 - \frac{1}{\gamma})\right) (e'_r - e'_s) + \bar{C}^*(1 - \frac{1}{\gamma}) \frac{\bar{Q}^{1-\theta}}{(1-\alpha) + \alpha \bar{Q}^{1-\theta}} e'_d - e'_{no} \right] \kappa A (I - \kappa A)^{-1} Z_t
$$